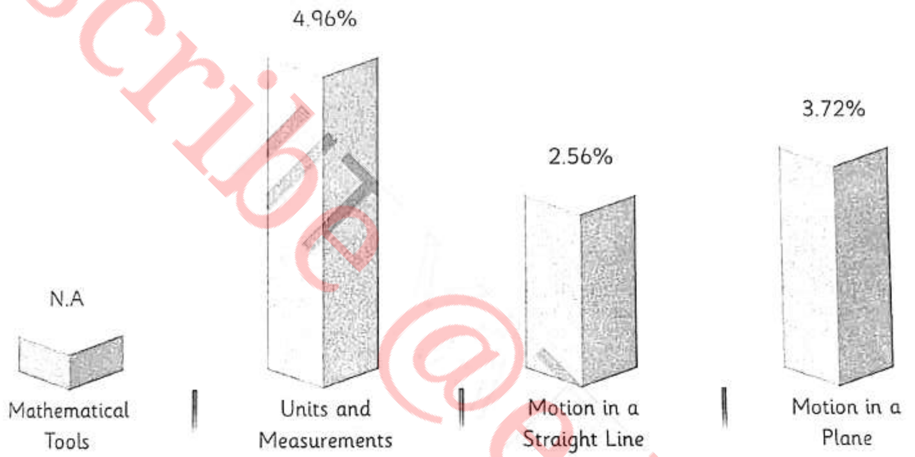
PHYSICS WALLAH

arjuna JEE



Chapter Wise Weightage (in %) Based on Past 6 Years JEE Main Papers

PHYSICS Module 1

Practice Gratitude In this module for the emotional well being we will practice gratitude along with our regular academic learning

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PREFACE

A highly skilled professional team of Physics Wallah (PW) works arduously to ensure that the students receive the best content for the JEE exam.

From the beginning, the whole content team comprising Associate faculties, DTP operators, Proofreaders and others are involved in shaping the material to their best knowledge and experience to produce powerful content for the students.

Faculties have adopted a new style of presenting the content in easy-to-understand language and have provided the team with their guidance and supervision throughout the creation of this Study Material.

Physics Wallah (PW) strongly believes in conceptual and fun-based learning. PW provides highly exam-oriented content to bring quality and clarity to the students.

A plethora of **JEE Study Material** is available in the market but PW professionals are continuously working to provide the supreme Study Material for our **JEE** students.

This Study Material adopts a multi-faceted approach to master and understanding the concepts by having a rich diversity of questions asked in the examination and equip the students with the knowledge for the competitive exam.

The main objective of the study material is to provide a large number of quality problems with varying cognitive levels to facilitate the teaching-learning of concepts that are presented through the book.

It has become popular among aspirants because of its easy-to-understand language and clear illustrations with diagrams, mnemonics, flow charts and tables.

Students can benefit themselves by attempting the exercise given in this Study Material for the self-assessment and also mastering the basic techniques of problem-solving.

The questions are strictly designed in accordance with the exam relevant topics that helps to develop examination temperament within the students.

Mastering the Physics Wallah (PW) study material curated by the PW team, the students can easily qualify for the exam with a Top Rank in the **JEE**.

In each chapter, for better understanding, questions have been classified according to the latest syllabus of **JEE**

- The nature and diversity of the questions help students to ace the examination
- Quality questions to strengthen the concept of the topic at the zenith level

BOOK FEATURES

- Detailed theory with Train Your Brain and Concept Application Questions
- "Aarambh" Section for Solved Miscellaneous Examples
- School Level Problems for School Going Students
- "Prarambh" Section for Topicwise, "Prabal" Section for JEE Main Level and "Parikshit" Section for Advance Level MCQs covered in exercises
- Match the Column, Comprehension Based, Numerical Type and Multicorrect questions covered for complete practice
- JEE Past Year Questions
- PW Challengers Problems for Difficult Level Questions
- Elaborated Solutions Booklet you can get separately

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PLANNER

CHAPTER NAMES	Start Date of Module	End Date of Module	Solve More PYQs	Revision Period	Re-revision Period (Before Test Series)
Mathematical Tools					
Units and Measurements					
Motion in a Straight Line					
Motion in a Plane					

PROGRESS TRACKING

CHAPTER NAMES	Solved Examples	School Level	Topicwise	Main Level	Advanced Level	PYQs	Challengers
Mathematical Tools	☐	☐	☐	☐	☐	☐	☐
Units and Measurements	☐	☐	☐	☐	☐	☐	☐
Motion in a Straight Line	☐	☐	☐	☐	☐	☐	☐
Motion in a Plane	☐	☐	☐	☐	☐	☐	☐

PREP POINTERS

Note down the topic you want to revise or section you want to re attempt while your preparation.

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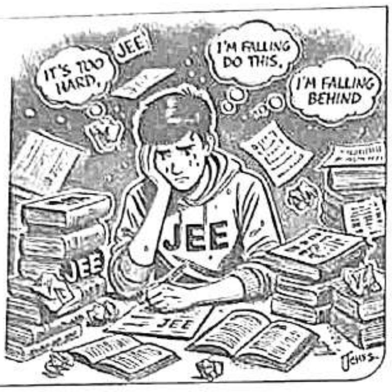
The Power of Gratitude: Transforming Challenges into Opportunities

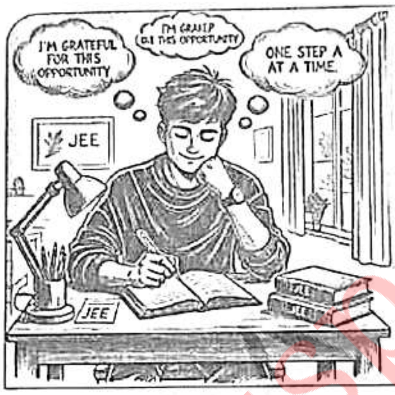
Gratitude is about recognizing and appreciating the good, even amid challenges. For a JEE aspirant, it means being thankful for opportunities to learn, support from teachers and family, and every bit of progress made. It's more than just saying "thank you" – it's developing a mindset that helps you focus on the positives, manage stress, boost motivation, and stay resilient throughout your preparation.

> *"When we focus on our gratitude, the tide of disappointment goes out and the tide of love rushes in."*
> **— Kristin Armstrong**

Without Gratitude

"Lacking Gratitude. Even the smallest challenges can feel overwhelming, making the journey of knowledge feel unfulfilling."




I'M GRATEFUL FOR THIS OPPORTUNITY
I'M GRATEFUL FOR THIS OPPORTUNITY
ONE STEP A AT A TIME.
JEE
JEE

With Gratitude

"Gratitude is your secret weapon to power through tough moments, turning every lesson into an opportunity to grow."

Scenario	*Without Gratitude*	*With Gratitude*
After a Difficult Mock Test	Feels disheartened, thinking, "I'll never be able to score well; this is too tough for me."	Reflects, "I'm thankful I found areas to improve. I can focus on these weaknesses to get better."
Feedback from a Teacher	Focuses on criticism, thinking, "They don't believe I can do it; I must be bad at this subject."	Sees the guidance, thinking, "I'm grateful for the constructive feedback; it will help me improve."
Studying Long Hours	Thinks, "I have to study so much; I miss out on all the fun things others are doing."	Thinks, "I'm grateful for the opportunity to learn and work towards my dream of becoming an engineer."
Facing a Tough Concept	Feels overwhelmed, saying, "I just don't get this; I'll never understand it."	Thinks, "I'm thankful for the challenge. Breaking this down step by step will help me master it."
Comparing Scores with Friends	Feels jealous and demotivated, thinking, "They are all ahead of me; I'll never catch up."	Reflects, "I'm grateful for friends who inspire me to push harder and learn from their success."
Missing a Social Event for Study	Thinks, "I always have to miss out because of this exam; it's unfair."	Reflects, "I'm thankful that I'm investing time now for a brighter future. There will be more chances for fun later."
Unexpected Challenges During Preparation	Feels defeated, saying, "Nothing goes my way; I might as well give up."	Thinks, "I'm grateful for the chance to adapt and learn from this. It's part of the journey."
Daily Study Routine	Thinks, "This is boring and repetitive; I'm tired of studying all the time."	Thinks, "I'm grateful I have a routine that keeps me on track. Each day brings me closer to my goal."

This table demonstrates how adopting a mindset of gratitude can help students navigate the stresses and challenges of exam preparation with more resilience and positivity. It highlights how a shift in perspective can transform obstacles into opportunities for growth and improvement.

CHAPTER 1

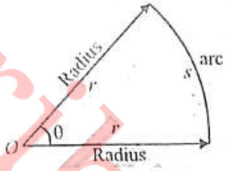
Mathematical Tools

RIGONOMETRY

Angle

It is a measure of change in direction.

$$\text{Angle }(\theta) = \frac{\text{Arc}(s)}{\text{Radius}(r)}$$



Angles measured in anticlockwise and clockwise direction are usually taken as positive and negative respectively.

System of measurement of an angle

(A) Sexagesimal system:

In this system, angle is measured in degrees.

In this system, 1 right angle = 90°, 1° = 60′ (arc minutes), 1′ = 60″ (arc seconds)

(B) Circular system:

In this system, angle is measured in radians. If arc = radius then θ = 1 rad

Relation between degrees and radians.

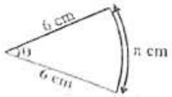
$$2\pi \text{ rad} = 360° \Rightarrow \pi \text{ rad} = 180° \Rightarrow 1 \text{ rad} = \frac{180°}{\pi} = 57.3°$$

To convert from degree to radian multiply by $\frac{\pi}{180°}$

To convert from radian to degree multiply by $\frac{180°}{\pi}$

Train Your Brain

Example 1: A circular arc of length π cm. Find angle subtended by it at the centre in radians and degrees.



Sol. $\theta = \frac{\pi \text{ cm}}{6 \text{ cm}} = \frac{\pi}{6}$ rad = 30°

Example 2: The moon's distance from the earth is 360000 km and its diameter subtends an angle of 42′ at the eye of the observer. The diameter of the moon in kilometers is

(*a*) 4400 (*b*) 1000

(*c*) 3600 (*d*) 8800

Sol. Here angle is very small so diameter ≈ arc

$$\theta = 42' = \left(42 \times \frac{1}{60}\right) = 42 \times \frac{1}{60} \times \frac{\pi}{180} = \frac{7\pi}{1800} \text{ rad}$$

$$\text{Diameter} = R\theta = 360000 \times \frac{7}{1800} \times \frac{22}{7} = 4400 \text{ km}$$

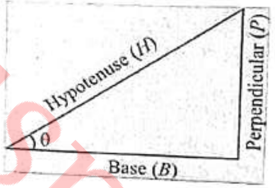
Concept Application

1. Convert 15° into radians

2. Convert $\frac{5\pi}{6}$ radians into degrees

Trigonometric Ratios (T-ratios)

Following ratios of the sides of a right angled triangle are known as trigonometrical ratios.



$\sin\theta = \frac{P}{H}$

$\cos\theta = \frac{B}{H}$

$\tan\theta = \frac{P}{B}$ $\qquad$ $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{H}{P}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{H}{B}$ $\qquad$ $\cot\theta = \frac{1}{\tan\theta} = \frac{B}{P}$

Trigonometric Identities

In figure, $P^2 + B^2 = H^2$ $\qquad$ Divide by H^2, $\left(\frac{P}{H}\right)^2 + \left(\frac{B}{H}\right)^2 = 1$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$

Divide by B^2: $\left(\frac{P}{B}\right)^2 + 1 = \left(\frac{H}{B}\right)^2 \Rightarrow 1 + \tan^2\theta = \sec^2\theta$

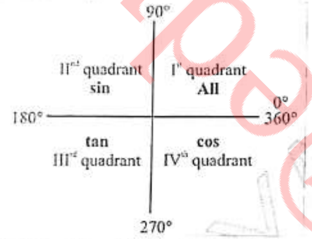
Divide by P^2: $1 + \left(\frac{B}{P}\right)^2 = \left(\frac{H}{P}\right)^2 \Rightarrow 1 + \cot^2\theta = \text{cosec}^2\theta$

Commonly Used Values of Trigonometric Functions

Angle (θ)	0°	30°	37°	45°	53°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{4}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{3}{5}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	$\frac{3}{4}$	1	$\frac{4}{3}$	$\sqrt{3}$	∞

Four Quadrants and ASTC Rule

In first quadrant, all trigonometric ratios are positive. In second quadrant, only sinθ and cosecθ are positive. In third quadrant, only tanθ and cotθ are positive.



In fourth quadrant, only cosθ and secθ are positive

Remember as **'Add Sugar To Coffee' or 'After School To College'.**

Trigonometrical Ratios of General Angles (Reduction Formulae)

(*i*) The value of a trigonometric function does not change if you add $2n\pi$ to its argument where n is an integer.

$\sin(2n\pi + \theta) = \sin\theta$; $\cos(2n\pi + \theta) = \cos\theta$

$\tan(2n\pi + \theta) = \tan\theta$

(*ii*) Trigonometric function of an angle $\left(\frac{n\pi}{2} + \theta\right)$ will remain same if n is even and sign of trigonometric function will be according to value of that function in quadrant.

$\sin(\pi - \theta) = +\sin\theta$ $\quad$ $\cos(\pi - \theta) = -\cos\theta$

$\tan(\pi - \theta) = -\tan\theta$

$\sin(\pi + \theta) = -\sin\theta$ $\quad$ $\cos(\pi + \theta) = -\cos\theta$

$\tan(\pi + \theta) = +\tan\theta$

$\sin(2\pi - \theta) = -\sin\theta$ $\quad$ $\cos(2\pi - \theta) = +\cos\theta$

$\tan(2\pi - \theta) = -\tan\theta$

(*iii*) Trigonometric function of an angle $\left(\frac{n\pi}{2} + \theta\right)$ will be changed into co-function if n is odd and sign of trigonometric function will be according to value of that function in quadrant.

$\sin\left(\frac{\pi}{2} + \theta\right) = +\cos\theta$ $\quad$ $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$

$\tan\left(\frac{\pi}{2} + \theta\right) = -\cot\theta$

$\sin\left(\frac{\pi}{2} - \theta\right) = +\cos\theta$ $\quad$ $\cos\left(\frac{\pi}{2} - \theta\right) = +\sin\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = +\cot\theta$

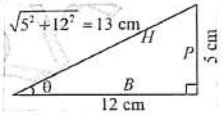
(*iv*) Trigonometric function of an angle $-\theta$ (negative angles)

$\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = +\cos\theta$, $\tan(-\theta) = -\tan\theta$

Train Your Brain

Example 3: The two shorter sides of right angled triangle are 5 cm and 12 cm. Let θ denote the angle opposite to the 5 cm side. Find sinθ, cosθ and tanθ.

Sol.

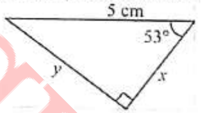


$$\sin\theta = \frac{P}{H} = \frac{5\text{cm}}{13\text{cm}} = \frac{5}{13}$$

$$\cos\theta = \frac{B}{H} = \frac{12\text{cm}}{13\text{cm}} = \frac{12}{13}$$

$$\tan\theta = \frac{P}{B} = \frac{5\text{cm}}{12\text{cm}} = \frac{5}{12}$$

Example 4: Find x, y and perimeter of the triangle shown



Sol.

$$\frac{y}{5} = \sin 53° = \frac{4}{5} \Rightarrow y = 4 \text{ cm}$$

and $\frac{x}{5} = \cos 53° = \frac{3}{5} \Rightarrow x = 3$ cm

Perimeter of the triangle $= x + y + 5 = 3 + 4 + 5 = 12$ cm

Example 5: Find the value of :

(*i*) sin30° + cos60°
(*ii*) sin 0° − cos 0°
(*iii*) tan 45°−tan 37°
(*iv*) sin 390°
(*v*) cos 405°
(*vi*) tan 420°
(*vii*) sin 150°
(*viii*) cos 120°
(*ix*) tan 135°
(*x*) sin 330°
(*xi*) cos 300°
(*xii*) sin(−30°)
(*xiii*) cos(−60°)
(*xiv*) tan(−45°)
(*xv*) sin(−150°)

Sol.

(*i*) $\sin 30° + \cos 60° = \frac{1}{2} + \frac{1}{2} = 1$

(*ii*) $\sin 0° - \cos 0° = 0 - 1 = -1$

(*iii*) $\tan 45° - \tan 37° = 1 - \frac{3}{4} = \frac{1}{4}$

(*iv*) $\sin 390° = \sin(360° + 30°) = \sin 30° = \frac{1}{2}$

(*v*) $\cos 405° = \cos(360° + 45°) = \cos 45° = \frac{1}{\sqrt{2}}$

(*vi*) $\tan 420° = \tan(360° + 60°) = \tan 60° = \sqrt{3}$

(*vii*) $\sin 150° = \sin(90° + 60°) = \cos 60° = \frac{1}{2}$ or

$\sin 150° = \sin(180° - 30°) = \sin 30° = \frac{1}{2}$

(*viii*) $\cos 120° = \cos(180° - 60°) = -\cos 60° = -\frac{1}{2}$

(*ix*) $\tan 135° = \tan(180° - 45°) = -\tan 45° = -1$

(*x*) $\sin 330° = \sin(360° - 30°) = -\sin 30° = -\frac{1}{2}$

(*xi*) $\cos 300° = \cos(360° - 60°) = \cos 60° = \frac{1}{2}$

(*xii*) $\sin(-30°) = -\sin 30° = -\frac{1}{2}$

(*xiii*) $\cos(-60°) = +\cos 60° = \frac{1}{2}$

(*xiv*) $\tan(-45°) = -\tan 45° = -1$

(*xv*) $\sin(-150°) = -\sin(150°) = -\sin(180° - 30°)$

$= -\sin 30° = -\frac{1}{2}$

Concept Application

3. Evaluate
   (*i*) $\sin(210°)$
   (*ii*) $\cos 120°$
   (*iii*) $\sin(330°)$
   (*iv*) $\tan(300°)$
4. If $\tan\theta$ is $\frac{5}{12}$, find
   (*i*) $\sin\theta$
   (*ii*) $\cos\theta$

Addition / Subtraction Formulae for Trigonometrical Ratios

(*i*) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

(*ii*) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(*iii*) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(*iv*) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Small Angle Approximation

If θ is small in radians then $\sin\theta \approx \theta$, $\cos\theta \approx 1$ and $\tan\theta \approx \theta$

Note : here θ must be in radian.

Smaller the value of θ, better will be approximation

Maximum and Minimum Values of Some useful Trigonometric Functions

(*i*) $-1 \le \sin\theta \le 1$

(*ii*) $-1 \le \cos\theta \le 1$

(*iii*) $-\sqrt{a^2 + b^2} \le a\cos\theta \pm b\sin\theta \le \sqrt{a^2 + b^2}$

Train Your Brain

Example 6: Find the value of

(*i*) $\sin 74°$ (*ii*) $\cos 106°$

(*iii*) $\sin 15°$ (*iv*) $\cos 75°$

Sol. (*i*) $\sin 74° = \sin(2 \times 37°) = 2\sin 37° \cos 37°$

$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

(*ii*) $\cos 106° = \cos(2 \times 53°) = \cos^2 53° - \sin^2 53°$

$$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9-16}{25} = -\frac{7}{25}$$

(*iii*) $\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45°$

$$\sin 30° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

(*iv*) $\cos 75° = \cos(45° + 30°) = \cos 45° \cos 30° - \sin 45°$

$$\sin 30° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Example 7: Find the approximate values of (*i*) sin 10° (*ii*) tan 20° (*iii*) cos 10°.

Sol. (*i*) $\sin 10° = \sin\left(10° \times \frac{\pi}{180°}\right) = \sin\frac{\pi}{18} \approx \frac{\pi}{18}$

(*ii*) $\tan 20° = \tan\left(20° \times \frac{\pi}{180°}\right) = \tan\frac{\pi}{9} \approx \frac{\pi}{9}$

(*iii*) $\cos 10° = \cos\left(10° \times \frac{\pi}{180°}\right) = \cos\frac{\pi}{18} = 1$

Example 8: Find maximum and minimum values of y:

(*i*) $y = 2\sin x$

(*ii*) $y = 4 - \cos x$

(*iii*) $y = 3\sin x + 4\cos x$

Sol. (*i*) $y_{\max} = 2(1) = 2$ and $y_{\min} = 2(-1) = -2$

(*ii*) $y_{\max} = 4 - (-1) = 4 + 1 = 5$ and $y_{\min} = 4 - (1) = 3$

(*iii*) $y_{\max} = \sqrt{3^2 + 4^2} = 5$ and $y_{\min} = -\sqrt{3^2 + 4^2} = -5$

Example 9: A ball is projected with speed u at an angle θ to the horizontal. The range R of the projectile is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

For which value of θ will the range be maximum for a given speed of projection? (Here g = constant)

(*a*) $\frac{\pi}{2}$ rad (*b*) $\frac{\pi}{4}$ rad (*c*) $\frac{\pi}{3}$ rad (*d*) $\frac{\pi}{6}$ rad

Sol. As $\sin 2\theta \le 1$ so range will be maximum if $\sin 2\theta = 1$.

Therefore $2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$ rad.

Example 10: The position of a particle moving along x-axis varies with time t according to equation $x = \sqrt{3}\ \sin\omega t - \cos\omega t$ where ω is constant. Find the region in which the particle is confined.

Sol. $\because x = \sqrt{3}\sin\omega t - \cos\omega t$

$$\therefore x_{max} = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = 2 \text{ and}$$

$$x_{min} = -\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = -2$$

Thus, the particle is confined in the region $-2 \le x \le 2$

Concept Application

5. Find the maximum and the minimum value of
   (*i*) $8 - 6 \cos x$
   (*ii*) $3 \sin x - 4 \cos x$
   (*iii*) $5 \sin(x) + 12 \cos x + 4$
6. Find the approximate value of
   (*i*) $\sin(19.1°)$ (*ii*) $\cos(5°)$
   (*iii*) $\tan(9°)$
7. Find the value of $\left[\sin 37° \approx \frac{3}{5}\right]$
   (*i*) $\cos(74°)$ (*ii*) $\sin(106°)$

QUADRATIC EQUATION

An algebraic equation of second order (highest power of the variable is equal to 2) is called a quadratic equation. The equation $ax^2 + bx + c = 0$....(i) is the general form of quadratic equation where $a \ne 0$. The general solution of above equation is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If values of x be x_1 and x_2 then $x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

and $x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Here x_1 and x_2 are called roots of equation (i). We can easily see that

sum of roots $x_1 + x_2 = -\frac{b}{a}$

and

product of roots $x_1 x_2 = \frac{c}{a}$

Train Your Brain

Example 11: Find roots of equation $2x^2 - x - 3 = 0$.

Sol. Compare this equation with standard quadratic equation $ax^2 + bx + c = 0$, we have $a = 2$, $b = -1$, $c = -3$.

Now from

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{1 + 24}}{4} = \frac{1 \pm 5}{4} \Rightarrow x = \frac{6}{4}, x = \frac{-4}{4} \Rightarrow x = \frac{3}{2}$$

$$\Rightarrow x = \frac{3}{2} \text{ or } x = -1$$

Concept Application

8. Find the root of quadratic equation
   (*i*) $x^2 + \frac{2}{3}x - \frac{5}{3} = 0$ (*ii*) $4x^2 + 6x - 12 = 0$
9. Find the sum and product of roots of equation
   (*i*) $\frac{5x^2}{3} - \frac{2}{3}x + 3 = 0$ (*ii*) $2x^2 - \frac{3}{2}x + 4 = 0$

ARITHMETIC PROGRESSION (AP)

General form : $a, a + d, a + 2d, ..., a + (n - 1)d$

Here a = first term, d = common difference

Sum of n terms $S_n = \frac{n}{2}[a + a + (n - 1)d] = \frac{n}{2}[2a + (n - 1)d]$

GEOMETRICAL PROGRESSION (GP)

General form : $a, ar, ar^2, ..., ar^{n-1}$ Here a = first term, r = common ratio

Sum of n terms $S_n = \frac{a(1 - r^n)}{1 - r}$

Sum of ∞ term $S_\infty = \frac{a}{1 - r}$

Train Your Brain

Example 12: Find the sum of first n natural numbers

Sol. $S_n = 1 + 2 + 3 + 4 + ... + n = \frac{n}{2}(1 + n) = \frac{n(n+1)}{2}$

Example 13: Find $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$upto ∞

Sol. Here, $a = 1$, $r = \frac{1}{2}$ So, $S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 2$

Concept Application

10. Find the sum of $1 + 3 + 5 + 7 + ... + 51$
11. Find the sum of the series $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24}$ ......∞
12. Find the 20^{th} term of series $1 + 5 + 9 + 13 +$ ......

BINOMIAL EXPRESSION

An algebraic expression containing two terms is called a binomial expression.

For example $(a + b)$, $(a + b)^3$, $(2x - 3y)^{-1}$, $\left(x + \frac{1}{y}\right)^2$ etc. are binomial expressions.

Binomial Theorem

$$(a + b)^n = a^n + na^{n-1}b^1 + \frac{n(n-1)}{2!}a^{n-2}b^2 + ...$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + ...$$

Binomial Approximation

If x is very small, then terms containing higher powers of x can be neglected so

$(1 + x)^n \approx 1 + nx \qquad (x << 1)$

$\frac{(1 + x)^n}{(1 + y)^m} \approx 1 + nx - my \qquad (x, y << 1)$

Train Your Brain

Example 14: Calculate the value of

(*i*) $\sqrt{0.99}$ (*ii*) $(1030)^{1/3}$ (*iii*) $\sqrt{\frac{101}{98}}$

Sol. (*i*) $\sqrt{0.99} = \left(1 - \frac{1}{100}\right)^{\frac{1}{2}} \approx \left(1 - \frac{1}{200}\right) \approx 0.995$

(*ii*) $(1030)^{\frac{1}{3}} = 10\left[1 + \frac{30}{1000}\right]^{\frac{1}{3}}$

$\approx 10\left[1 + \frac{10}{1000}\right] \approx 10.10$

(*iii*) $\sqrt{\frac{101}{98}} = \frac{(1.01)^{\frac{1}{2}}}{(1-0.02)^{\frac{1}{2}}} \approx 1 + \frac{1}{2}\times 0.01 + \frac{1}{2}\times 0.02 \approx 1.015$

Example 15: The mass m of a body moving with a velocity v is given by $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ where m_0 = rest mass of body = 10 kg and c = speed of light = 3×10^8 m/s. Find the value of m at $v = 3 \times 10^7$ m/s.

Sol.

$$m = m_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2} = 10\left[1 - \left(\frac{3\times10^7}{3\times10^8}\right)^2\right]^{-1/2} = 10\left[1 - \frac{1}{100}\right]^{-1/2}$$

$$\approx 10\left[1 - \left(-\frac{1}{2}\right)\left(\frac{1}{100}\right)\right] \approx 10 + \frac{10}{200} \approx 10.05 \text{ kg}$$

Concept Application

13. Calculate the approximate value of

    (*i*) $\sqrt{52}$ (*ii*) $(130)^{\frac{1}{3}}$ (*iii*) $\sqrt{\frac{49}{47}}$

14. The value of acceleration due to gravity at height h above the surface of earth is given by $g' = g\left(\frac{R}{R+h}\right)^2$ where g = 9.81 m/s² and R = 6400 km.

    Find the acceleration due to gravity at height of 64 km above the surface of earth.

CO-ORDINATE GEOMETRY

To specify the position of a point in space, we use right handed rectangular axes coordinate system. This system consists of (*i*) origin (*ii*) axis or axes. If point is known to be on a given line or in a particular direction, only one coordinate is necessary to specify its position. If it is in a plane, two coordinates are required. If it is in space three coordinates are needed.

Origin

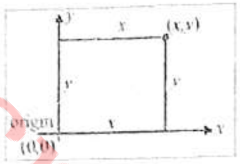
This is any fixed point which is convenient to you. All measurement are taken w.r.t. this fixed point.

Axis or Axes

Any fixed direction passing through origin and convenient to you can be taken as an axis. If the position of a point or position of all the points under consideration always happen to be in a particular direction, then only one axis is required. This is generally called the x-axis. If the positions of all the points under consideration are always in a plane, two perpendicular axes are required. These are generally called x and y-axis. If the points are distributed in a space, three perpendicular axes are taken which are called x, y and z-axis

Position of a point in *xy* plane

The position of a point is specified by its distances from origin along (or parallel to) x and y-axis as shown in figure. Here x-coordinate and y-coordinate is called abscissa and ordinate respectively



Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Train Your Brain

Example 16: Find the value of a if distance between the points (−9 cm, a cm) and (3 cm, 3cm) is 13 cm.

Sol. By using distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\Rightarrow 13 = \sqrt{[3-(-9)]^2 + [3-a]^2}$$

$$\Rightarrow 13^2 = 12^2 + (3-a)^2 \Rightarrow (3-a)^2 = 13^2 - 12^2 = 5^2$$

$$\Rightarrow (3-a) = \pm 5 \Rightarrow a = -2 \text{ cm or } 8 \text{ cm}$$

Example 17: A dog wants to catch a cat. The dog follows the path whose equation is $y - x = 0$ while the cat follows the path whose equation is $x^2 + y^2 = 8$. The coordinates of possible points of catching the cat are:

(*a*) (2, −2)
(*b*) (2, 2)
(*c*) (−2, 2)
(*d*) (−2, −2)

Sol. Let catching point be (x_1, y_1) then, $y_1 - x_1 = 0$ and $x_1^2 + y_1^2 = 8$

Therefore, $2x_1^2 = 8 \Rightarrow x_1^2 = 4 \Rightarrow x_1 = \pm 2$; So possible points are (2, 2) and (−2, −2).

Example 18: Distance between two points (8, −4) and $(0, a)$ is 10. All the values are in the same unit of length. Find the positive value of a.

Sol. From distance formula $(8-0)^2 + (-4-a)^2 = 100$

$$\Rightarrow (4+a)^2 = 36 \Rightarrow a = 2 \text{ or } -10$$

Concept Application

15. Find the distance of point (−12, 5) from the origin.
16. Find the distance between points
    (*i*) (5, 4) and (6, 7)
    (*ii*) (−7, 3) and (7, −6)
17. Find the value of a if distance between $(a, 3)$ cm and (−2, 6) cm is 5 cm.

FUNCTION AND GRAPH

Function

Physics involves study of natural phenomena and describes them in terms of several physical quantities. A mathematical formulation of interdependence of these physical quantities is necessary for a concise and precise description of the phenomena. These mathematical formulae are expressed in form of equations and known as function.

Thus, a function describing a physical process expresses an unknown physical quantity in terms of one or more known physical quantities. We call the unknown physical quantity as dependent variable and the known physical quantities as independent variables. For the sake of simplicity, we consider a function that involves a dependent variable y and only one independent variable x. It is denoted $y = f(x)$ and is read as y equals to f of x. Here $f(x)$ is the value of y for a given x. Following are some examples of functions.

$$y = 2x + 1,\ y = 2x^2 + 3x + 1,\ y = \sin x,\ y = \ell n\,(2x + 1)$$

Knowledge of the dependent variable for different values of the independent variable, and how it changes when the independent variable varies in an interval is collectively known as behavior of the function.

GRAPH OF A FUNCTION

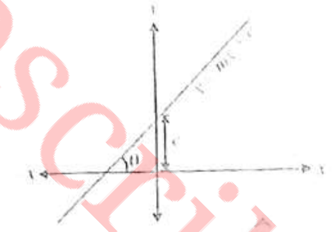
Graph of a function is the diagrammatic representation of a function and allows us to visualize it. To plot a graph the dependent variable (here y) is usually taken on the ordinate and the independent variable (here x) on the abscissa. Graph being an alternative way to represent a function does not require elaborate calculations and explicitly shows behavior of the function in a concerned interval.

Graphs of some commonly used Functions

Linear, parabolic, trigonometric and exponential functions are the most common in use

(i) Straight line Equation and its Graph

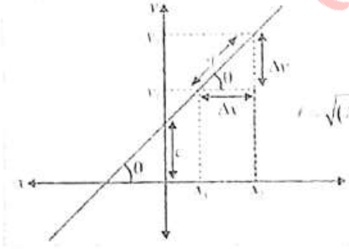
When the dependent variable y varies linearly with the independent variable x, the relationship between them is represented by a linear equation of the type given below $y = mx + c$. The equation is also shown in graph by an arbitrary line



Here m & c are known as slope of the line and intercept on the y-axis, respectively.

Slope:

Slope of a line is a quantitative measure to express the inclination of the line. It is expressed by ratio of change in ordinate to change in abscissa.



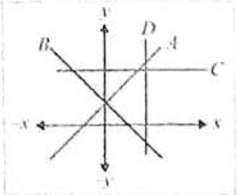
$l = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

Slope of a line $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x}$ = slope of tangent

When the x and the y axes are scaled identically, slope equals to tangent of the angle, which the line makes with the positive x-axis.

$$m = \tan\theta$$

It is positive if y increases with increase in x, negative if y decreases with increase in x, zero if y remains unchanged with change in x and infinite if y changes but x remains unchanged. For these cases the line is inclined up, inclined down, parallel to x-axis and parallel to y-axis respectively as shown in the adjoining figure by lines A, B, C and D respectively.

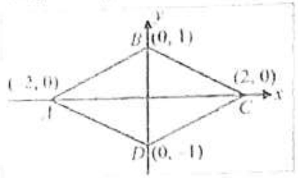


Intercept

It is equal to the value of ordinate y, where the line cuts the y axis. It may be positive, negative or zero for lines crossing the positive y-axis, negative y-axis and passing through the origin respectively.

Train Your Brain

Example 19: A parallelogram $ABCD$ is shown in figure



Column-I		Column-II	
(A)	Equation of side AB	(P)	$2y + x = 2$
(B)	Equation of side BC	(Q)	$2y - x = 2$
(C)	Equation of side CD	(R)	$2y + x = -2$
(D)	Equation of side DA	(S)	$2y - x = -2$
		(T)	$y + 2x = 2$

Sol. (A)-Q, (B)-P, (C)-S, (D)-R

For side AB : $m = \dfrac{1-0}{0-(-2)} = \dfrac{1}{2}$, $c = 1 \Rightarrow y = \dfrac{1}{2}x + 1$

For side BC : $m = \dfrac{0-1}{2-0} = \dfrac{-1}{2}$, $c = 1 \Rightarrow y = \dfrac{-1}{2}x + 1$

For side CD : $m = \dfrac{-1-0}{0-2} = \dfrac{1}{2}$, $c = -1 \Rightarrow y = \dfrac{1}{2}x - 1$

For side DA : $m = \dfrac{0-(-1)}{-2-0} = -\dfrac{1}{2}$, $c = -1 \Rightarrow y = \dfrac{-1}{2}x - 1$

Example 20: Frequency f of a simple pendulum depends on its length ℓ and acceleration g due to gravity according to the following equation $f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{\ell}}$

Graph between which of the following quantities is a straight line?

(a) f on the ordinate and ℓ on the abscissa

(b) f on the ordinate and $\sqrt{\ell}$ on the abscissa

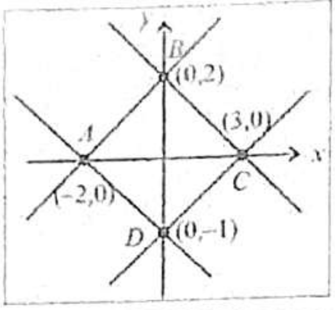
(c) f^2 on the ordinate and ℓ on the abscissa

(d) f^2 on the ordinate and $1/\ell$ on the abscissa

Sol. $f^2 \propto 1/\ell$

Concept Application

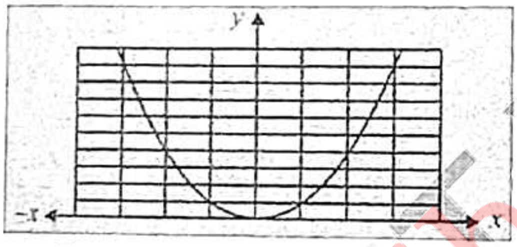
18. Four lines are drawn in the figure below. Match the entries in Column-I with entries in Column-II



Column-I		Column-II	
A.	Equation of line AB	P.	$3y - x + 3 = 0$
B.	Equation of line BC	Q.	$2y + x + 2 = 0$
C.	Equation of line CD	R.	$y = x + 2$
D.	Equation of line AD	S.	$3y + x = 2$
		T.	$3y + 2x = 6$

(ii) Quadratic equation and its graph

A function of the form $y = ax^2 + bx + c$ is known as quadratic equation. The graph of the quadratic function is a U-shaped curve is called a parabola. The simplest parabola has the form $y = ax^2$. Its graph is shown in the following figure.

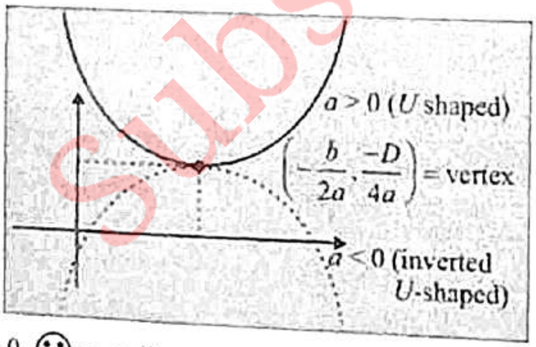


In general

$y = ax^2 + bx + c$

$= a\left(x + \frac{b}{2a}\right)^2 + \left(\frac{-D}{4a}\right)$

where $D = b^2 - 4ac$



$a > 0$ ☺ ⇒ smiling parabola. (parabola opens upwards)

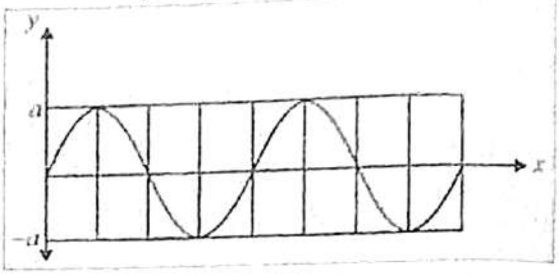
$a < 0$ ☹ ⇒ sad parabola. (parabola opens downwards)

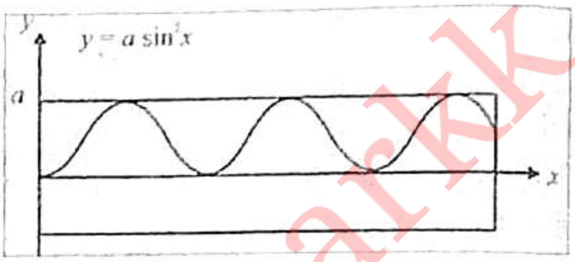
Vertex can lie in any quadrant depending on value of a, b and c.

Graph of some trigonometric functions

Among all the trigonometric functions, sinusoidal function, which includes both sine and cosine is most commonly in use.

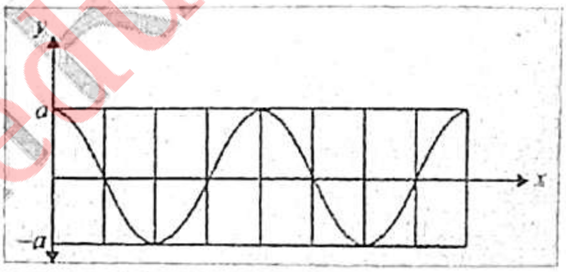
Sine Function $y = a \sin x$

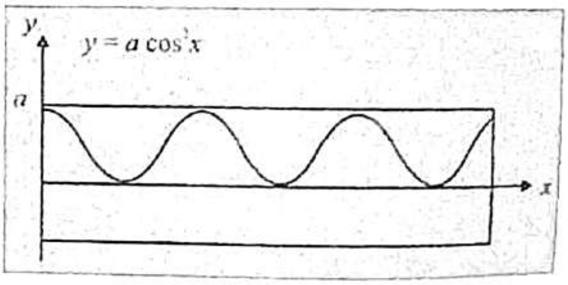




Here, a is known as the amplitude and equals the maximum magnitude of y. In the adjoining figure graph of a sine function is shown, which has amplitude a units.

Cosine Function $y = a \cos x$

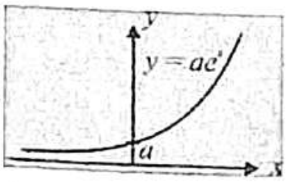


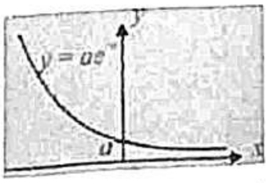


Here, a is known as the amplitude and equals the maximum magnitude of y. In the adjoining figure graph of a cosine function is shown, which has amplitude a units.

(iii) Exponential function and its graph

Behavior of several physical phenomena is described by exponential function to the base e. Here e is known as Euler's Number. $e = 2.718218$



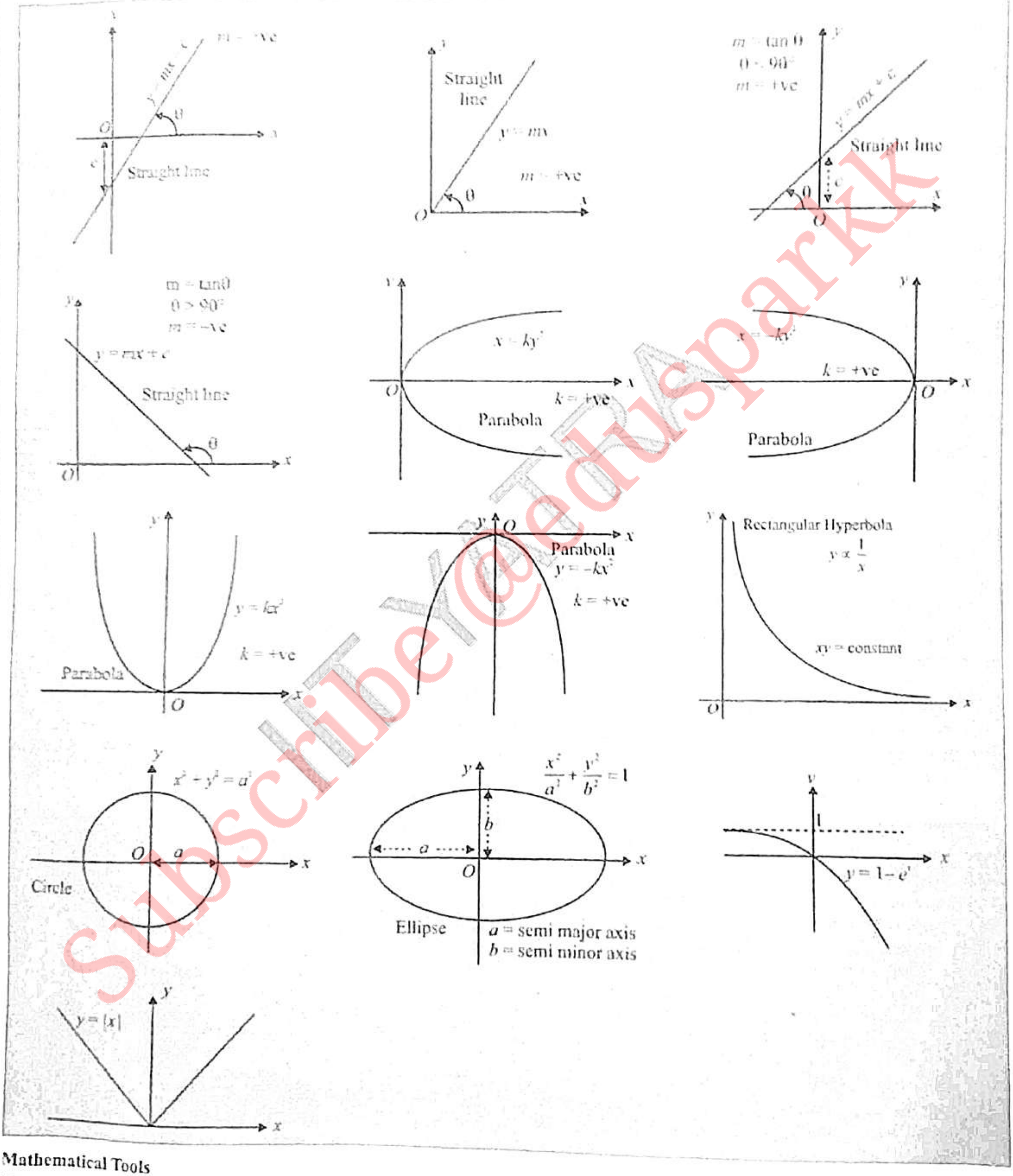


Most commonly used exponential function has the form $y = ae^x$, $y = ae^{-x}$. In the adjoining figure graph of this function is shown.

(iv) Circle and Ellipse

Circle : $x^2 + y^2 = a^2$ Ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Some standard graphs & their equations


y
m = +ve
y = mx + c
θ
O
x
c
Straight line
y
Straight line
y = mx
m = +ve
θ
O
x
m = tan θ
θ < 90°
m = +ve
y
y = mx + c
Straight line
c
θ
O
x
m = tanθ
θ > 90°
m = −ve
y
y = mx + c
Straight line
θ
O
x
y
x = ky²
O
x
k = +ve
Parabola
y
x = −ky²
k = +ve
O
x
Parabola
y
y = kx²
k = +ve
Parabola
O
x
y
O
x
Parabola
y = −kx²
k = +ve
y
Rectangular Hyperbola
y ∝ 1/x
xy = constant
O
x
y
x² + y² = a²
O
a
x
Circle
y
x²/a² + y²/b² = 1
b
a
O
x
Ellipse
a = semi major axis
b = semi minor axis
y
1
x
y = 1 − eˣ
y
y = |x|
x

Logarithm

Logarithm is the exponent or power to which a base must be raised to yield a given number. Expressed mathematically, x is the logarithm of n to the base b if $b^x = n$ in which case one writes $x = \log_b n$. For example $2^5 = 32$, therefore 5 is the logarithm of 32 to base 2 or $5 = \log_2 32$. Similarly since $10^3 = 1000$ then $3 = \log_{10} 1000$.

Logarithms with base 10 are common and are written simply as log.

Logarithm laws

- $\log_a a = 1$
- $\log mn = \log m + \log n$
- $\log\left(\frac{m}{n}\right) = \log m - \log n$
- $\log m^n = n \log m$

Logarithms with base e denoted by ln are called natural logarithms where e (Euler's number) is an irrational number approximately equal to 2.71828. The relation between natural logarithm and common logarithm is given by

$\log_e m = 2.303 \log_{10} m$

$\ln 2 = 0.69$

$\log 2 = 0.3010$

$\ln e = 1$

$\ln 1 = 0$

Train Your Brain

Example 21: Calculate the value of $\log_{10} 12$

(Given $\log_{10} 2 = 0.301$, $\log_{10} 3 = 0.477$)

Sol. $\log_{10} 12 = \log_{10}(4 \times 3) = \log_{10} 4 + \log_{10} 3$

$= \log_{10} 2^2 + \log_{10} 3 = 2\log_{10} 2 + \log_{10} 3$

$= 2 \times 0.301 + 0.477 = 1.079$

Concept Application

19. Calculate the value of

(i) $\log_{10} 16$ (ii) $\log_{10} 4 + \log_{10} 12$

(iii) $\log_{10} 64$ (iv) $\log_{10} 3 - \log_{10} 24$

DIFFERENTIATION

(i) **Finite difference:** The finite difference between two values of a physical quantity is represented by Δ notation.

For example: Difference in two values of y is written as Δy as given in the table below

y_2	100,	100	100
y_1	50	99	99.5
$\Delta y = y_2 - y_1$	50	1	0.5

(ii) **Infinitely small difference:** The infinitely small difference means very-very small difference. And this difference is represented by 'd' notation instead of 'Δ'.

For example infinitely small difference in the value of y is written as 'dy' if $y_2 = 100$ and $y_1 = 99.99999999$

then $dy = 0.000000..........00001$

DEFINITION OF DIFFERENTIATION

Another name for differentiation is derivative. Suppose y is a function of x or $y = f(x)$

Differentiation of y with respect to x is denoted by symbol $f'(x)$ where $f'(x) = \frac{dy}{dx}$

dx is very small change in x and dy is corresponding very small change in y.

NOTATION: There are many ways to denote the derivative of a function $y = f(x)$. Besides $f'(x)$, the most common notations are these:

y'	"y prime" or "y dash"	Nice and brief but does not name the independent variable
$\frac{dy}{dx}$	"dy by dx"	Names the variables and used d for derivative
$\frac{df}{dx}$	"df by dx"	Emphasizes the function's name.
$\frac{d}{dx} f(x)$	"d by dx of f"	Emphasizes the idea that differentiation is an operation performed on f.
$d(f)$	"d of f"	A common operator notation.
$\dot{y}$	"y dot"	One of Newton's notations, now common for time derivatives i.e. $\frac{dy}{dt}$.

SLOPE OF A LINE

It is the tan of angle made by a line with the positive direction of x-axis, measured in anticlockwise direction.

Slope = tan θ

(In 1st quadrant tan θ is +ve and 2nd quadrant tan θ is –ve)

In figure (a)slope is positive

In figure (b)slope is negative

$\theta < 90°$ (1st quadrant)

$\theta > 90°$ (2nd quadrant)

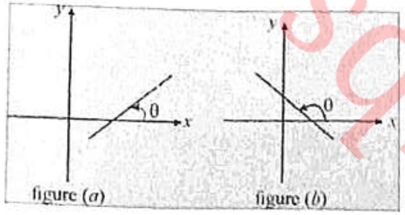
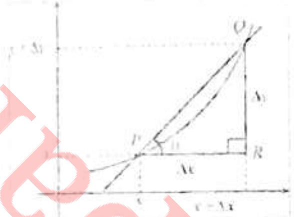


figure (a) figure (b)

AVERAGE RATE OF CHANGE

Given an arbitrary function $y = f(x)$ we calculate the average rate of change of y with respect to x over the interval $(x, x + \Delta x)$ by dividing the change in value of y, i.e. $\Delta y = f(x+\Delta x) - f(x)$, by length of interval Δx over which the change occurred.



The average rate of change of y with respect to x over the interval $[x, x+\Delta x] = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$

Geometrically, $\frac{\Delta y}{\Delta x} = \frac{QR}{PR} = \tan\theta =$ Slope of the line PQ therefore we can say that average rate of change of y with respect to x is equal to slope of the line joining P and Q

THE DERIVATIVE OF A FUNCTION

We know that, average rate of change of y w.r.t. x is

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

If the limit of this ratio exists as $\Delta x \to 0$, then it is called the derivative of given function $f(x)$ and is denoted as

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Derivatives with Respect to Time

In physics, we are often looking at how things change over time:

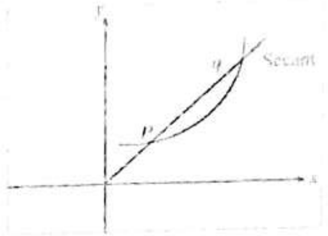
1. Velocity is the derivative of position with respect to time
$$v(t) = \frac{d}{dt}(x(t))$$
2. Acceleration is the derivative of velocity with respect to time:
$$a(t) = \frac{d}{dt}(v(t)) = \frac{d^2}{dt^2}(x(t))$$
3. Momentum (usually denoted p) is mass times velocity, and force (F) is mass times acceleration, so the derivative of momentum is $\frac{dp}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} = ma = F$

GEOMETRICAL MEANING OF DIFFERENTIATION

The geometrical meaning of differentiation is very much useful in the analysis of graphs in physics. To understand the geometrical meaning of derivatives we should have knowledge of secant and tangent to a curve.

Secant and Tangent to a Curve

A secant to a curve is a straight line, which intersects the curve at any two points.



TANGENT

A tangent is a straight line, which touches the curve at a particular point. Tangent is a limiting case of secant which intersects the curve at two overlapping points.

In the figure (a) shown, if value of Δx is gradually reduced then the point Q will move nearer to the point P. If the process is continuously repeated (figure (b)) value of Δx will be infinitely small and secant PQ to the given curve will become a tangent at point P

Therefore $\lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right) = \frac{dy}{dx} = \tan\theta$

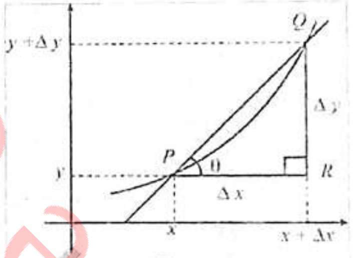


Figure (a)

we can say that differentiation of y with respect to x, i.e. $\left(\frac{dy}{dx}\right)$ is equal to slope of the tangent at point $P(x, y)$ or $\tan\theta = \frac{dy}{dx}$

(From fig. a, the average rate of change of y from x to $x+\Delta x$ is identical with the slope of secant PQ.)

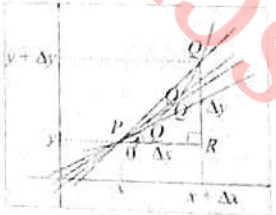


Figure (b)

THEOREMS OF DIFFERENTIATION

1. If c is constant, then $\frac{d}{dx}(c) = 0$

2. If $y = ku$, where k is a constant and u is a function of x, then

$$\frac{dy}{dx} = \frac{d}{dx}(ku) = k\frac{du}{dx}$$

3. If $y = u \pm v \pm w$, where u, v and w are functions of x, then

$$\frac{dy}{dx} = \frac{d}{dx}(u \pm v \pm w)$$

$$= \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$$

4. If $y = uv$ where u and v are function of x, then

$$\frac{dy}{dx} = \frac{d}{dx}(uv)$$

$$= u\frac{dv}{dx} + v\frac{du}{dx}$$

5. If $y = \frac{u}{v}$, where u and v are functions of x, then

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right)$$

$$= \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

6. If $y = x^n$ where n is a real number, then

$$\frac{dy}{dx} = \frac{d}{dx}(x^n) = nx^{n-1}$$

Formulae for differential coefficients of trigonometric, logarithmic and exponential functions

1. $\frac{d}{dx}(\sin x) = \cos x$
2. $\frac{d}{dx}(\cos x) = -\sin x$
3. $\frac{d}{dx}(\tan x) = \sec^2 x$
4. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
5. $\frac{d}{dx}(\sec x) = \sec x \tan x$
6. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$
7. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$
8. $\frac{d}{dx}(e^x) = e^x$

Train Your Brain

Example 22: Find $\frac{dy}{dx}$, when

(i) $y = \sqrt{x}$

(ii) $y = x^5 + x^3 + 7$

(iii) $y = x^2 + 4x^{-1/2} - 3x^{-2}$

Sol. (i) Here, $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

(ii) Here, $y = x^5 + x^3 + 7$

$$\frac{dy}{dx} = \frac{d}{dx}(x^5 + x^3 + 7) = \frac{d}{dx}(x^5) + \frac{d}{dx}(x^3) + \frac{d}{dx}(7)$$

$$= 5x^4 + 3x^2 + 0 = 5x^4 + 3x^2$$

(iii) Here, $y = x^2 + 4x^{-1/2} - 3x^{-2}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 4x^{-1/2} - 3x^{-2})$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$$

$$= \frac{d}{dx}(x^2) + 4\frac{d}{dx}(x^{-1/2}) - 3\frac{d}{dx}(x^{-2})$$

$$= 2x + 4\left(-\frac{1}{2}\right)x^{-3/2} - 3(-2)x^{-3} = 2x - 2x^{-3/2} + 6x^{-3}$$

Example 23: If $3y = 4x^2 - 5$ find $\frac{dy}{dx}$

Sol. $y = \frac{4}{3}x^2 - \frac{5}{3} \Rightarrow \frac{dy}{dx} = \frac{8x}{3}$

Example 24: $\sqrt{y} = x - 1$, find $\frac{dy}{dx}$

Sol. $y = x^2 - 2x + 1 \Rightarrow \frac{dy}{dx} = 2x - 2$

Concept Application

20. If acceleration $= \frac{dv}{dt}$. Find acceleration at $t = 1$ sec from $v = 3t^2 - 1$

21. $y = \sqrt{x} - 3x^3$, find $\frac{dy}{dx}$

22. $y = \frac{1}{x^2} - 2x$, find $\frac{dy}{dx}$

23. $y = \frac{3x-5}{x^2}$ find $\frac{dy}{dx}$

24. $x = 9y^2$ find $\frac{dy}{dx}$

Find $\frac{dy}{dx}$ for the following

25. $y = x^{7/2}$

26. $y = x^{-3}$

27. $y = x$

28. $y = x^5 + x^3 + 4x^{1/2} + 7$

29. $y = 5x^4 + 6x^{3/2} + 9x$

30. $y = ax^2 + bx + c$

31. $y = 3x^5 - 3x - \frac{1}{x}$

32. Given $S = t^2 + 5t + 3$, then $\frac{dS}{dt} = ?$

33. Given $S = ut + \frac{1}{2}at^2$, where u and a are constants. Obtain the value of $\frac{dS}{dt}$.

34. The area of a blot of ink is growing such that after t seconds, its area is given by $A = (3t^2 + 7)\ \text{cm}^2$. Calculate the rate of increase of area at $t = 5$ seconds.

35. The area of a circle is given by $A = \pi r^2$, where r is the radius. What is the rate of increase of area w.r.t. rate of change of radius.

Obtain the differential coefficient (differentiation) of the following:

36. $(x - 1)(2x + 5)$ 37. $(9x^3 - 8x + 7)(3x^2 + 5)$

38. If $t = \sqrt{x} - 1$, then the velocity at $t = 2$ sec is

39. If $S = 3t^2$, then double differentiation of S with respect to t is

40. Velocity of a body is given by $v = 3t^2 - 4t$, then rate of change of velocity w.r.t. to time at $t = 1$ sec is

41. Acceleration is given by $a = \frac{dv}{dt}$, then acceleration at $t = 10$ sec from $v = 3t^2 + t$

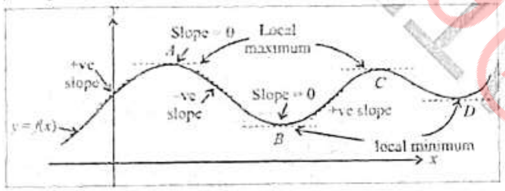
Find dy/dx for the following

42. $y = \frac{1}{2x+1}$ 43. $y = \frac{3x+4}{4x+5}$ 44. $y = \frac{x^2}{x^3+1}$

MAXIMA AND MINIMA

Finding maxima and minima of a function using derivatives:-

A maximum is a high point and minimum is low point of a function (see figure below)



In a smoothly changing function a maximum or a minimum is always where function flattens out or where slope of tangent line is zero. We know slope $= \frac{dy}{dx}$. So a function reaches its maximum or minimum value when $\frac{dy}{dx} = 0$.

In the neighbourhood of maximum (point A), slope changes from positive to zero at point A and then becomes negative as x increases which means $\frac{d}{dx}(\text{slope}) < 0 \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} < 0$

In the neighbourhood of minimum (point B), slope changes from negative to zero and then becomes positive as x increases which means

$$\frac{d}{dx}(\text{slope}) > 0 \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} > 0$$

Second Derivative Test:

When a function's slope $\left(\frac{dy}{dx}\right) = 0$ at a point and its second derivative at that point is

(*i*) less than zero, it is a local maximum

(*ii*) greater than zero, it is a local minimum

Train Your Brain

Example 25: What is the minimum value of y for the curve $y = -8x^2 + x^4$

Sol. $y = -8x^2 + x^4$

$$\frac{dy}{dx} = -16x + 4x^3 = -x(16 - 4x^2)$$

The function will have a maximum or minimum value when $\frac{dy}{dx} = 0$

$\Rightarrow x(16 - 4x^2) = 0$

$\Rightarrow x = 0$ or $x = \pm 2$

Now

$$\frac{d^2y}{dx^2} = -16 + 12x^2$$

At $x = 0$, $\frac{d^2y}{dx^2} = -16$ (maximum)

At $x = \pm 2$, $\frac{d^2y}{dx^2} = -16 + 48 = +32$ (minimum)

So function has minimum value at $x = \pm 2$

$y_{min} = -8 \times 4 + 16 = -16$

Example 26: A ball is thrown vertically upward in the air. Its height y at any time t is given by $y = 10t - 5t^2$ where y is in meters and t is in seconds. What is the maximum height attained by the ball?

Sol. $y = 10t - 5t^2$

$\frac{dy}{dt} = 10 - 10t = 0$

$\Rightarrow t = 1$ sec

$\frac{d^2y}{dt^2} = -10$ (maximum)

ball attains maximum height at $t = 1s$

$y_{max} = 10 \times 1 - 5 \times 1^2 = 5m.$

Concept Application

45. Find the minimum value of

(*i*) $y = 1 + x^2 - 2x$ (*ii*) $y = 5x^2 - 2x + 1$

46. The position of a particle moving along the y-axis is given by $y = 3t^2 - t^3$, where y is in meters and t is in sec. The time when particle attains its maximum positive y position is

(*a*) 1.5 sec (*b*) 4 sec

(*c*) 2 sec (*d*) 3 sec

INTEGRATION

(i) **Integration is the inverse process of differentiation:** Integration is the process of finding the function, whose derivative is given. For this reason, the process of integration is the inverse process of differentiation.

Consider a function $f(x)$, whose derivative w.r.t. x is another function $f'(x)$ i.e. $\frac{d}{dx}(f(x)) = f'(x)$

If differentiation of $f(x)$ w.r.t. x is equal to $f'(x)$, then $f(x) + c$ is called the integration of $f'(x)$, where c is called the constant of integration.

Symbolically, it is written as $\int f'(x)dx = f(x) + c$

Here, $f'(x)\ dx$ is called element of integration and $\int$ is called indefinite integral operator. Let us proceed to obtain integral of x^n w.r.t. x. $\frac{d}{dx}(x^{n+1}) = (n+1)x^n$

Since the process of integration is the inverse process of differentiation,

$\int (n+1)x^n dx$ or $(n+1)\int x^n dx$

or $\int x^n dx = \frac{x^{n+1}}{n+1}$

The above formula holds for all values of n, except $n = -1$.

It is because, for $n = -1$, $\int x^n dx = \int x^{-1} dx = \int \frac{1}{x} dx$

Since $1/x$ is differential coefficient of $\log_e x$

i.e. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$ $\therefore$ $\int \frac{1}{x} dx = \log_e x$

Similarly, the formulae for integration of some other functions can be obtained if we know the differential coefficients of various functions.

(ii) **Few basic formulae of integration:** Following are a few basic formulae of integration:

1. $\int 1 dx = x + C$
2. $\int a\ dx = ax + C$
3. $\int x^n\ dx = \left(\left(x^{n+1}\right)/(n+1)\right) + C; n \neq -1$
4. $\int \sin x\ dx = -\cos x + C$
5. $\int \cos x\ dx = \sin x + C$
6. $\int \sec^2 x\ dx = \tan x + C$
7. $\int \csc^2 x\ dx = -\cot x + C$
8. $\int \sec x\ (\tan x)dx = \sec x + C$
9. $\int \csc x\ (\cot x)dx = -\csc x + C$
10. $\int (1/x)dx = \ln|x| + C$
11. $\int e^x dx = e^x + C$

Train Your Brain

Example 27: Integrate w.r.t. x

(i) $x^{11/2}$ (ii) x^{-7} (iii) $x^{p/q}$

Sol. (i) $\int x^{11/2}dx = \frac{x^{\frac{11}{2}+1}}{\frac{11}{2}+1} + c = \frac{2}{13}x^{13/2} + c$

(ii) $\int x^{-7}dx = \frac{x^{-7+1}}{-7+1} + c = -\frac{1}{6}x^{-6} + c$

(iii) $\int x^{\frac{p}{q}} dx = \frac{x^{\frac{p}{q}+1}}{\frac{p}{q}+1} + c = \frac{q}{p+q}x^{(p+q)/q} + c$

Example 28: Find integration w.r.t. x, for $\left(\frac{\sqrt{x}}{x^2}\right)$

Sol. $\int \frac{\sqrt{x}}{x^2} dx = \int x^{-2+\frac{1}{2}} dx = \int x^{\frac{-3}{2}} dx = \int \frac{x^{\frac{-3}{2}+1}}{-\frac{3}{2}+1}$

$= \frac{-2}{\sqrt{x}} + c$

Example 29: $\int (x+1)^2\ dx$

Sol. $\int x^2 dx + 2\int x dx + 1\int dx$

$\frac{x^3}{3} + 2\frac{x^2}{2} + x + c = \frac{x^3}{3} + x^2 + x + c$

Concept Application

47. $\int (1-3x^3)dx$
48. $\int \frac{6x^2+4}{x^2} dx$
49. $\int (\sin x + \cos x)\,dx$
50. $\int \left(\frac{1}{x^3} - \cos x\right) dx$
51. $\int \cos 3x\ dx$
52. $\int \left(\frac{1}{x^2} + 3\sin 2x\right) dx$

(iii) **Definite integrals:** When a function is integrated between a lower limit and an upper limit, it is called a definite integral.

If $\frac{d}{dx}(f(x)) = f'(x)$, then $\int f'(x)dx$ is called indefinite integral

and $\int_a^b f'(x)dx$ is called definite integral

Here, a and b are called lower and upper limits of the variable x.

After carrying out integration, the result is evaluated between upper and lower limits as explained below:

$$\int_a^b f'(x)dx = |f(x)|_a^b = f(b) - f(a)$$

Train Your Brain

Example 30: Evaluate the integral $\int_1^5 x^2 dx$

Sol. $\int_1^5 x^2 dx = \left|\frac{x^3}{3}\right|_1^5 = \frac{1}{3}\left|x^3\right|_1^5 = \frac{1}{3}((5)^3 - (1)^3)$

$= \frac{1}{3}(125 - 1) = \frac{124}{3}$

Example 31: $\int_0^{\pi}(\sin x + \cos x)\,dx$

Sol. $\int_0^{\pi}\sin x dx + \int_0^{\pi}\cos x dx$

$= \left|-\cos x\right|_0^{\pi} + \left|\sin x\right|_0^{\pi}$

$= -[\cos\pi - \cos 0] + [\sin\pi - \sin 0] = -(-2) + 0 = 2$

Concept Application

53. $\int_{x=-1}^{x=1}(ax^2 + b)dx$

54. $\int_{x=0}^{x=2}\frac{2x}{4x^2+1}dx$

55. $\int_{x=0}^{\pi/2}\sin(3x-1)dx$

56. $\int_{x=\pi}^{2\pi}\cos(2x-30^\circ)dx$

57. $\int_R^{\infty}\frac{GMm}{r^2}dr$

58. $\int_{r_1}^{r_2} -k\frac{q_1q_2}{r^2}dr$

59. $\int_u^v M\upsilon d\upsilon$

60. $\int_0^{\pi} x^{-1/2}dx$

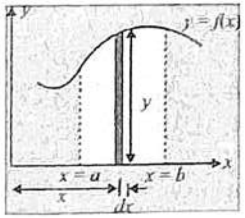
61. $\int_0^{\pi/2}\sin x\,dx$

62. $\int_0^{\pi/2}\cos x\,dx$

63. $\int_{-\pi/2}^{\pi/2}\cos x\,dx$

Application of integration

- Area under the curve :



Area of small element = $ydx = f(x)dx$

If we sum up all areas between $x = a$ and $x = b$ then $\int_a^b f(x)dx$ = shaded area between curve and x-axis.

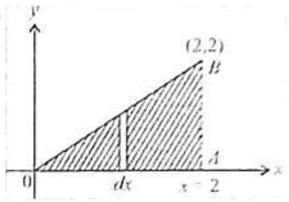
- Average value

If $y = f(x)$ then $\bar{y}$ or $y_{average} = \frac{\text{Area under the curve}}{\text{interval}} = \frac{\int_a^b f(x)dx}{b-a}$

Train Your Brain

Example 32: Find the area bounded by $y = x$ and x-axis between $x = 0$ and $x = 2$.

Sol.



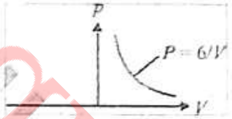
$\int ydx = \int_0^2 xdx$

$\frac{1}{2}x^2\Big|_0^2 = 2$ unit

This result can also be obtained by using geometry

$\Delta OAB = \frac{1}{2}\times 2\times 2 = 2$ units

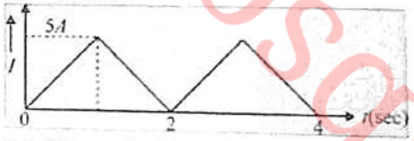
Example 33: A gas expands its volume from V to 3V as shown in figure. Calculate the work in this process if $W = \int pdv$.



(*a*) 6.6 (*b*) 6.9 (*c*) 3 (*d*) 3.5

Sol. (*a*) $W = \int_V^{3V} pdv \Rightarrow \int_V^{3V}\frac{6}{v}dv = [6\log_e v]_V^{3V} = 6.6\,\text{J}$

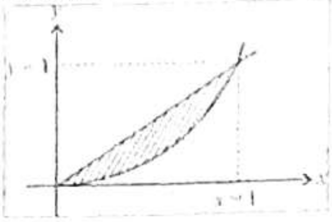
Example 34: Calculate average value of current from $t = 0$ to $t = 4$ seconds.



(*a*) 2.5 (*b*) 3 (*c*) 3.5 (*d*) 2

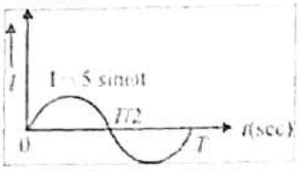
Sol. (*a*) $I_{average} = \frac{\int_0^4 Idt}{\int_0^4 dt} = \frac{\text{total area}}{\text{total time}} = \frac{5\times 2}{4} = 2.5$ amp

Example 35: Find the shaded area bounded by line $y = x$ and $y = x^2$ as shown in figure



Sol. $A = \int_0^1 x\,dx - \int_0^1 x^2\,dx = \frac{x^2}{2}\Big|_0^1 - \frac{x^3}{3}\Big|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ units

Example 36: Calculate average value of current from $t = 0$ to $t = T$ seconds ($T = 2\pi/\omega$)



(*a*) 0 (*b*) 1 (*c*) 5 (*d*) 2

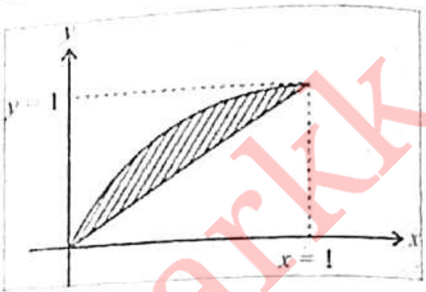
Sol. (*a*) $I_{average} = \frac{\int_0^T 5\sin\omega t}{\int_0^T dt} = \frac{5}{\omega T}[-\cos\omega t]_0^T$

$$= -\frac{5}{\omega T}[\cos\omega T - \cos 0] = 0 \qquad \left[\omega = \frac{2\pi}{T}\right]$$

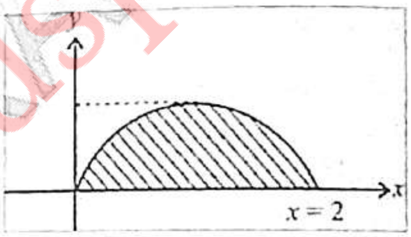
Concept Application

64. Find the area bounded by the curve $y = 4 - x^2$ f
$x = -2$ to $x = 2$

65. Find the shaded area bounded by the curve $y = \sqrt{x}$ a
$y = x$ as shown in figure?



66. Find the area bounded by $y = 2x - x^2$ and x-axis betw
$x = 0$ to $x = 2$ as shown in figure.



REFRAME NEGATIVE SITUATIONS

- Identify your unique skills and qualities. Focusing on what you're good at can help you overcome feelings of inadequacy.
- List your strengths and achievements, then refer to the list whenever you doubt yourself.

Prarambh (Topicwise)

TRIGONOMETRY

1. Change degree into radian

(a) 160° (b) 135° (c) 75° (d) 65°

(e) 225° (f) 250° (g) 310°

2. Change radian into degree

(a) $\frac{\pi}{4}$ (b) $\frac{7\pi}{2}$

(c) $\frac{3\pi}{8}$ (d) $\frac{2\pi}{3}$

(e) $\frac{3\pi}{4}$

3. Evaluate -

(a) $\cos 15^\circ$ (b) $\tan 15^\circ$ (c) $\sin 53^\circ$ (d) $\cos 53^\circ$

(e) $\tan 37^\circ$ (f) $\tan 53^\circ$ (g) $\sin 53^\circ - \cos 37^\circ$

4. Evaluate:-

(a) $\frac{\sin 135^\circ}{\cos 120^\circ}$ (b) $\frac{\sin 120^\circ}{\cos 15^\circ}$

(c) $\sin 105^\circ$ (d) $\sin 300^\circ$

(e) $\cos 240^\circ$ (f) $\sin^2(20^\circ) + \sin^2(70^\circ)$

(g) $\sin 225^\circ$ (h) $\sin 315^\circ$

(i) $\cos 270^\circ$

5. Evaluate:-

(a) $2\sin 15^\circ \cos 15^\circ$ (b) $\sin 22.5^\circ \cos 22.5^\circ$

(c) $\tan 75^\circ$ (d) $\sin^2 22.5^\circ$

6. Evaluate:-

(a) $\cos\left(\frac{5\pi}{4}\right)$ (b) $\sin\left(\frac{2\pi}{3}\right)$

(c) $\sin\left(\frac{3\pi}{4}\right)$ (d) $\tan\left(\frac{7\pi}{6}\right)$

7. $\sqrt{1+\sin\theta}$ is equal to

(a) $(\sin\theta + \cos\theta)$ (b) $\sin\theta - \cos\theta$

(c) $\sin\frac{\theta}{2} + \cos\frac{\theta}{2}$ (d) $\sin\frac{\theta}{2} - \cos\frac{\theta}{2}$

8. $\sqrt{1+\cos\theta}$ is equal to

(a) $\sqrt{2}\sin\frac{\theta}{2}$ (b) $\sqrt{2}\cos\frac{\theta}{2}$

(c) $\frac{1}{\sqrt{2}}\sin\frac{\theta}{2}$ (d) $\frac{1}{\sqrt{2}}\cos\frac{\theta}{2}$

9. If $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} = \frac{7}{3}$,, then $\tan\theta =$

(a) $\frac{3}{5}$ (b) $\frac{5}{2}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

ALGEBRA

10. The equation whose roots are the squares of the roots of the equation $ax^2 + bx + c = 0$ is

(a) $a^2x^2 + b^2x + c^2 = 0$

(b) $a^2x^2 - (b^2 - 4ac)x + c^2 = 0$

(c) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

(d) $a^2x^2 + (b^2 - ac)x + c^2 = 0$

11. The value of 'a' for which one root of quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as other is

(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

12. If the roots of the equation $ax^2 + bx + c = 0$ are in the ratio $m : n$, then

(a) $mnb^2 = ac(m+n)^2$ (b) $b^2(m+n) = mn$

(c) $m + n = b^2mn$ (d) $mnc^2 = ab(m+n)^2$

13. The quadratic equation whose roots are the x and y intercepts of the line passing through (1, 1) and making a triangle of area A with axes may be-

(a) $x^2 + Ax + 2A = 0$ (b) $x^2 - 2Ax + 2A = 0$

(c) $x^2 - Ax + 2A = 0$ (d) None of these

AP AND GP SERIES

14. Find the sum of first 20 natural Number

(a) 210 (b) 200 (c) 220 (d) 230

15. 30th term of the A.P: 10, 7, 4, …, is

(a) 97 (b) 77 (c) −77 (d) −87

16. The sum of the first five multiples of 3 is:

(a) 45 (b) 55 (c) 65 (d) 75

17. Find sum of $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32}$ ….. ∞.

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

18. Find sum of $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ up to ∞.

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

19. Find the sum of the geometric series:

4 − 12 + 36 − 108 + …………. to 10 terms

(a) 59048 (b) −30421

(c) −59048 (d) 30421

BINOMIAL APPROXIMATION

20. Find approximate the value of the $\sqrt{0.95}$
(a) 1 (b) 0.998 (c) 0.90 (d) 0.975

21. Find approximate value of the $\sqrt{104}$
(a) 10.2 (b) 12 (c) 13.5 (d) 15

22. Find approximate value of the $(102)^2$
(a) 11600 (b) 9200 (c) 10400 (d) 12400

23. Find approximate value of the $(4.04)^3$
(a) 60.08 (b) 75.63 (c) 65.92 (d) 55.72

24. Find approximate value of the $(9.6)^4$
(a) 4200 (b) 3600 (c) 2100 (d) 8400

FUNCTIONS & GRAPHS

25. The equation of straight line having slope $\sqrt{3}$ and y intercept of −2 will be
(a) $y = \sqrt{3}x + 2$
(b) $y = \sqrt{3}x - 2$
(c) $y = -\sqrt{3}x - 2$
(d) $y = -\sqrt{3}x + 2$

26. The equation of line is $2x - 3y = 6$ its x and y intercepts are
(a) −3, 2 (b) 3, −2 (c) 2, −3 (d) −2, 3

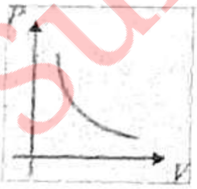
27. The equation of line making an angle 135° with the positive x-axis and passing through a point (2, 3) will be :
(a) $y = x + 5$
(b) $y = x - 5$
(c) $y = x - 1$
(d) $y = -x + 5$

28. The minimum distance of origin from line $3x - 4y - 5 = 0$ is
(a) 1 (b) 2 (c) 3 (d) 4

29. The distance between points (2, 3, −7) and (−2, 0, 5) is
(a) 5 (b) 13 (c) $\sqrt{145}$ (d) $\sqrt{119}$

30. The points where the line $y = x$ intersect the curve $x^2 + y^2 = 32$ is
(a) $4\sqrt{2}, 4\sqrt{2}$
(b) 4, 4
(c) $-4\sqrt{2}, -4\sqrt{2}$
(d) −2, −2

31. P – V graph for ideal gas at constant temperature is {ideal gas eq^n $PV = nRT$}

(a)


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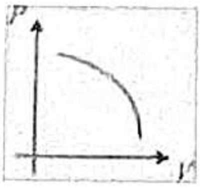
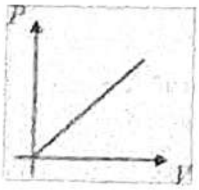


(c)


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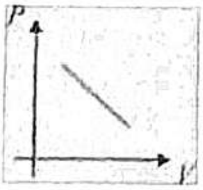
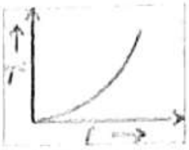
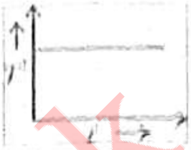
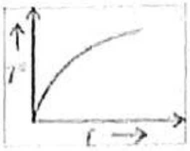


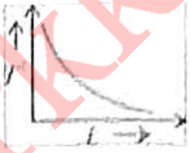
32. Time period of oscillation of a pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$ then graph between T & l is
(a) Straight line
(b) Parabola
(c) Ellipse
(d) Rectangular hyperbola

33. Which of the following curve is related to function $T^2 \propto l$

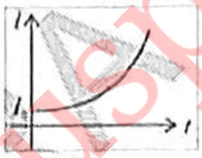
(a)


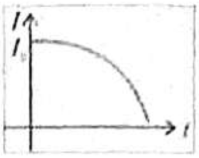
(b)


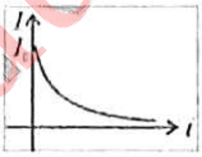
(c)


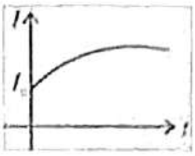
(d)


34. The variation of current flow in a circuit is given as $I = I_0 e^{-t/RC}$. The graph representing I vs t will be

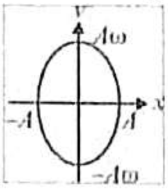
(a)


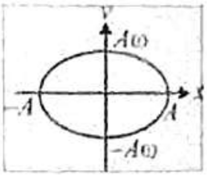
(b)


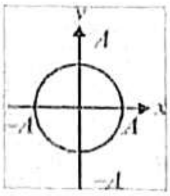
(c)


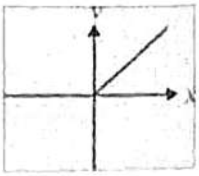
(d)


35. Velocity of a particle undergoing SHM is $v = \omega\sqrt{A^2 - x^2}$ where $\omega > 1$. The graph representing v versus x will be

(a)


(b)


(c)


(d)


LOGARITHMS

36. log 25 + log 4 − log 5 is equal to
(a) log 20 (b) log 25 (c) log 15 (d) log 10

37. $\log_e 8$ is equal to
(a) $\log_e 2$ (b) $2\log_e 2$ (c) $3\log_e 2$ (d) $4\log_e 2$

38. $\log_e 15$ is equal to
(a) $\log_e 3 + \log_e 5$
(b) $\log_e 5 - \log_e 3$
(c) $\log_e 10 + \log_e 5$
(d) $\log_e 10 - \log_e 5$

39. $\log_3 x^2 = 4$, find the value of x
(a) 3 (b) 5 (c) 7 (d) 9

40. If $\log_{10} x - \log_{10}(y) = 2$, find the value of $\frac{x}{y}$

(a) 100 (b) 25 (c) 50 (d) 75

DIFFERENTIATION

41. $f(x) = \cos x + \sin x$ then $f(\pi/2)$ will be

(a) 2 (b) 1 (c) 3 (d) 0

Direction (No. 42 to 44): Derivative of given function w.r.t corresponding independent variable is:

42. $y = x^2 + x + 8$

(a) $\frac{dy}{dx} = 2x+1$ (b) $\frac{dy}{dx} = x+1$

(c) $\frac{dy}{dx} = 2x-1$ (d) $\frac{dy}{dx} = x-1$

43. $s = 5t^3 - 3t^5$

(a) $\frac{ds}{dt} = 15t^2 - 15t^4$ (b) $\frac{ds}{dt} = 15t^4 - 15t^2$

(c) $\frac{ds}{dt} = 15t^4 - 15t^2$ (d) $\frac{ds}{dt} = 15t^2 - 15t^4$

44. $y = 5\sin x$

(a) $\frac{dy}{dx} = 3\cos x$ (b) $\frac{dy}{dx} = 5\cos x$

(c) $\frac{dy}{dx} = 5\sin x$ (d) $\frac{dy}{dx} = 3\sin x$

Direction (No. 45 to 48): First derivative and second derivative of given functions w.r.t. Corresponding independent variable is:

45. $y = 6x^2 - 10x - 5x^{-2}$

(a) $12x - 10 + 10x^{-3}, 12 - 30x^{-4}$

(b) $10x - 12 + 20x^{-3}, 15 - 30x^{-4}$

(c) $12x - 10 + 15x^{-3}, 12 - 30x^{-4}$

(d) $10x - 15 + 12x^{-3}, 12 - 30x^{-4}$

46. $r = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4}$

(a) $12\theta^{-2} - 12\theta^{-4} + 4\theta^{-5}, 24\theta^{-3} - 48\theta^{-5} - 20\theta^{-6}$

(b) $-12\theta^{-2} + 12\theta^{-4} - 4\theta^{-5}, 24\theta^{-3} - 48\theta^{-5} + 20\theta^{-6}$

(c) $-6\theta^{-2} + 12\theta^{-4} - 8\theta^{-5}, 12\theta^{-3} - 24\theta^{-5} + 10\theta^{-6}$

(d) $-8\theta^{-2} + 12\theta^{-4} - 6\theta^{-5}, 24\theta^{-3} - 24\theta^{-5} + 10\theta^{-6}$

47. $w = 3z^7 - 7z^3 + 21z^2$

(a) $21z^6 - 21z^2 - 42z, 126z^5 + 42z - 42$

(b) $14z^6 - 28z^2 + 22z, 120z^5 - 21z + 42$

(c) $28z^6 - 14z^2 + 42z, 122z^5 - 42z + 21$

(d) $21z^6 - 21z^2 + 42z, 126z^5 - 42z + 42$

48. $y = \sin x + \cos x$

(a) $\cos x - \cos x, -\sin x - \sin x$

(b) $\sin x - \sin x, -\sin x - \cos x$

(c) $\cos x - \sin x, -\sin x - \cos x$

(d) $\sin x + \cos x, -\cos x - \cos x$

Direction (No. 49 to 51): Derivative of given function w.r.t the independent variable x is:

49. $y = x\sin x$

(a) $\sin x + x\cos x$ (b) $\sin x - x\cos x$

(c) $\cos^2 x - x\sin^2 x$ (d) $\sin^2 x - x\cos^2 x$

50. $y = e^x \ln x$

(a) $e^x \ln x - \frac{e^x}{x}$ (b) $e^x \ln x - \frac{e^x}{x^2}$

(c) $e^x \ln x + \frac{e^x}{x^2}$ (d) $e^x \ln x + \frac{e^x}{x}$

51. $y = (x-1)(x^2+x+1)$

(a) $\frac{dy}{dx} = 3x$ (b) $\frac{dy}{dx} = 3x^2$

(c) $\frac{dy}{dx} = 2x^2$ (d) $\frac{dy}{dx} = 2x$

Direction (No. 52 to 54): Derivative of given function w.r.t the independent variable is

52. $y = \frac{\sin x}{\cos x}$

(a) $\sec^2 x$ (b) $\sec x$ (c) $\sec^2 2x$ (d) $\sec^2 2x$

53. $y = \frac{2x+5}{3x-2}$

(a) $y' = \frac{-19}{(3x-2)^2}$ (b) $y' = \frac{19}{(3x-2)^2}$

(c) $y' = \frac{19}{(3x-2)}$ (d) $y' = \frac{-19}{(3x+2)^2}$

54. $z = \frac{2x+1}{x^2-1}$

(a) $\frac{-2x^2-2x+2}{(x^2+1)^2}$ (b) $\frac{-2x^2-2x-2}{(x^2-1)^2}$

(c) $\frac{-2x^2-2x+2}{(x-1)^2}$ (d) $\frac{-2x^2-2x-2}{(x^2-1)}$

Direction (No. 55 to 59): $\frac{dy}{dx}$ for following functions is:

55. $y = (2x-1)^5$

(a) $10(2x+1)^5$ (b) $10(2x+1)^4$

(c) $10(2x-1)^5$ (d) $10(2x-1)^4$

56. $y = (4-3x)^9$

(a) $-8(4-3x)^9$ (b) $-27(4-3x)^4$

(c) $-27(4-3x)^8$ (d) $-27(4-3x)^3$

57. $y = \left(1-\frac{x}{7}\right)^{-7}$

(a) $\left(1-\frac{x}{7}\right)^{8}$ (b) $\left(1-\frac{x}{7}\right)^{-8}$ (c) $\left(1-\frac{x}{7}\right)^{-5}$ (d) $\left(1-\frac{x}{7}\right)^{-6}$

58. $y = \sin(x) + \ln(x^2) + e^{2x}$

(a) $\cos(x) + \frac{2}{x} - 2e^{2x}$ (b) $\cos(x) + \frac{2}{x} + 2e^{2x}$

(c) $\sin(x) + \frac{2}{x} + 2e^{2x}$ (d) $\sin(x) + \frac{2}{x} - 2e^{2x}$

59. $y = 2\sin(\omega x + \phi)$ where ω and ϕ constants

(a) $2\omega\cos(\omega x + \phi)$ (b) $2\omega\cos(\omega x - \phi)$

(c) $\omega\cos(\omega x + \phi)$ (d) $2\omega\text{cosec}(\omega x + \phi)$

DIFFERENTIATION AS A RATE MEASUREMENT

60. Suppose that the radius r and area $A = \pi r^2$ of a circle are differentiable functions of t. equation that relates dA/dt to dr/dt is

(a) $\frac{dA}{dt} = \pi r\frac{dr}{dt}$ (b) $\frac{dA}{dt} = \pi r^2\frac{dr}{dt}$

(c) $\frac{dA}{dt} = 2\pi r^2\frac{dr}{dt}$ (d) $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

61. $y = 2u^3$, $u = 8x - 1$. Find $\frac{dy}{dx}$

(a) $48(8x-1)^2$ (b) $48(8x+1)^2$

(c) $48(8x-1)$ (d) $48(8x+1)$

62. $y = \sin u$, $u = 3x + 1$. Find $\frac{dy}{dx}$

(a) $3\cos(3x-1)$ (b) $3\cos(3x+1)$

(c) $3\sin(3x-1)$ (d) $3\sin(3x+1)$

63. $y = \sin u$, $u = \cos x$. Find $\frac{dy}{dx}$.

(a) $-\cos u.\sin x$ (b) $-\cos u.\sqrt{1-u^2}$

(c) $-\sin x.\sqrt{1-y^2}$ (d) all of above

64. $y = 3t^2 - 1$, $x = t^2$. Find $\frac{dy}{dx}$.

(a) 3 (b) 2 (c) 1/3 (d) 1/2

MAXIMA & MINIMA

65. Maximum and minimum values of function $2x^3 - 15x^2 + 36x + 11$ is

(a) 39, 38 (b) 93, 83 (c) 45, 42 (d) 59, 58

66. Find out minimum/maximum value of $y = 1 - x^2$ also find out those points where value is minimum/maximum.

(a) max 2, $x = -1$ (b) max 1, $x = 0$

(c) min 1, $x = -1$ (d) min 2, $x = 0$

67. For $y = (x-2)^2$, what is the maximum/minimum value and the point at which y is maximum/minimum?

(a) max 2, $x = 0$ (b) max 0, $x = 0$

(c) min 1, $x = -1$ (d) min 0, $x = 2$

68. Particle's position as a function of time is given by $x = -t^2 + 4t + 4$ find the maximum value of position co-ordinate of particle.

(a) 2 (b) 4 (c) -8 (d) 8

69. Find out minimum/maximum value of $y = 2x^3 - 15$ also find out those points where value is minimum

(a) max $= 39$ at $x = 2$, min $= 39$ at $x = 2$

(b) max $= 39$ at $x = 3$, min $= 38$ at $x = 2$

(c) max $= 39$ at $x = 2$, min $= 38$ at $x = 3$

(d) max $= 39$ at $x = 2$, min $= 38$ at $x = 2$

70. Determine the position where potential energy minimum if $U(x) = 100 - 50x + 1000x^2$ J.

(a) 0.25×10^{-2} (b) 2.5×10^{-2}

(c) 2.5×10^{-1} (d) 250×10^{-2}

71. Find out minimum/maximum value of $y = 4x^2 - 2x +$ find out those points where value is minimum/m

(a) min $= \frac{11}{4}$, $x = \frac{1}{2}$ (b) max $= \frac{11}{4}$, $x = \frac{1}{4}$

(c) min $= \frac{11}{4}$, $x = \frac{1}{4}$ (d) max $= \frac{11}{4}$, $x = \frac{1}{2}$

72. $x + y = 10$ then find the maximum value of $f = xy$

(a) 25 (b) 30 (c) 15 (d) 35

73. If $\ell + r = 12$ here ℓ is length of cylinder and r is ra cylinder then find maximum value of volume of cyl

(a) 156π (b) 350π (c) 256π (d) 250π

74. If $\ell \times b = 32$ here ℓ is length of rectangle and b is wi rectangle then find minimum periphery of rectangle

(a) $8\sqrt{2}$ (b) $2\sqrt{16}$ (c) $32\sqrt{2}$ (d) $16\sqrt{2}$

INTEGRATION

75. $\int(2x)\,dx$ will be

(a) $x^2 + C$ (b) $2x + C$ (c) $2x^2 + C$ (d) $-x^2 +$

76. $\int(x^2)\,dx$ will be

(a) $x + C$ (b) $2x + C$ (c) $\frac{x^3}{3} + C$ (d) $\frac{x^2}{2}$

77. $\int(x^2 - 2x + 1)\,dx$ will be

(a) $\frac{x^3}{3} - x^2 - x + C$ (b) $\frac{x^3}{3} - x^2 + x + C$

(c) $\frac{x^3}{3} + x^2 - x + C$ (d) $\frac{x^3}{3} + x^2 + x + C$

78. $\int(-3x^{-4})dx$ will be

(a) $x^{-3} + C$ (b) $x^3 + C$

(c) $-3x^{-3} + C$ (d) $3x^{-3} + C$

79. $\int(x^{-4})dx$ will be

(a) $\frac{1}{3}x^{-3} + C$ (b) $\frac{1}{3}x^{-5} + C$

(c) $\frac{1}{2}x^{-2} + C$ (d) $-\frac{1}{3}x^{-3} + C$

80. $\int\left(\frac{5}{x^2}\right)dx$ will be

(a) $-\frac{5}{x}+C$ (b) $\frac{5}{x}+C$ (c) $\frac{x}{5}+C$ (d) $\frac{x}{5}+C$

81. $\int\left(\frac{3}{2}\sqrt{x}\right)dx$ will be

(a) $\sqrt{x^2}+C$
(b) $3\sqrt{x^3}+C$
(c) $2\sqrt{x^3}+C$
(d) $\sqrt{x^3}+C$

82. $\int\left(\frac{3}{2\sqrt{x}}\right)dx$ will be

(a) $2\sqrt{x^3}+C$
(b) $3\sqrt{x}+C$
(c) $\sqrt{x^3}+C$
(d) $\sqrt{x^4}+C$

83. $\int\left(\frac{4}{3}\sqrt[3]{x}\right)dx$ will be

(a) $x^{4/3}+C$ (b) $x^{3/4}+C$ (c) $x^{2/3}+C$ (d) $x^{1/3}+C$

84. $\int\left(\frac{1}{3\sqrt[3]{x}}\right)dx$ will be

(a) $\frac{x^{\frac{3}{4}}}{2}+C$
(b) $\frac{x^{\frac{2}{3}}}{3}+C$
(c) $x^{\frac{2}{3}}+C$
(d) $\frac{x^{\frac{2}{3}}}{2}+C$

85. $\int\left(\sqrt[3]{x}+\frac{1}{\sqrt[3]{x}}\right)dx$ will be

(a) $\frac{3x^{\frac{4}{2}}}{4}+\frac{3x^{\frac{2}{4}}}{2}+C$
(b) $\frac{3x^{\frac{4}{2}}}{4}+\frac{3x^{\frac{2}{1}}}{2}+C$
(c) $\frac{3x^{\frac{3}{4}}}{4}+\frac{3x^{\frac{3}{2}}}{2}+C$
(d) $\frac{3x^{\frac{4}{3}}}{4}+\frac{3x^{\frac{2}{3}}}{2}+C$

86. $\int\left(\frac{1}{2}x^{-1/2}\right)dx$ will be

(a) $x^{2/3}+C$
(b) $x^{1/2}+C$
(c) $x^{3/1}+C$
(d) $x^{2/1}+C$

87. $\int\left(-\frac{1}{2}x^{-3/2}\right)dx$ will be

(a) $x^{-1/2}+C$
(b) $x^{+1/2}+C$
(c) x^2+C
(d) $x^{-2/1}+C$

88. $\int(3\sin x)\,dx$ will be

(a) $+3\cos x+C$
(b) $+4\cos x+C$
(c) $-3\cos x+C$
(d) $-4\cos x+C$

89. $\int\left(\frac{1}{3x}\right)dx$ will be

(a) $\frac{1}{3}\ln x+C$
(b) $\frac{3}{1}\ln x+C$
(c) $\frac{2}{3}\ln x+C$
(d) $\frac{1}{2}\ln x+C$

90. $\int\sin 3x\,dx$, will be (use, $u = 3x$)

(a) $-\cos 3x+C$
(b) $\frac{1}{3}\cos 3x+C$
(c) $-\frac{1}{2}\cos 3x+C$
(d) $-\frac{1}{3}\cos 3x+C$

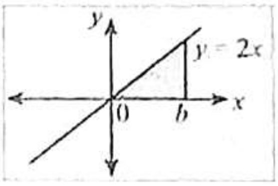
91. $\int_{2}^{1}5\,dx$ will be

(a) 15 (b) 16 (c) 17 (d) 18

92. $\int_{-4}^{-1}\frac{\pi}{2}d\theta$ will be

(a) $\frac{3\pi}{2}$ (b) $\frac{2\pi}{2}$ (c) 3π (d) 2π

93. Use a definite integral to find the area of the region between the given curve and the x-axis on the interval $[0, b]$ $y = 2x$



(a) b^2 (b) b^3 (c) $2b^2$ (d) $\frac{b^3}{3}$

94. $\int_{-2}^{+2}(t^2-1)\,dt$.

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$

95. A particle moves along x-axis with acceleration $a = 6(t-1)$, where t is in seconds. If the particle is initially at the origin and moves along positive x-axis with $v_0 = 2$ ms^{-1}, find average acceleration of particle from $t = 0$ to $t = 10$ sec.

(a) 30 (b) 24 (c) 15 (d) 20

96. A body is moving along x axis as $v = 2t + 3t^2 + 2$ here v is velocity and t is time in second then find average velocity when particle moves from $t = 0$ to $t = 5$ second.

(a) 25 (b) 40 (c) 32 (d) 30

97. Current is flowing in conductor as $i = 6t + 9t^2$ here t is time and i is current then find average current in conductor $t = 0$ to $t = 10$ sec.

(a) 50 A (b) 330 A (c) 200 A (d) 420 A

98. Find the area of the curve y = sin$x$ between 0 and π.

(a) 2 sq. unit
(b) 5 sq. unit
(c) 4 sq. unit
(d) 10 sq. unit

99. Find the area of the region bounded by the two parabolas $y = x^2$ and $y^2 = x$.

(a) 1/3 sq. units
(b) 1/5 sq. units
(c) 1/2 sq. units
(d) 1/4 sq. units

PW Challengers

INTEGER TYPE QUESTIONS

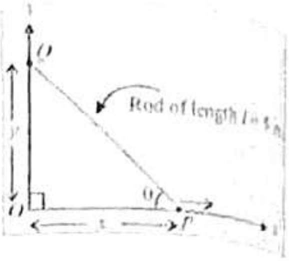
1. A person in a boat 'a' miles from the nearest point of the beach, wishes to reach as quickly as possible a point 'b' miles from that point along the shore. The ratio of his rate of walking to his rate of rowing is sec α. Then he should land at a distance of x mile from his destination to reach the destination in minimum time. Take $b = 10$ miles, $a = \frac{7}{\sqrt{3}}$ miles and $\alpha = 30°$. Find x.

2. Find $\left|\frac{dy}{dx}\right|$ at (0, 3) for given equation $y^2e^{2x} = 3y + x^2$

SUBJECTIVE TYPE QUESTIONS

3. Acceleration of a particle is defined as $a = \left(\frac{dv}{dt}\right)$. If $v^2 = \frac{2gl\cot\theta + 4gl(1-\operatorname{cosec}\theta)}{(1+2\cos^2\theta)}$ where, g and l are constants and $\frac{d\theta}{dt} = -\left(\frac{v}{l}\sin^2\theta\right)$ then find the magnitude of acceleration of particle at $\theta = 45°$.

4. A rod of length $l = 5$ m is slipping on two perpendicular surfaces as shown in figure.



Point P on the rod is moving along x axis with speed 5 m/s when point P is at a distance of 3 m from 'O'. Find the speed of point Q at given instant in m/s (given speed along x axis is $\frac{dx}{dt}$ and speed along y axis is $\frac{dy}{dt}$).

5. If $x + y + z = \pi$
Prove that
$$\cot\left(\frac{x}{2}\right) + \cot\left(\frac{y}{2}\right) + \cot\left(\frac{z}{2}\right) = \cot\left(\frac{x}{2}\right)\cdot\cot\left(\frac{y}{2}\right)\cdot\cot\left(\frac{z}{2}\right)$$

6. Find $\frac{dz}{dx}$ at $(x, y, z) = (1, 0, 1)$ for given equation $x^2\cos(y) = \sin(y^3 + 4z)$.

7. Evaluate $\int_0^{\pi/6}[2\cos(2x) - 3\sin(3x)]dx$

8. Evaluate $\int \tan^3\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)dx$

9. Evaluate $\int_0^{\infty}\frac{x\,dx}{(1+x)(1+x^2)}$

ANSWER KEY

CONCEPT APPLICATION

1. $\frac{\pi}{12}$
2. [150°]
3. (*i*) $-\frac{1}{2}$ (*ii*) $\frac{-1}{2}$ (*iii*) $-\frac{1}{2}$ (*iv*) $-\sqrt{3}$
4. (*i*) $\frac{5}{13}$ (*ii*) $\frac{12}{13}$
5. (*i*) 14, 2 (*ii*) 5, −5 (*iii*) 17, −9
6. (*i*) 0.33 (*ii*) 1 (*iii*) 0.158
7. (*i*) $\frac{7}{25}$ (*ii*) $\frac{24}{25}$
8. (*i*) $1, -\frac{5}{3}$ (*ii*) $\frac{-3 \pm \sqrt{57}}{4}$
9. (*i*) $\frac{2}{5}, \frac{9}{5}$ (*ii*) $\frac{3}{4}, 2$
10. [676]
11. $\frac{2}{3}$
12. [77]
13. (*i*) 7.21 (*ii*) 5.067 (*iii*) 1.02
14. 9.61m/s^2
15. [13]
16. (*i*) $\sqrt{10}$ (*ii*) $\sqrt{277}$
17. [2 or −6]
18. (A)-R, (B)-T, (C)-P, (D)-Q

19. (i) 1.204 (ii) 1.678 (iii) 1.806 (iv) 0.903 20. [6 m/s²] 21. $\frac{1}{2\sqrt{x}}-6x$ 22. $-2\left[\frac{1}{x^3}+1\right]$

23. $\frac{-3}{x}+\frac{10}{x^3}$ 24. $1-\frac{1}{6\sqrt{x}}$ 25. $\frac{7}{2}x^{5/2}$ 26. $3x^{-4}$ 27. [1] 28. $5x^4+3x^2+2x^{-1/2}$ 29. $20x^3+9x^{1/2}+9$

30. $2ax+b$ 31. $15x^4-3+1/x^2$ 32. $2t+5$ 33. $u+at$ 34. 30 cm² 35. $2\pi r$ 36. $4x+3$

37. $216x^7-144x^5+105x^4+135x^2-40$ 38. [6] 39. [6] 40. [2] 41. [61] 42. $\frac{-2}{(2x+1)^2}$

43. $\frac{1}{(4x+5)^2}$ 44. $\frac{2x-x^4}{(x^3+1)^2}$ 45. (i) 0 (ii) $\frac{4}{5}$ 46. (c) 47. $x-\frac{3}{4}x^4+C$ 48. $6x-\frac{4}{x}+C$

49. $\sin x-\cos x+C$ 50. $-\left(\frac{1}{2x^2}+\sin x\right)+C$ 51. $\frac{\sin 3x}{3}+C$ 52. $-\left[\frac{1}{x}+\frac{3}{2}\cos 2x\right]+C$ 53. $\frac{2a}{3}+2b$

54. $\frac{1}{4}\log_e 17$ 55. $-\frac{1}{3}\left[\cos\left(\frac{3\pi}{2}-1\right)-\cos(1)\right]$ 56. $\frac{1}{2}[\sin(2B-30)-\sin(2A-30)]$

57. $\frac{GMm}{R}$ 58. $kq_1q_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$ 59. $\frac{1}{2}M(v^2-u^2)$ 60. [∞] 61. [1] 62. [1] 63. [2]

64. $\frac{32}{3}$ units 65. $\frac{1}{6}$ 66. $\frac{4}{3}$ units

PRARAMBH (TOPICWISE)

1. (a) $\frac{8\pi}{9}$, (b) $\frac{3\pi}{4}$, (c) $\frac{5\pi}{12}$, (d) $\frac{13\pi}{36}$, (e) $\frac{5\pi}{4}$, (f) $\frac{25\pi}{18}$, (g) $\frac{31\pi}{18}$ 2. (a) 45°, (b) 630°, (c) 108°, (d) 120°, (e) 135°

3. (a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$, (b) $\frac{(\sqrt{3}-1)^2}{2}$, (c) $\frac{4}{5}$, (d) $\frac{3}{5}$, (e) $\frac{3}{4}$, (f) $\frac{4}{3}$, (g) 0 4. (a) $-\sqrt{2}$, (b) $\frac{\sqrt{6}}{\sqrt{3}+1}$, (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$, (d) $\frac{-\sqrt{3}}{2}$, (e) $\frac{-1}{2}$, (f) 1,

(g) $\frac{-1}{\sqrt{2}}$, (h) $\frac{-1}{\sqrt{2}}$, (i) 0 5. (a) $\frac{1}{2}$, (b) $\frac{1}{2\sqrt{2}}$, (c) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$, (d) $\frac{\sqrt{2}-1}{2\sqrt{2}}$ 6. (a) $-\frac{1}{\sqrt{2}}$, (b) $\frac{\sqrt{3}}{2}$, (c) $\frac{1}{\sqrt{2}}$, (d) $\frac{1}{\sqrt{3}}$

7. (c) 8. (b) 9. (b) 10. (c) 11. (a) 12. (a) 13. (b) 14. (a) 15. (c) 16. (a)
17. (c) 18. (a) 19. (c) 20. (d) 21. (a) 22. (c) 23. (c) 24. (d) 25. (b) 26. (c)
27. (d) 28. (a) 29. (b) 30. (b) 31. (a) 32. (b) 33. (a) 34. (c) 35. (a) 36. (a)
37. (c) 38. (a) 39. (d) 40. (b) 41. (b) 42. (c) 43. (d) 44. (b) 45. (a) 46. (b)
47. (d) 48. (c) 49. (a) 50. (d) 51. (b) 52. (a) 53. (a) 54. (b) 55. (b) 56. (d)
57. (b) 58. (b) 59. (a) 60. (d) 61. (a) 62. (b) 63. (a) 64. (a) 65. (a) 66. (b)
67. (d) 68. (d) 69. (c) 70. (b) 71. (c) 72. (a) 73. (c) 74. (d) 75. (a) 76. (c)
77. (b) 78. (a) 79. (d) 80. (a) 81. (d) 82. (b) 83. (a) 84. (d) 85. (d) 86. (b)
87. (a) 88. (c) 89. (a) 90. (d) 91. (a) 92. (a) 93. (a) 94. (b) 95. (b) 96. (c)
97. (b) 98. (a) 99. (a)

PW CHALLENGERS

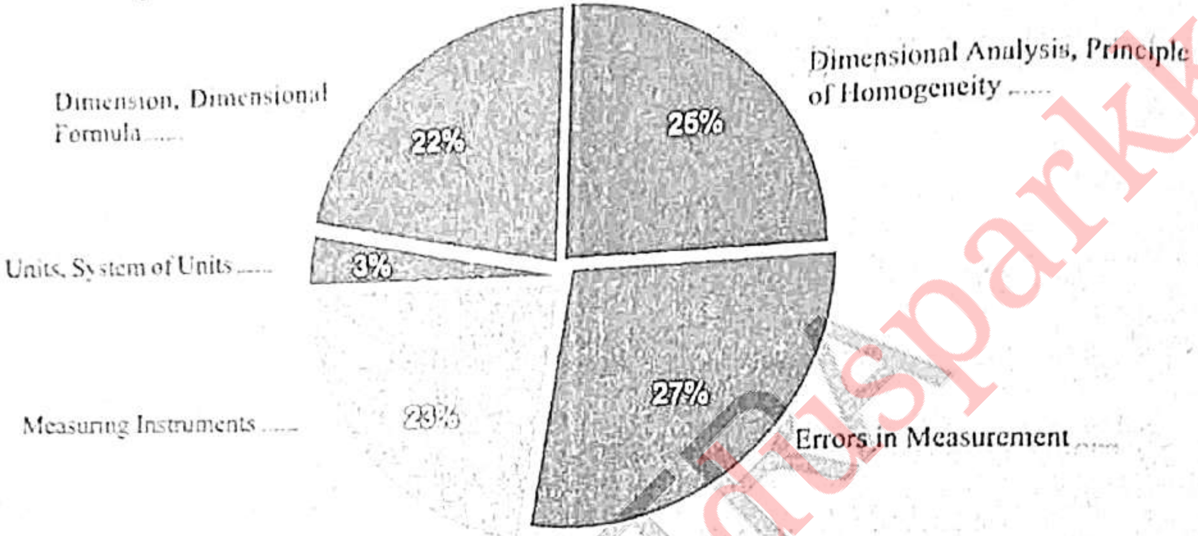
1. [3] 2. [6] 3. $\frac{g}{4}$ 4. 3.75 6. $\frac{1}{2\cos(4)}$ 7. $\left(\frac{\sqrt{3}}{2}-1\right)$

8. $\frac{1}{2\cos^4\left(\frac{x}{2}\right)}+\cos^2\left(\frac{x}{2}\right)-\frac{3}{\cos^2\left(\frac{x}{2}\right)}-6\ln\left(\cos\left(\frac{x}{2}\right)\right)+C$ 9. $\frac{\pi}{4}$

CHAPTER 2

Units and Measurements

Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

PHYSICAL QUANTITIES

All quantities that can be measured are called physical quantities. e.g. time, length, mass, force, work done, etc. In physics we study about physical quantities and their inter relationships.

Fundamental Quantities

A set of physical quantities which are completely independent of each other but all other physical quantities can be expressed in terms of these physical quantities is called set of Fundamental Quantities.

The Fundamental Quantities that are currently being accepted by the scientific community are mass, time, length, current, temperature, luminous intensity and amount of substance.

Derived Physical Quantities

The physical quantities that can be expressed in terms of fundamental physical quantities are called derived physical quantities. E.g. speed = distance/time.

MEASUREMENT

Measurement is the comparison of a quantity with a standard of the same physical quantity. In the past different countries followed different standards.

UNITS

All physical quantities are measured with respect to standard magnitude of the same physical quantity and these standards are called UNITS. e.g. second, meter, kilogram, etc.

Four basic properties of units are:

1. They must be well defined.
2. They should be easily available and reproducible.
3. They should be invariable e.g. step as a unit of length is not invariable.
4. They should be accepted to all.

SYSTEM OF UNITS

1. **FPS or British Engineering system:** In this system length, mass and time are taken as fundamental quantities and their base units are foot (ft), pound (lb) and second (s) respectively.
2. **CGS or Gaussian system:** In this system the fundamental quantities are length, mass and time and their respective units are centimetre (cm), gram (g) and second (s).
3. **MKS system:** In this system also the fundamental quantities are length, mass and time but their fundamental units are meter (m), kilogram (kg) and second (s) respectively.

Table: Units of some physical quantities in different systems

Type of physical Quantity	Physical Quantity	System		
		CGS	MKS	FPS
Fundamental	Length	cm	m	ft
	Mass	g	kg	lb
	Time	s	s	s

4. **International system (SI) of units:** This system is modification over the MKS system. Besides the three base units of MKS system four other fundamental and two supplementary units are also included in this system.

Table: SI base quantities and their units

S.No.	Physical quantity	unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3	Time	second	s
4.	Temperature	kelvin	K
5.	Electric current	ampere	A
6.	Luminous Intensity	candela	cd
7.	Amount of substance	mole	mol

Supplementary Units

In SI, two supplementary units are also defined viz.

(i) radian (rad) for plane angle and

(ii) steradian (sr) for solid angle.

Quantity	SI Unit	
	Name	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

(i) **Radian:** Radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

(ii) **Steradian:** Steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere which is equal in area to the square of the radius of the sphere.

Physical Quantity (SI Unit)	Definition
Length (m)	The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299792458 when expressed in the unit m s^{-1}, where the second is defined in terms of the caesium frequency Δv_{Cs}.
Mass (kg)	The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be 6.62607015×10^{-34} when expressed in the unit J s, which is equal to kg m^2 s^{-1}, where the metre and the second are defined in terms of c and Δv_{Cs}.
Time (s)	The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency Δv_{Cs}, the unperturbed ground state hyperfine transition frequency of the caesium-133 atom, to be 9192631770 when expressed in the unit Hz, which is equal to s^{-1}.
Electric Current (A)	The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602176634\times10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of Δv_{Cs}.
Thermodynamic Temperature (K)	The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be 1.380649×10^{-23} when expressed in the unit J K^{-1}, which is equal to kg m^2 s^{-2} k^{-1}, where the kilogram, metre and second are defined in terms of h, c and Δv_{Cs}.
Amount of substance (mole)	The mole, symbol mol, is the SI unit of amount of substance. One mole substance contains exactly $6.02214076 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number. The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.
Luminous Intensity (cd)	The candela, symbol cd, is the SI unit of luminous intensity in given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd}, to be 683 when expressed in the unit lm W^{-1}, which is equal to cd sr W^{-1}, or cd sr kg^{-1} $m^{-2}s^{3}$, where the kilogram, metre and second are defined in terms of h, c and Δv_{Cs}.

DIMENSIONS OF PHYSICAL QUANTITIES

All the physical quantities of interest can be derived from the base quantities. The power (exponent) of base quantity that enters into the expression of a physical quantity, is called the dimension of the quantity in that base. To make it clear, consider the physical quantity force.

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$= \text{Mass} \times \frac{\text{Length / Time}}{\text{Time}} = \text{Mass} \times \text{Length} \times (\text{Time})^{-2}$$

So the dimensions of force are 1 in mass, 1 in length and –2 in time. Thus

$$[\text{Force}] = MLT^{-2}$$

Similarly, energy has dimensional formula given by

$$[\text{Energy}] = ML^2T^{-2}$$

i.e. energy has dimensions 1 in mass, 2 in length and –2 in time. Such an expression for a physical quantity in terms of base quantities is called dimensional formula

Physical quantity can be further of four types:

1. Dimensionless constant i.e. 1, 2, 3, π etc.
2. Dimensionless variable i.e. angle θ etc.
3. Dimensional constant i.e. G, h etc.
4. Dimensional variable i.e. F, v, etc.

The variables inside any mathematical function are always dimensionless. Eg., in log x, sin x, e^x etc., x is dimensionless.

ADVANCED LEARNING

Table: Units and dimensions of some physical quantities

Quantity	SI Unit	Dimension
Density	kg m^{-3}	ML^{-3}
Force	newton (N)	ML/T^2
Work	joule (J)(= N m)	ML^2/T^2
Energy	joule(J)	ML^2/T^2
Power	Watt (W) (= J/s)	ML^2/T^3
Momentum	kg m/s	ML/T
Gravitational constant	N m^2/kg^2	L^3/MT^2
Angular velocity	radian/s	T^{-1}
Angular acceleration	radian/s^2	T^{-2}
Angular momentum	kg m^2/s	ML^2/T
Moment of inertia	kg m^2	ML^2
Torque	N m	ML^2/T^2
Angular frequency	radian/s	T^{-1}
Frequency	hertz (Hz)	T^{-1}
Period	s	T
Surface Tension	N/m	M/T^2
Coefficient of viscosity	N s/m^2	M/LT
Wavelength	m	L
Intensity of wave	W/m^2	M/T^3
Temperature	Kelvin (K)	K
Specific heat capacity	J/kg K	L^2/T^{2}K
Stefan's constant	W/m^2 K^4	M/T^{3}K^4
Heat	J	ML^2/T^2
Thermal conductivity	W/m-K	ML/T^{3}K
Current density	A/m^2	I/L^2
Electrical conductivity	1/Ω m(= mho/m)	I^{2}T^3/ML^3
Electric dipole moment	C m	LIT
Electric field	V/m (=N/C)	ML/IT^3
Potential (voltage)	Volt (V) (=J/C)	ML^2/IT^3
Electric flux	V m	ML^3/IT^3
Capacitance	Farad (F)	I^{2}T^4/ML^2
Electromotive force	Volt (V)	ML^2/IT^3
Resistance	ohm (Ω)	ML^2/I^{2}T^3
Permittivity of space	C^2/N m^2	I^{2}T^4/ML^3
Permeability of space	N/A^2	ML/I^{2}T^2
Magnetic field	Tesla (T) (= Wb/m^2)	M/IT^2
Magnetic flux	Weber (Wb)	ML^2/IT^2
Magnetic dipole moment	A m^2	IL^2
Inductance	Henry (H)	ML^2/I^{2}T^2

Dimensional Equation

Whenever the dimension of a physical quantity is equated with its dimensional formula, we get a dimensional equation.

Principle of Homogeneity

The magnitude of a physical quantity may be added or subtracted from each other only if they have the same dimension. Also the dimension on both sides of an equation must be same. This is called as principle of homogeneity.

Train Your Brain

Example 1: The distance covered by a particle in time t is given by $x = a + bt + ct^2 + dt^3$; find the dimensions of a, b, c and d.

Sol. The equation contains five terms. All of them should have the same dimensions. Since $[x]$ = length, each of the remaining four must have the dimension of length.

Thus, $[a]$ = length = L

$[bt] = \text{L}$, or $[b] = [\text{LT}^{-1}]$

$[ct^2] = \text{L}$, or $[c] = [\text{LT}^{-2}]$

and $[dt^3] = \text{L}$ or $[d] = [\text{LT}^{-3}]$

Example 2: Calculate the dimensional formula of energy from the equation $E = \frac{1}{2}mv^2$.

Sol. Dimensionally, E = mass × (velocity)^2, since $\frac{1}{2}$ is a number and has no dimension.

$$\text{or, } [E] = [\text{M}] \times \left[\frac{\text{L}}{\text{T}}\right]^2 = [\text{ML}^2\text{T}^{-2}].$$

Example 3: Kinetic energy of a particle moving along elliptical trajectory is given by $K = \alpha s^2$ where s is the distance travelled by the particle. Determine dimensions of α.

Sol. $K = \alpha s^2 \Rightarrow \alpha = \frac{K}{s^2}$

$$[\alpha] = \left[\frac{(\text{ML}^2\text{T}^{-2})}{(\text{L}^2)}\right]$$

$$[\alpha] = [\text{M}^1\text{L}^0\text{T}^{-2}]$$

$$[\alpha] = [\text{M T}^{-2}]$$

Example 4: The position of a particle at time t, is given by the equation, $x(t) = \frac{v_0}{\alpha}(1 - e^{-\alpha t})$, where v_0 is a constant and $\alpha > 0$. The dimensions of v_0 and α are respectively.

(*a*) $[\text{M}^0\text{L}^1\text{T}^0]$ and $[\text{T}^{-1}]$
(*b*) $[\text{M}^0\text{L}^1\text{T}^{-1}]$ and $[\text{T}]$
(*c*) $[\text{M}^0\text{L}^1\text{T}^{-1}]$ and $[\text{T}^{-1}]$
(*d*) $[\text{M}^1\text{L}^1\text{T}^{-1}]$ and $[\text{LT}^{-2}]$

Sol. $[\alpha][t] = [M^0L^0T^0]$ and $[v_0] = [x][\alpha]$

$[\alpha] = [M^0L^0T^{-1}] \quad = [M^0L^1T^{-1}]$

Example 5: If $\frac{v + A\sqrt{s}}{Bt^2} = F$; find the dimension of A and B where F = force, v = velocity, s = displacement and t = time

Sol. Using principle of homogeneity, $[A\sqrt{x}] = [v]$

$[A][L^{\frac{1}{2}}] = [LT^{-1}]$

$[A] = [L^{\frac{1}{2}}T^{-1}]$

Also, we can write, $B = \frac{v + A\sqrt{x}}{Ft^2}$

$[B] = \frac{[LT^{-1}]}{[(MLT^{-2})(T^2)]} \Rightarrow [B] = [M^{-1}LT^{-1}]$

Concept Application

1. If $\frac{v}{A} - Bx = at$, find the dimension of A and B where v = velocity, a = acceleration, t = time.
2. If $\frac{v}{t^2} = A\sqrt{x} - B$ where v is velocity and x is displacement. Find the dimension of A and B.
3. If $A\sin\left(\frac{B}{\sqrt{t}} + C\right)$ is equation of displacement of a body as a function of time t. Find dimensions of A, B, C.
4. If displacement, $x = \frac{A}{t^2} - Bt$ where t is time. Find dimension of $[A \times B]$.

DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

Checking the Dimensional Consistency of Equations

It is based on principle of homogeneity, which states that a given physical relation is dimensionally correct if the dimensions of the various terms on either side of the relation are the same.

Remark:

- Powers are dimensionless.
- The input and output of all trigonometric and logarithmic functions must be dimensionless. Eg., $\sin\theta$, e^{θ}, $\cos\theta$, $\log\theta$ give dimensionless value and in above expression θ is dimensionless.
- We can add or subtract quantity having same dimensions only.

Train Your Brain

Example 6: Check the accuracy of the relation

$$T = 2\pi\sqrt{\frac{L}{g}}$$

for a simple pendulum using dimensional analysis.

Sol. From principle of homogeneity of dimension, the dimensions of LHS = The dimension of RHS

Now, $[T] = [M^0L^0T^1]$

The dimensions of

$$\text{RHS} = \left(\frac{\text{dimension of length}}{\text{dimension of acceleration}}\right)^{1/2}$$

($\because$ 2π is a dimensionless constant)

$$[\text{RHS}] = \left[\frac{L}{LT^{-2}}\right]^{1/2} = \left[T^2\right]^{1/2} = [T] = [M^0L^0T^1] = [\text{LHS}]$$

So, equation is correct.

Example 7: Check whether the given relation $\frac{Fv^2}{t} = \text{KE}$ (Kinetic Energy) is dimensionally correct? Here F = force, v = velocity and t = time?

Sol. $$[\text{LHS}] = \left[\frac{Fv^2}{t}\right] = \frac{\left[MLT^{-2}\right]\times\left[L^2T^{-2}\right]}{[T]} = \left[ML^3T^{-5}\right]$$

$$[\text{RHS}] = [\text{KE}] = \left[\frac{1}{2}mv^2\right] = \left[M\times L^2T^{-2}\right] = \left[ML^2T^{-2}\right]$$

$\therefore$ [LHS] $\neq$ [RHS], so the given relation is incorrect dimensionally.

Concept Application

5. Consider the following equation: $a = qvbt^2 + c\left(\frac{t}{x}\right)^2$,

   where a, b, c are constants (not necessarily dimensionless) and q, v, x and t represent charge, velocity, distance and time respectively. For the equation to be dimensionally correct,

   (*a*) $\left[\frac{a}{c}\right]^{\frac{1}{2}} = \left[LT^{-1}\right]$ (*b*) $\left[\frac{a}{c}\right]^{\frac{1}{2}} = \left[L^{-1}T^{-1}\right]$

   (*c*) $\left[\frac{b}{c}\right] = \left[L^{-3}A^{-1}\right]$ (*d*) $\left[\frac{a}{b}\right] = [LTA]$

6. Consider the statements below.

   (i) A dimensionally consistent equation is a physically correct equation.

   (ii) A dimensionally consistent equation may or may not be correct

   (iii) A dimensionally inconsistent equation is an incorrect equation.

   (iv) A dimensionally inconsistent equation may or may not be incorrect.

   The correct statement(s) is/are

   (*a*) (i), (iii) and (iv) (*b*) (ii) and (iv)

   (*c*) only (iii) (*d*) (ii) and (iii)

Deducing Relation Among the Physical Quantities

If we know the various factors on which a physical quantity depends, then we can find a relation among different factors by using principle of homogeneity.

Train Your Brain

Example 8: Find an expression for the time period T of a simple pendulum. The time period T may depend upon (i) mass m of the bob of the pendulum, (ii) length ℓ of pendulum, (iii) acceleration due to gravity g at the place where the pendulum is suspended.

Sol. Let (i) $T \propto m^a$ (ii) $T \propto \ell^b$ (iii) $T \propto g^c$

Combining all the three factors, we get

$T \propto m^a \ell^b g^c$ or $T = Km^a \ell^b g^c$ ...(i)

where K is a dimensionless constant of proportionality.

Writing down the dimensions on either side of equation (i), we get

$[T] = [M^a][L^b][LT^{-2}]^c = [M^a L^{b+c} T^{-2c}]$

Comparing dimensions, $a = 0$, $b + c = 0$, $-2c = 1$

$\therefore a = 0, c = -1/2, b = 1/2$

From equation (i) $T = Km^0 \ell^{1/2} g^{-1/2}$

or $T = K\left(\frac{\ell}{g}\right)^{1/2} = K\sqrt{\frac{\ell}{g}}$

The value of K, as found by experiment or mathematical investigation, comes out to be 2π.

$\therefore T = 2\pi\sqrt{\frac{\ell}{g}}$

Example 9: When a solid sphere moves through a liquid, the liquid opposes the motion with a force F. The magnitude of F depends on the coefficient of viscosity η of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.

Sol. Suppose the formula is $F = k\,\eta^a r^b v^c$

Then, $[MLT^{-2}] = [ML^{-1}T^{-1}]^a [L]^b \left[\frac{L}{T}\right]^c$

$= [M^a L^{-a+b+c} T^{-a-c}]$

Equating the exponents of M, L and T from both sides,

$a = 1$

$-a + b + c = 1$

$-a - c = -2$

Solving these, $a = 1$, $b = 1$ and $c = 1$

Thus, the formula for F is $F = k\eta rv$.

Example 10: If P is the pressure of a gas and ρ is its density, then find the dimension of velocity in terms of P and ρ.

(a) $P^{1/2}\rho^{-1/2}$ (b) $P^{1/2}\rho^{1/2}$

(c) $P^{-1/2}\rho^{1/2}$ (d) $P^{-1/2}\rho^{-1/2}$

Sol. Method - I

$[P] = [ML^{-1}T^{-2}]$ ...(i)

$[\rho] = [ML^{-3}]$ ...(ii)

Dividing eq. (i) by (ii)

$[P\rho^{-1}] = [L^2T^{-2}]$

$\Rightarrow [LT^{-1}] = [P^{1/2}\rho^{-1/2}]$

$\Rightarrow [v] = [P^{1/2}\rho^{-1/2}]$

Method - II

$v \propto P^a \rho^b$

$v = kP^a \rho^b$

$[LT^{-1}] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b$

$\Rightarrow a = \frac{1}{2}, b = -\frac{1}{2}$ (Equating dimensions)

$\Rightarrow [v] = [P^{1/2}\rho^{-1/2}]$

Example 11: Find relationship between speed (v) of sound in a medium, the elastic constant (E) and the density of the medium (ρ).

Sol. Let the speed depends upon elastic constant and density according to the relation

$v \propto E^a \rho^b$

or $v = KE^a\rho^b$ ...(i)

where K is a dimensionless constant of proportionality.

Considering dimensions of the quantities

$[v] = M^0 L T^{-1}$

$[E] = \frac{[\text{stress}]}{[\text{strain}]} = \frac{[\text{force}]/[\text{area}]}{[\Delta\ell]/[\ell]} = \frac{[M^1L^1T^{-2}]/[L^2]}{[L^1]/[L^1]} = [M^1L^{-1}T^{-2}]$

$\therefore [E^a] = [M^a L^{-a} T^{-2a}]$

$[\rho] = [\text{mass}]/[\text{volume}] = [M]/[L^3] = [M^1L^{-3}T^0]$

$\therefore [\rho^b] = [M^b L^{-3b} T^0]$

Equating the dimensions of the LHS and RHS quantities of equation (i), we get

$[M^0 L^1 T^{-1}] = [M^a L^{-a} T^{-2a}] \times [M^b L^{-3b} T^0]$

or $[M^0 L^1 T^{-1}] = [M^{a+b} L^{-a-3b} T^{-2a}]$

Comparing the individual dimensions of M, L and T

$a + b = 0,$ ...(ii)

$-a - 3b = 1,$ and ...(iii)

$-2a = -1$ ...(iv)

Solving we get

$a = \frac{1}{2}, b = -\frac{1}{2}$

Therefore the required relation is

$v = K\sqrt{\frac{E}{\rho}}$.

Example 12: Pressure (P) acting due to a fluid kept in a container depends on, weight of liquid (w), Area of cross-section of container (A) and density of fluid (ρ). Establish a formula of pressure (P).

Sol. According to the question

$P \propto w^x A^y \rho^z$

$P = K[w^x A^y \rho^z]$

Now writing the dimension of each quantity on either side.

$[ML^{-1}T^{-2}] = K[MLT^{-2}]^x[L^2]^y[ML^{-3}]^z$

$[ML^{-1}T^{-2}] = K[M]^{x+z}[L]^{x+2y-3z}[T]^{-2x}$

Now comparing the powers

For M, $1 = x + z$ ...(i)

L, $-1 = x + 2y - 3z$ ...(ii)

T, $-2 = -2x$ ...(iii)

$\Rightarrow$ Solving we get, $x = 1, y = -1, z = 0$

$\therefore P = KwA^{-1}\rho^0$

Concept Application

7. If c is the velocity of light, h is Planck's constant and G is Gravitational constant are taken as fundamental quantities, then the dimensional formula of mass is

8. Taking frequency f, velocity (v) and Density (ρ) to be the fundamental quantities then the dimensional formula for momentum will be

   (*a*) $(\rho v^4 f^{-3})$ (*b*) $(\rho v^3 f^{-1})$

   (*c*) $(\rho v f^2)$ (*d*) $(\rho^2 v^2 f^2)$

9. If momentum (P), mass (M) and time (T) are chosen as fundamental quantities the dimensional formula for length is ________.

   (*a*) $[P^1T^1M^1]$ (*b*) $[P^1T^1M^2]$

   (*c*) $[P^1T^1M^{-1}]$ (*d*) $[P^2T^2M^1]$

10. For the equation $F = A^a v^b d^c$ where F is force, A is area, v is velocity and d is density, with the dimensional analysis gives the following values for exponents.

    (*a*) $a = 1, b = 2, c = 1$

    (*b*) $a = 2, b = 1, c = 1$

    (*c*) $a = 1, b = 1, c = 2$

    (*d*) $a = 0, b = 1, c = 1$

Conversion From One System of Units to Other

It is based on the fact that,

Numerical value × unit = constant

So on changing unit, numerical value will also change. If n_1 and n_2 are the numerical values of a given physical quantity and u_1 and u_2 be the units respectively in two different systems of units, then

$$n_1u_1 = n_2u_2$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

LIMITATIONS OF DIMENSIONAL ANALYSIS

(*i*) It supplies no information about dimensionless constants and the nature (vector and scalar) of physical quantities

(*ii*) This method is applicable only if relation is of product type. It fails in the case of exponential and trigonometric relations.

(*iii*) It does not predict numerical correctness of formula.

Train Your Brain

Example 13: Convert 1 newton (SI unit of force) into dyne (CGS unit of force)

Sol. The dimensional equation of force is $[F] = [M^1L^1T^{-2}]$

Therefore if n_1, u_1 and n_2, u_2 corresponds to SI and CGS numerical value and unit respectively, then

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 1\left[\frac{kg}{g}\right]\left[\frac{m}{cm}\right]\left[\frac{s}{s}\right]^{-2} = 1\times(1000)(100)(1) = 10^5$$

Example 14: A calorie is a unit of heat or energy and it equals about 4.2 J, where 1 J = 1 kg m²/s². Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β metre, the unit of time is γ second. Show that a calorie has a magnitude $4.2\ \alpha^{-1}\beta^{-2}\gamma^2$ in terms of the new units.

Sol. 1 cal = 4.2 kg m²s⁻²

SI	New system
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1$ kg	$M_2 = \alpha$ kg
$L_1 = 1$ m	$L_2 = \beta$ metre
$T_1 = 1$ s	$T_2 = \gamma$ second

Dimensional formula of energy is $[ML^2T^{-2}]$

Comparing with $[M^aL^bT^c]$.

We find that $a = 1, b = 2, c = -2$

$$\text{Now, } n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 4.2\left[\frac{1\ kg}{\alpha\ kg}\right]^1\left[\frac{1\ m}{\beta\ m}\right]^2\left[\frac{1\ s}{\gamma\ s}\right]^{-2} = 4.2\ \alpha^{-1}\beta^{-2}\gamma^2$$

Example 15: Young's modulus of steel is 19 × 10¹⁰ N/m². Express it in dyne/cm². Here dyne is the CGS unit of force.

Sol. The unit of Young's modulus is N/m².

This suggest that it has dimensions of $\frac{\text{Force}}{(\text{Distance})^2}$

$$\text{Thus, } [Y] = \left[\frac{F}{L^2}\right] = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}].$$

N/m² is in SI units, so, $1\ N/m^2 = (1\ kg)(1\ m)^{-1}(1\ s)^{-2}$

and $1\ dyne/cm^2 = (1\ g)(1\ cm)^{-1}(1\ s)^{-2}$ so, $\frac{1\ N/m^2}{1\ dyne/cm^2}$

$$= \left(\frac{1\ kg}{1\ g}\right)\left(\frac{1\ m}{1\ cm}\right)^{-1}\left(\frac{1\ s}{1\ s}\right)^{-2} = 1000 \times \frac{1}{100} \times 1 = 10$$

or, $1\ N/m^2 = 10\ dyne/cm^2$

or, $19 \times 10^{10}\ N/m^2 = 19 \times 10^{11}\ dyne/cm^2$.

Example 16: The dimensional formula for viscosity of fluids is $[\eta] = [M^1L^{-1}T^{-1}]$. Find how many poise (CGS unit of viscosity) is equal to 1 poiseuille (SI unit of viscosity).

Sol. $[\eta] = [M^1L^{-1}T^{-1}]$

1 CGS units = $g\ cm^{-1}\ s^{-1}$

1 SI units = $kg\ m^{-1}\ s^{-1}$

$= (1000\ g)(100\ cm)^{-1}\ s^{-1} = 10\ g\ cm^{-1}\ s^{-1}$

Thus, 1 Poiseuilli = 10 poise

Concept Application

11. If minute is the unit of time, 10 ms^{-2} is the unit of acceleration and 100 kg is the unit of mass, the new unit of work in joule is

(*a*) 10^5 (*b*) 10^6

(*c*) 6×10^9 (*d*) 36×10^6

12. The magnitude of force is 100 N. What will be its value if the units of mass and time are doubled and that of length is halved?

(*a*) 25 (*b*) 100

(*c*) 200 (*d*) 400

13. The value of universal gravitational constant G in CGS system is $6.67 \times 10^{-8}\ dyne\ cm^2\ g^{-2}$. Its value in SI system is

(*a*) $6.67 \times 10^{-11} Nm^2\ kg^{-2}$ (*b*) $6.67 \times 10^{-5} Nm^2\ kg^{-2}$

(*c*) $6.67 \times 10^{-10} Nm^2\ kg^{-2}$ (*d*) $6.67 \times 10^{-9} Nm^2\ kg^{-2}$

14. Which equation cannot be derived using dimensional analysis from the following? (k is a dimensionless constant and x does not necessarily represent distance. Remaining variables follow standard meaning)

(*a*) $x = 2at^2$ (*b*) $x = k\sin(\omega t)$

(*c*) $x = \frac{r^2\rho g}{\eta}$ (*d*) All of these

MEASUREMENT OF LENGTH

You are already familiar with some direct methods for the measurement of length. For example, a metre scale is used for lengths from 10^{-3} m to 10^2 m. A vernier callipers is used for lengths to an accuracy of 10^{-4} m. A screw gauge and a spherometer can be used to measure lengths as less as 10^{-5} m. To measure lengths beyond these ranges, we make use of some special indirect methods.

Range of Lengths

The sizes of the objects we come across in the universe vary over a very wide range. These may vary from the size of the order of 10^{-15} m of the proton to the size of the order of 10^{26} m of the extent of the observable universe.

We also use certain special length units for short and large lengths. These are

1 fermi = 1 f = 10^{-15} m

1 angstrom = 1 Å = 10^{-10} m (It is used mainly in measuring wavelength of light)

1 astronomical unit = 1 AU (average distance of the Sun from the Earth) = 1.496×10^{11} m

1 light year = 1 ly = 9.46×10^{15} m (distance that light travels with velocity of $3 \times 10^8\ m\ s^{-1}$ in 1 year)

1 parsec = 3.08×10^{16} m (Parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second)

MEASUREMENT OF LARGE DISTANCES

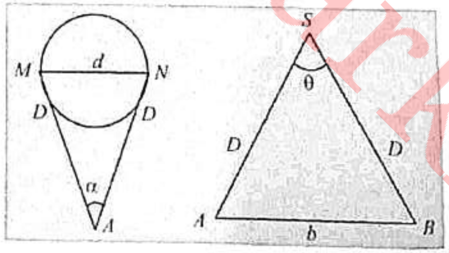
Parallax Method

Large distances such as the distance of a planet or a star from the earth cannot be measured directly with a metre scale. An important method in such cases is the parallax method. When you hold a pencil in front of you against some specific point on the background (a wall) and look at the pencil first through your left eye A (closing the right eye) and then look at the pencil through your right eye B (closing the left eye), you would notice that the position of the pencil seems to change with respect to the point on the wall. This is called parallax.

The distance between the two points of observation is called the basis. In this example, the basis is the distance between the eyes. To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions (observatories) A and B on the Earth, separated by distance $AB = b$ at the same time as shown in figure. We measure the angle between the two directions along which the planet is viewed at these two points. The ΔASB in figure represented by symbol θ is called the parallax angle or parallactic angle.

As the planet is very far away, $\frac{b}{D} << 1$ and therefore, θ is very small. Then we approximately take AB as an arc of length b of a circle with centre at S and the distance D as the radius $AS = BS$ so that $AB = b = D\theta$ where θ is in radians.

$$D = \frac{b}{\theta} \qquad ...(i)$$



Having determined D, we can employ a similar method to determine the size or angular diameter of the planet. If d is the diameter of the planet and α the angular size of the planet (the angle subtended by d at the earth), we have

$$\alpha = d/D \qquad \text{(ii)}$$

The angle α can be measured from the same location on the earth. It is the angle between the two directions when two diametrically opposite points of the planet are viewed through the telescope. Since D is known, the diameter d of the planet can be determined using equation (ii).

ESTIMATION OF VERY SMALL DISTANCES

Size of a Molecule

To measure a very small size, like that of a molecule (10^{-8} m to 10^{-10} m), we have to adopt special methods. We cannot use a screw gauge or similar instruments. Even a microscope has certain limitations. An optical microscope uses visible light to 'look' at the system under investigation. As light has wave like features, the resolution to which an optical microscope can be used is the wavelength of light.

For visible light the range of wavelengths is from about 4000 Å to 7000 Å (1 angstrom = 1 Å = 10^{-10} m). Hence an optical microscope cannot resolve particles with sizes smaller than this. Instead of visible light, we can use an electron beam. Electron beams can be focused by properly designed electric and magnetic fields. The resolution of such an electron microscope is limited finally by the fact that electrons can also behave as waves.

The wavelength of an electron can be as small as a fraction of an angstrom. Such electron microscopes with a resolution of 0.6 Å have been built. They can almost resolve atoms and molecules in a material. In recent times, tunneling microscopy has been developed in which again the limit of resolution is better than an angstrom. It is possible to estimate the sizes of molecules.

MEASUREMENT OF MASS

Mass is a basic property of matter. It does not depend on the temperature, pressure or location of the object in space. The SI unit of mass is kilogram (kg). It is defined by taking the fixed numerical value of the Plank Constant h to be $6.62607015 \times 10^{-34}$ when expressed in the unit of Js which is equal to $kg\,m^2s^{-1}$, where the metre and the second are defined is terms of C and $\Delta\nu\,cs$.

While dealing with atoms and molecules, the kilogram is an inconvenient unit. In this case, there is an important standard unit of mass, called the **unified atomic mass unit** (u), which has been established for expressing the mass of atoms as

1 unified atomic mass unit = 1u

$= \left(\frac{1}{12}\right)$ of the mass of an atom of carbon-12 isotope $\left({}^{12}_{6}C\right)$

including the mass of electrons

$= 1.66 \times 10^{-27}$ kg

Mass of commonly available objects can be determined by a common balance like the one used in a grocery shop. Large masses in the universe like planets, stars, etc., based on Newton's law of gravitation can be measured by using gravitational method (See Chapter 8). For measurement of small masses of atomic/sub-atomic particles etc., we make use of mass spectrograph in which radius of the trajectory is proportional to the mass of a charged particle moving in uniform electric and magnetic field.

Range of Masses

The masses of the objects, we come across in the universe, vary over a very wide range. These may vary from tiny mass of the order of 10^{-30} kg of an electron to the huge mass of about 10^{55} kg of the known universe. Table gives the range and order of the typical masses of various objects.

Table: Range and order of lengths

Object	Length (m)
Size of a proton	10^{-15}
Size of atomic nucleus	10^{-14}
Size of hydrogen atom	10^{-10}
Length of typical virus	10^{-8}
Wavelength of light	10^{-7}
Size of red blood corpuscle	10^{-5}
Thickness of a paper	10^{-4}
Height of the Mount Everest above sea level	10^{4}
Radius of the Earth	10^{7}
Distance of moon from the Earth	10^{9}
Distance of the Sun from the Earth	10^{11}
Distance of Pluto from the Sun	10^{13}
Size of our galaxy	10^{21}
Distance to Andromeda galaxy	10^{22}
Distance to the boundary of observable universe	10^{26}

Table: Range and order of masses

Object	Mass(kg)
Electron	10^{-30}
Proton	10^{-27}
Uranium atom	10^{-25}
Red blood cell	10^{-13}
Dust particle	10^{-9}
Rain drop	10^{-6}
Mosquito	10^{-5}
Grape	10^{-3}
Human	10^{2}
Automobile	10^{3}
Boeing 747 aircraft	10^{8}
Moon	10^{23}
Earth	10^{25}
Sun	10^{30}
Milky way galaxy	10^{41}
Observable Universe	10^{55}

MEASUREMENT OF TIME

To measure any time interval we need a clock. We now use an **atomic standard of time**, which is based on the periodic vibrations produced in a cesium atom. This is the basis of the **caesium clock**, sometimes called **atomic clock**, used in the national standards. Such standards are available in many laboratories. In the caesium atomic clock, the second is taken as the time needed for 9,192,631,770 vibrations of the radiation corresponding to the transition between the two hyperfine levels of the ground state of caesium-133 atom. The vibrations of the caesium atom regulate the rate of this caesium atomic clock just as the vibrations of a balance wheel regulate an ordinary wristwatch or the vibrations of a small quartz crystal regulate a quartz wristwatch.

The caesium atomic clocks are very accurate. In principle they provide portable standard. The national standard of time interval 'second' as well as the frequency is maintained through four cesium atomic clocks. A caesium atomic clock is used at the National Physical Laboratory (NPL), New Delhi to maintain the Indian standard of time.

In our country, the NPL has the responsibility of maintenance and improvement of physical standards, including that of time, frequency, etc. Note that the Indian Standard Time (IST) is linked to this set of atomic clocks. The efficient caesium atomic clocks are so accurate that they impart the uncertainty in time realisation as $\pm 1 \times 10^{-15}$, i.e. 1 part in 10^{15}. This implies that the uncertainty gained over time by such a device is less than 1 part in 1015; they lose or gain no more than 32 μs in one year. In view of the tremendous accuracy in time measurement, the SI unit of length has been expressed in terms the path length light travels in certain interval of time (1/299, 792, 458 of a second).

The time interval of events that we come across in the universe vary over a very wide range.

You may notice that there is an interesting coincidence between the numbers. Note that the ratio of the longest and shortest lengths of objects in our universe is about 1041. Interestingly enough, the ratio of the longest and shortest time intervals associated with the events and objects in our universe is also about 10^{41}. The ratio of the largest and smallest masses of the objects in our universe is about $(10^{41})^2$. Is this a curious coincidence between these large numbers purely accidental?

ORDER OF MAGNITUDE

If a number P can be expressed as

$$P = A \times 10^x$$

where $0.5 \le A < 5$, then x is called order of magnitude of the number.

SI Prefixes: The magnitudes of physical quantities vary over a wide range. The mass of an electron is 9.1×10^{-31} kg and that of our earth is about 6×10^{24} kg. Standard prefixes for certain power of 10. Table shows these prefixes

Power of 10	Prefix	Symbol
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deca	da
–1	deci	d
–2	centi	c
–3	milli	m
–6	micro	μ
–9	nano	n
–12	pico	p
–15	femto	f

Train Your Brain

Example 17: The Sun's angular diameter is measured to be 1920". The distance D of the Sun from the Earth is 1.496×10^{11} m. What is the diameter of the Sun?

Sol. Sun's angular diameter α.

$$= 1920''$$

$$= 1920 \times 4.85 \times 10^{-6}\text{ rad} = 9.31 \times 10^{-3}\text{ rad}$$

Sun's diameter $d = \alpha D$

$$= (9.31 \times 10^{-3}) \times (1.496 \times 10^{11})\text{ m} = 1.39 \times 10^{9}\text{ m}$$

Example 18: The moon is observed from two diametrically opposite points on the earth with angle subtended = 1°54' given diameter of earth = 1.276×10^7 m. Find the distance of moon from the earth.

Sol. From parallax method,

$$\theta = \frac{b}{D}$$

We have, $\theta = 1°54' = 60' + 54' = 114'$

Also $1' = 2.91 \times 10^{-4}$ rad

$$\therefore \theta = 114 \times 2.91 \times 10^{-4} = 0.033\text{ rad}$$

$$\therefore D = \frac{1.2760 \times 10^7}{0.033} = 3.8 \times 10^8\text{ m}$$

Example 19: If the size of a nucleus (in the range of 10^{-15} to 10^{-11}m) is scaled up to the tip of a sharp pin, what roughly is the size of an atom? Assume tip of the pin to be in the range 10^{-5} m to 10^{-4} m.)

Sol. The size of a nucleus is in the range of 10^{-15} m and 10^{-14} m. The tip of a sharp pin is taken to be in the range of 10^{-5} m and 10^{-4} m.

Thus we are scaling up by a factor of 10^{10}. An atom roughly of size 10^{-10} m will be scaled up to a size of 1m. Thus a nucleus in an atom is as small in size as the tip of a sharp pin placed at the centre of a sphere of radius about a metre long.

Concept Application

15. Considering the distance between sun and moon to be 1.5×10^{11} m, the angular diameter of the sun as observed from the moon is close to (Take the radius of sun to be 7×10^{8} m.)
(*a*) 214 rad (*b*) 107 rad
(*c*) 0.005 rad (*d*) 0.009 rad

16. The mass of an object is 75.2×10^4 kg. The order of magnitude of the mass is
(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 7

ERROR ANALYSIS IN EXPERIMENTS

Significant Figures or Digits

The significant figures (SF) in a measurement are the figures or digits that are known with certainty plus one that is uncertain.

Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is its accuracy and vice versa.

1. **Rules to find out the number of significant figures:**

   I - Rule: All the non-zero digits are significant E.g. 1984 has 4 SF.

   II - Rule: All the zeros between two non-zero digits are significant. E.g. 10806 has 5 SF.

   III - Rule: All the zeros to the left of first non-zero digit are not significant. E.g.00103 has 3 SF.

   IV - Rule: If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. E.g. 0.002308 has 4 SF.

   V - Rule: The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. E.g. 01.020 has 4 SF.

   VI - Rule: The trailing zeros in a number without a decimal point are not significant e.g. 010100 has 3 SF.

   VII - Rule: When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in $x = 12.3 = 1.23 \times 10^1 = .123 \times 10^2 = 0.0123 \times 10^3 = 123 \times 10^{-1}$, each term has 3 SF only. (**Note:** It has 3 significant figure in each expression.)

2. **Rules for arithmetical operations with significant figures:**

   I - Rule: In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. E.g. $12.587 - 12.5 = 0.087 = 0.1$ (∵ second term contain lesser i.e. one decimal place)

   II - Rule: In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. E.g. $5.0 \times 0.125 = 0.625 = 0.62$

To avoid the confusion regarding the trailing zeros of the numbers without the decimal point the best way is to report every measurement in **scientific notation** (in the power of 10). In this notation every number is expressed in the form $a \times 10^b$, where a is the base number between 1 and 10 and b is any positive or negative exponent of 10. The base number a is written in decimal form with the decimal after the first digit. While counting the number of SF only base number is considered (Rule VII).

Note: The change in the unit of measurement of a quantity does not affect the number of SF. For example in 2.308 cm = 23.08 mm = 0.02308 m = 23080 μm each term has 4 SF.

ROUNDING OFF

To represent the result of any computation containing more than one uncertain digit, it is rounded off to appropriate number of significant figures.

Rules for rounding off the numbers:

I - Rule: If the digit to be rounded off is more than 5, then the preceding digit is increased by one. e.g. 6.87≈ 6.9

II - Rule: If the digit to be rounded off is less than 5, then the preceding digit is unaffected and is left unchanged. e.g. 3.94 ≈ 3.9

III - Rule: If the digit to be rounded off is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even. e.g. 14.35 ≈ 14.4 and 14.45 ≈ 14.4

Train Your Brain

Example 20: Write down the number of significant figures in the following.
(*i*) 165 (*ii*) 2.05
(*iii*) 34.000 m (*iv*) 0.005
(*v*) 0.02340 N m^{-1} (*vi*) 26900

Sol. (*i*) 3 SF (following rule I)
(*ii*) 3 SF (following rules I and II)
(*iii*) 5 SF (following rules I and V)
(*iv*) 1 SF (following rules I and IV)
(*v*) 4 SF (following rules I, IV and V)
(*vi*) 3 SF (see rule VI)

Example 21: The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Sol. Length (ℓ) = 4.234 m, Breadth (b) = 1.005 m

Thickness (t) = 2.01 cm = 2.01×10^{-2} m

Therefore, area of the sheet = $2(\ell \times b + b \times t + t \times \ell)$

$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)\,m^2$

$= 2(4.3604739)\,m^2 = 8.720978\,m^2$

Since area can contain a maximum of 3 SF (Rule II of arithmetic operations) therefore, rounding off, we get

Area = 8.72 m^2

Like wise volume = $l \times b \times t$

$= 4.234 \times 1.005 \times 0.0201\,m^3 = 0.0855289\,m^3$

Since volume can contain 3 SF, therefore, rounding off,

Volume = 0.0855 m^3

Example 22: Find the number of significant figures in each

(*i*) 36.72 (*ii*) 0.003303

Sol. (*i*) It has four significant figures. All non-zero digits are significant.

(*ii*) Here first three zeros are insignificant but zero between 3's is significant. So it has four significant figures.

Example 23: Round off following values to four significant figures.

(*i*) 36.879 (*ii*) 1.0084

(*iii*) 11.115 (*iv*) 11.1250

(*v*) 11.1251

Sol. The following values can be rounded off to four significant figures as follows:

(*i*) 36.879 ≈ 36.88 ($\because$ 9 > 5 $\therefore$ 7 is increased by one i.e. I Rule)

(*ii*) 1.0084 ≈ 1.008 ($\because$ 4 < 5 $\therefore$ 8 is left unchanged i.e. II Rule)

(*iii*) 11.115 ≈ 11.12 ($\because$ last 1 is odd it is increased by one i.e. III Rule)

(*iv*) 11.1250 ≈ 11.12 ($\because$ 2 is even it is left unchanged i.e. III Rule)

(*v*) 11.1251 ≈ 11.13 ($\because$ 51 > 50 $\therefore$ 2 is increased by one i.e. I Rule)

Concept Application

17. The number of significant figures in 0.0006032 is
    (*a*) 7 (*b*) 4
    (*c*) 5 (*d*) 2

18. The radius of disc is 1.2 cm, its area according to idea of significant figures is ______
    (*a*) 4.5216 cm^2 (*b*) 4.521 cm^2
    (*c*) 4.52 cm^2 (*d*) 4.5 cm^2

19. The number of significant figures in 5.69×10^{15} kg is
    (*a*) 1 (*b*) 2
    (*c*) 3 (*d*) 4

20. When 57.986 is rounded off to 4 significant figures, then it becomes ______
    (*a*) 58 (*b*) 57
    (*c*) 57.90 (*d*) 57.99

ERRORS IN MEASUREMENT

The difference between the true value and the measured value of a quantity is known as the error of measurement.

Classification of Errors

Errors may arise from different sources and are usually classified as follows:

Systematic or controllable errors: Systematic errors are the errors whose causes are known. They can be either positive or negative. Systematic errors can further be classified into three categories:

(*i*) **Instrumental errors:** These errors are due to imperfect design or erroneous manufacture or misuse of the measuring instrument. These can be reduced by using more accurate instruments.

(*ii*) **Environmental errors:** These errors are due to the changes in external environmental conditions such as temperature, pressure, humidity, dust, vibrations or magnetic and electrostatic fields.

(*iii*) **Observational errors:** These errors arise due to improper setting of the apparatus or carelessness in taking observations. This can be reduced by proper setting of the instrument before we start using it.

(*iv*) **Personal Error:** This type of error may creep if the experimenter is not careful enough while performing the experiment and noting down the readings.

Random errors: These errors are due to unknown causes. Therefore they occur irregularly and are variable in magnitude and sign. Since the causes of these errors are not known precisely they can not be eliminated completely. For example, when the same person repeats the same observation in the same conditions, he may get different readings different times.

Random errors can be reduced by repeating the observation a large number of times and taking the arithmetic mean of all the observations. This mean value would be very close to the most accurate reading.

Note: If the number of observations is made n times then the random error reduces to $\left(\frac{1}{n}\right)$ times. E.g. If the random error in the arithmetic mean of 100 observations is 'x' then the random error in the arithmetic mean of 500 observations will be $\frac{x}{5}$

REPRESENTATION OF ERRORS

Errors can be expressed in the following ways:

1. **Mean Absolute Error:** The mean of measurements of a physical quantity ($a_1, a_2, \ldots a_n$) is expressed as

$$a_m = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n} = \sum_{i=1}^{n} \frac{a_i}{n}$$

a_m is taken as the true value of a quantity if the same is not known.

The absolute error in the measurements are expressed as

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$\ldots\ldots\ldots\ldots$$
$$\Delta a_n = a_m - a_n$$

Mean absolute error is defined as

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n} = \sum_{i=1}^{n} \frac{\Delta a_i}{n}$$

Final result of measurement may be written as:

$$a = a_m \pm \overline{\Delta a}$$

2. **Relative Error or Fractional Error:** It is given by

$$\frac{\overline{\Delta a}}{a_m} = \frac{\text{Mean absolute Error}}{\text{Mean value of measurement}}$$

3. **Percentage Error** $= \dfrac{\overline{\Delta a}}{a_m} \times 100\%$

Train Your Brain

Example 24: The period of oscillation of a simple pendulum in an experiment is recorded as 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. Find (i) mean time period (ii) absolute error in each observation and percentage error.

Sol. (*i*) Mean time period is given by

$$\bar{T} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$= \frac{13.12}{5} = 2.62\text{s}$$

(*ii*) The absolute error in each observation is

$2.62 - 2.63 = -0.01$, $2.62 - 2.56$
$= 0.06$, $2.62 - 2.42$
$= 0.20$, $2.62 - 2.71$
$= -0.09$, $2.62 - 2.80$
$= -0.18$

Mean absolute error, $\overline{\Delta T} = \dfrac{\Sigma |\Delta T|}{5}$

$$= \frac{0.01 + 0.06 + 0.2 + 0.09 + 0.18}{5} = 0.11\text{sec}$$

∴ Percentage error is

$$= \frac{\overline{\Delta T}}{\bar{T}} \times 100 = \frac{0.11}{2.62} \times 100 = 4.2\%$$

Example 25: In an experiment the values of refractive indices of glass material recorded as 1.56, 1.53, 1.54, 1.45, 1.44 and 1.43 in repeated measurements. Find

(*i*) Mean value of μ
(*ii*) Mean absolute error
(*iii*) Relative error
(*iv*) Percentage error

Sol. (*i*) Mean value of μ

$$\bar{\mu} = \frac{\mu_1 + \mu_2 + \ldots\ldots + \mu_n}{n}$$

$$\bar{\mu} = \frac{1.56 + 1.54 + 1.53 + 1.45 + 1.44 + 1.43}{6} = 1.491 \approx 1.50$$

(*ii*) Absolute error $|\Delta\mu|$

$$|\Delta\mu_1| = |\bar{\mu} - \mu_1| = 0.06$$
$$|\Delta\mu_2| = |\bar{\mu} - \mu_2| = 0.04$$
$$|\Delta\mu_3| = |\bar{\mu} - \mu_3| = 0.03$$
$$|\Delta\mu_4| = |\bar{\mu} - \mu_4| = 0.05$$
$$|\Delta\mu_5| = |\bar{\mu} - \mu_5| = 0.06$$
$$|\Delta\mu_6| = |\bar{\mu} - \mu_6| = 0.07$$

∴ mean absolute error

$$|\overline{\Delta\mu}| = \frac{|\Delta\mu_1| + |\Delta\mu_2| + \ldots\ldots\ldots\ldots |\Delta\mu_6|}{6}$$

$$= \frac{0.06 + 0.04 + 0.03 + 0.05 + 0.06 + 0.07}{6}$$

$$= 0.051$$

∴ Reading $= \bar{\mu} \pm |\overline{\Delta\mu}| = 1.50 \pm 0.05$

(*iii*) Relative error $= \pm \dfrac{0.05}{1.50} = \pm 0.033$

(*iv*) Relative percentage error = Relative error × 100%
= ± 0.033 × 100%
= ± 3.3% (approx)

Concept Application

21. The diameter of a wire as measured by a screw gauge was found to be 1.004 cm, 1.000 cm. Find the absolute error in first reading.

22. The diameter of a thick wire measured by a screw-gauge.
$D_1 = 1.006$ cm, $D_2 = 1.004$ cm, $D_3 = 1.002$ cm
Find mean absolute error and write the reading.

COMBINATION OF ERRORS

(*i*) **In Sum:** If $Z = A + B$, then $\Delta Z = \Delta A + \Delta B$.
Maximum fractional error in this case is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A + B} + \frac{\Delta B}{A + B}$$

i.e. when two physical quantities are added then the maximum absolute error in the result is the sum of the absolute errors of the individual quantities.

(*ii*) **In Difference:** If $Z = A - B$, then maximum absolute error is $\Delta Z = \Delta A + \Delta B$ and maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A-B} + \frac{\Delta B}{A-B}$$

(*iii*) **In Product:** If $Z = AB$, then the maximum fractional error,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

where $\Delta Z/Z$ is known as fractional error.

(*iv*) **In Division:** If $Z = A/B$, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(*v*) **In Power:** If $Z = A^n$ then $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

In more general form if $Z = \frac{A^x B^y}{C^q}$

then the maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = x\frac{\Delta A}{A} + y\frac{\Delta B}{B} + q\frac{\Delta C}{C}$$

Applications:

1. For a simple pendulum, $T \propto l^{1/2}$

$$\Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

2. For a sphere, area and volume are given as

$$A = 4\pi r^2, V = \frac{4}{3}\pi r^3$$

So, relative errors in them are given as

$$\frac{\Delta A}{A} = 2\frac{\Delta r}{r} \text{ and } \frac{\Delta V}{V} = 3\frac{\Delta r}{r}$$

3. When two resistors R_1 and R_2 are connected

(*i*) In series $R_s = R_1 + R_2 \Rightarrow \Delta R_s = \Delta R_1 + \Delta R_2$

$$\Rightarrow \frac{\Delta R_s}{R_s} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$$

(*ii*) In parallel,

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{\Delta R_p}{R_p^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

Train Your Brain

Example 26: In an experiment of simple pendulum, the errors in the measurement of length of the pendulum (L) and time period (T) are 3% and 2% respectively. The maximum percentage error in the value of L/T^2 is

(*a*) 5% (*b*) 7%

(*c*) 8% (*d*) 1%

Sol. (*b*) Maximum percentage in the value of L/T^2

$$= \frac{\Delta L}{L} \times 100\% + 2\frac{\Delta T}{T} \times 100\%$$

$$= 3 + 2 \times 2 = 7\%$$

Example 27: If $X = \frac{A^2\sqrt{B}}{C}$, then

(*a*) $\Delta X = \Delta A + \Delta B + \Delta C$

(*b*) $\frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$

(*c*) $\frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{2B} + \frac{\Delta C}{C}$

(*d*) $\frac{\Delta X}{X} = \frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$

Sol. $\because X = A^2B^{1/2}C^{-1}$

$$\therefore \frac{\Delta X}{X} = \frac{2\Delta A}{A} + \frac{\Delta B}{2B} + \frac{\Delta C}{C}$$

Example 28: A body travels uniformly a distance (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Calculate its velocity with error limits. What is the percentage error in velocity?

Sol. Given distance, $s = (13.8 \pm 0.2)$ m

and time $t = (4.0 \pm 0.3)$ s

$$\text{Velocity } v = \frac{s}{t} = \frac{13.8}{4.0} = 3.45\,\text{ms}^{-1} = 3.5\,\text{ms}^{-1}$$

$$\frac{\Delta v}{v} = \pm\left(\frac{\Delta s}{s} + \frac{\Delta t}{t}\right) = \pm\left(\frac{0.2}{13.8} + \frac{0.3}{4.0}\right) = \pm\left(\frac{0.8 + 4.14}{13.8 \times 4.0}\right)$$

$\therefore \Delta v = \pm 0.0895 \times v = \pm 0.0895 \times 3.45 = \pm 0.3087 = \pm 0.31$

Hence $v = (3.5 \pm 0.31)\ \text{ms}^{-1}$

Percentage error in velocity

$$= \frac{\Delta v}{v} \times 100 = \pm 0.0895 \times 100 = \pm 8.95\% = \pm 9\%$$

Example 29: The heat generated in a circuit is given by $Q = I^2 Rt$, where I is current, R is resistance and t is time. If the percentage errors in measuring I, R and t are 2%, 1% and 1% respectively, then the maximum error in measuring heat will be

(*a*) 2 % (*b*) 3 % (*c*) 4 % (*d*) 6 %

Sol. The percentage error in heat is given as

$$\frac{\Delta Q}{Q}\% = 2\frac{\Delta I}{I}\% + \frac{\Delta R}{R}\% + \frac{\Delta t}{t}\%$$

Substituting the values, maximum possible percentage error is

$$= 2 \times 2\% + 1 \times 1\% + 1 \times 1\% = 6\%$$

Example 30: Given: Resistance, $R_1 = (8 \pm 0.4)\ \Omega$ and Resistance, $R_2 = (8 \pm 0.6)\ \Omega$. What is the net resistance when R_1 and R_2 are connected in series?

(*a*) $(16 \pm 0.4)\ \Omega$ (*b*) $(16 \pm 0.6)\ \Omega$

(*c*) $(16 \pm 1.0)\ \Omega$ (*d*) $(16 \pm 0.2)\ \Omega$

Sol. $R_1 = (8 \pm 0.4)\Omega$, $R_2 = (8 \pm 0.6)\ \Omega$

$R_s = R_1 + R_2 = (16 \pm 1.0)\Omega$

Example 31: The following observations were taken for determining surface tension of water by capillary tube method. Diameter of capillary, $D = 1.25 \times 10^{-2}$ m and rise of water in capillary, $h = 1.45 \times 10^{-2}$ m.

Taking $g = 9.80$ ms^{-2} and using the relation $T = (rgh/2) \times 10^3$ Nm^{-1}, what is the possible error in surface tension T?

(*a*) 2.4 % (*b*) 15 %
(*c*) 1.6 % (*d*) 0.15 %

Sol. Given $T = (rgh/2) \times 10^3$ Nm^{-1},

$D = 1.25 \times 10^{-2}$ m, $h = 1.45 \times 10^{-2}$ m,

$g = 9.80$ ms^{-2}

$$\frac{\delta T}{T} = \frac{\delta r}{r} + \frac{\delta h}{h} + \frac{\delta g}{g} \Rightarrow \frac{\delta T}{T} = \frac{0.01}{1.25} + \frac{0.01}{1.45} + \frac{0.01}{9.8}$$

after applying the above values in this relation we get $\delta T\% = 1.6\%$

Concept Application

23. If the length of cylinder is measured to be 4.28 cm with an error of 0.01 cm. The percentage error in the measured length is approximately.

(*a*) 0.4% (*b*) 0.5% (*c*) 0.2 % (*d*) 0.1 %

24. The pressure on square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in measurement of force and length are respectively 4% and 2% then the maximum error in measurement of pressure is

(*a*) 1% (*b*) 2% (*c*) 6 % (*d*) 8%

25. The length and breadth of rectangular object are 25.2 cm and 16.8 cm respectively and have been measured to an accuracy of 0.1 cm. Relative error and percentage error in the area of the object are

(*a*) 0.01 and 1% (*b*) 0.02 and 2%
(*c*) 0.03 and 3% (*d*) 0.04 and 4%

26. The error in the measurement of length of a simple pendulum is 0.1% and error in the time period is 2%. The possible maximum error in the quantity having dimensional formula LT^{-2} is

(*a*) 1.1% (*b*) 2.1%
(*c*) 4.1% (*d*) 6.1%

MEASURING INSTRUMENTS

Measurement is an important aspect of physics. Whenever we want to know about a physical quantity, we take its measurement first of all. Instruments used in measurement are called measuring instruments.

Least Count: The least value of a quantity, which the instrument can measure accurately, is called the least count of the instrument.

Error: The measured value of the physical quantity is usually different from its true value. The result of every measurement by any measuring instrument is an approximate number, which contains some uncertainty. This uncertainty is called error. Every calculated quantity, which is based on measured values, has an error.

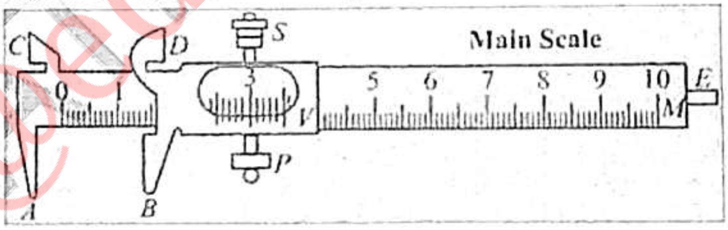
Accuracy and Precision: The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured.

VERNIER CALLIPERS

It is a device used to measure accurately upto 0.1 mm. There are two scales in the vernier callipers, vernier scale and main scale. The main scale is fixed whereas the vernier scale is movable along the main scale.

Its main parts are as follows:

Main scale: It consists of a steel metallic strip *M*, graduated in cm and mm at one edge and in inches and tenth of an inch at the other edge on same side. It carries fixed jaws *A* and *C* projected at right angle to the scale as shown in figure.



Vernier Scale: A vernier *V* slides on the strip *M*. It can be fixed in any position by screw *S*. It is graduated on both sides. The side of the vernier scale which slides over the mm side has ten divisions over a length of 9 mm, i.e., over 9 main scale divisions and the side of the vernier scale which slides over the inches side has 10 divisions over a length of 0.9 inch, i.e., over 9 main scale divisions.

Movable Jaws: The vernier scale carries jaws *B* and *D* projected at right angle to the main scale. These are called movable jaws. When vernier scale is pushed towards *A* and *C*, then as *B* touches *A*, straight side of *D* will touch straight side of *C*. In this position, in case of an instrument free from errors, zeros of vernier scale will coincide with zeros of main scales, on both the cm and inch scales.

The object whose length or external diameter is to be measured is held between the jaws *A* and *B*, while the straight edges of *C* and *D* are used for measuring the internal diameter of a hollow object.

Metallic Strip: There is a thin metallic strip *E* attached to the back side of *M* and connected with vernier scale. When the jaws *A* and *B* touch each other, the edge of strip *E* touches the edge of *M*. When the jaws *A* and *B* are separated, *E* moves outwards. The strip *E* is used for measuring the depth of a vessel.

Determination of Least Count (Vernier Constant)

Note the value of the main scale division and count the number n of vernier scale divisions. Slide the movable jaw till the zero of vernier scale coincides with any of the mark of the main scale and find the number of divisions $(n - 1)$ on the main scale coinciding with n divisions of vernier scale. Then

$$n \text{ V.S.D.} = (n - 1) \text{ M.S.D. or } 1 \text{ V.S.D.} = \left(\frac{n-1}{n}\right) \text{M.S.D.}$$

$$1 \text{ V.C.} = \text{L.C.} = 1 \text{ M.S.D.} - 1 \text{ V.S.D.} = \left(1 - \frac{n-1}{n}\right) \text{M.S.D.}$$

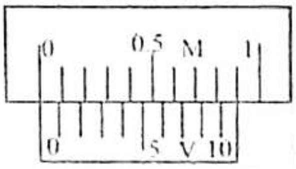
$$= \frac{1}{n} \text{ M.S.D.}$$

Determination of Zero Error and Zero Correction

For this purpose, movable jaw B is brought in contact with fixed jaw A.

One of the following situations will arise.

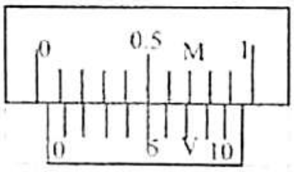
(*i*) Zero of Vernier scale coincides with zero of main scale



In this case, zero error and zero correction, both are nil.

Actual length = observed (measured) length.

(*ii*) Zero of vernier scale lies on the right of zero of main scale



Here 5^{th} vernier scale division is coinciding with any main sale division and zero of vernier is ahead of N^{th} main scale division.

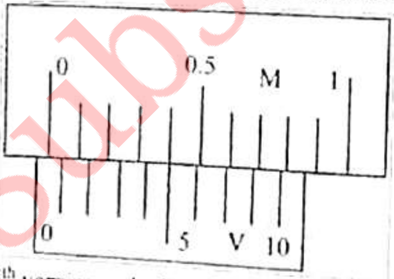
Hence, $N = 0$, $n = 5$, L.C. = 0.01 cm.

Zero error $= N + n \times (\text{L.C.}) = 0 + 5 \times 0.01 = +0.05$ cm

Zero correction = −0.05 cm

Actual length will be 0.05 cm less than the observed (measured) length.

(*iii*) Zero of the vernier scale lies left of the main scale.



Here, 5^{th} vernier scale division is coinciding with any main scale division.

In this case, zero of vernier scale lies on the right of −0.1 cm reading on main scale

Hence, $N = -0.1$ cm, $n = 5$, L.C. = 0.01 cm

Zero error $= N + n \times (\text{L.C.})$

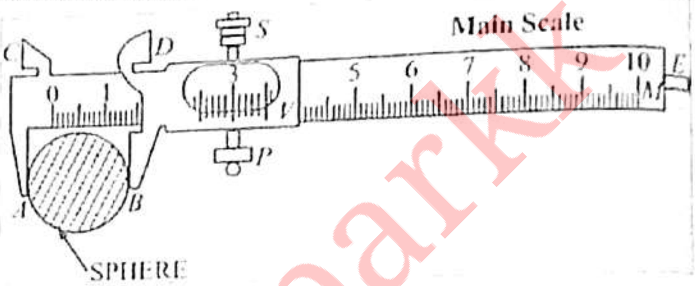
$= -0.1 + 5 \times 0.01 = -0.05$ cm

Zero correction = +0.05 cm

Actual length will be 0.05 cm more than the observed (measured) length.

Measurement Using Vernier Calliper

Let us measure the diameter of a small spherical/cylindrical body using a vernier calliper. Insert the object between the jaws as shown in the figure below.



If with the body between the jaws, the zero of vernier scale lies ahead of N^{th} division of main scale, then main scale reading (M.S.R.) = N.

If n^{th} division of vernier scale coincides with any division of main scale, then vernier scale reading (V.S.R.)

$= n \times$ (L.C.) (L.C. is least count of vernier calliper)

$= n \times$ (V.C.) (V.C. is vernier constant of vernier calliper)

Total reading,

T.R. = M.S.R. + V.S.R.

$= N + n \times$ (V.C.)

Train Your Brain

Example 32: The least count of vernier callipers is 0.1 mm. The main scale reading before the zero of the vernier scale is 10 and the zeroth division of the vernier scale coincides with the main scale division. Given that each main scale division is 1 mm, what is the measured value?

Sol. Length measured with vernier callipers

= reading before the zero of vernier scale + number of vernier divisions coinciding

with any main scale division × least count

$= 10 \text{ mm} + 0 \times 0.1 \text{ mm} = 10 \text{ mm} = 1.00$ cm

Example 33: A vernier callipers has its main scale of 10 cm equally divided into 200 equal parts. Its vernier scale of 25 divisions coincides with 12 mm on the main scale. The least count of the instrument is

(*a*) 0.020 cm
(*b*) 0.002 cm
(*c*) 0.010 cm
(*d*) 0.001 cm

Sol. 10 cm divided in 200 divisions, so

$$1 \text{ division} = \frac{10}{200} = 0.05 \text{ cm.}$$

Now, $25V = 24S$

$\Rightarrow V = \frac{24}{25}S$

$LC = S - V$

$= S - \frac{24}{25}S = \frac{1}{25}S$

$\because 1S = 0.05$ cm, thus

So, $LC = \frac{0.05}{25} = 0.002$ cm

Example 34: One centimetre on the main scale of vernier callipers is divided into ten equal parts. If 10 divisions of vernier scale coincide with 8 small divisions of the main scale, the least count of the callipers is

(*a*) 0.005 cm (*b*) 0.05 cm

(*c*) 0.02 cm (*d*) 0.01 cm

Sol. Now, $10V = 8S$

$\Rightarrow V = \frac{8}{10}S$

Now, $LC = S - V$

$= S - \frac{8}{10}S = \frac{2}{10}S = \frac{1}{5}S$

But $1S = 0.1$ cm, thus

$LC = \frac{0.1}{5} = 0.02$ cm

Concept Application

27. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°), find the least count of the instrument (in min.).

28. The diameter of a cylinder is measured using a vernier callipers with no zero error. It is found that, the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 division equivalent to 2.45 cm. If 24th division of the vernier scale exactly coincides with one of the main scale division, the diameter of the cylinder is

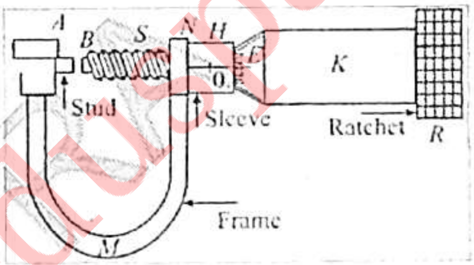
(*a*) 5.112 cm (*b*) 5.124 cm

(*c*) 5.136 cm (*d*) 5.148 cm

29. The main scale of vernier calliper is calibrated in mm and 19 divisions of main scale are equal in length to 20 divisions of vernier scale. In measuring the diameter of a cylinder by this instrument, the main scale reads 35 division and 4th division of vernier scale coincides with a M.S.D. Find LC in (cm).

SCREW GAUGE

This instrument (shown in figure) works on the principle of micrometer screw. It consists of a U-shaped frame *M*. At one end of it is fixed a small metal piece *A* of gun metal. It is called stud and it has a plane face. The other end *N* of *M* carries a cylindrical hub *H*. The hub extends few millimetre beyond the end of the frame. On the tubular hub along its axis, a line is drawn known as reference line. On the reference line graduations are in millimetre and half millimeter depending upon the pitch of the screw. This scale is called linear scale or pitch scale. A nut is threaded through the hub and the frame *N*. Through the nut moves a screw *S* made of gun metal. The front face *B* of the screw, facing the plane face *A*, is also plane. A hollow cylindrical cap *K* is capable of rotating over the hub when screw is rotated. It is attached to the right hand end of the screw. As the cap is rotated the screw either moves in or out. The bevelled surface *E* of the cap *K* is divided into 50 or 100 equal parts. It is called the circular scale or head scale. Right hand end *R* of *K* is milled for proper grip.



In most of the instrument the milled head *R* is not fixed to the screw head but turns it by a spring and ratchet arrangement such that when the body is just held between faces *A* and *B*, the spring yields and milled head *R* turns without moving in the screw.

In an accurately adjusted instrument when the faces *A* and *B* are just touching each other, the zero marks of circular scale and pitch scale exactly coincide.

Determination of Least Count of Screw Gauge

Note the value of linear (pitch) scale division. Rotate screw to bring zero mark on circular (head) scale on reference line. Note linear scale reading i.e. number of divisions of linear scale uncovered by the cap.

Now give the screw a few known number of rotations. (one rotation completed when zero of circular scale again arrives on the reference line). Again note the linear scale reading. Find difference of two readings on linear scale to find distance moved by the screw.

Then, pitch of the screw

$$= \frac{\text{Distance moved in } n \text{ rotation}}{\text{No. of full rotation } (n)}$$

Now count the total number of divisions on circular (head) scale. Then, least count is

$$LC = \frac{\text{Pitch}}{\text{Total number of divisions on the circular scale}}$$

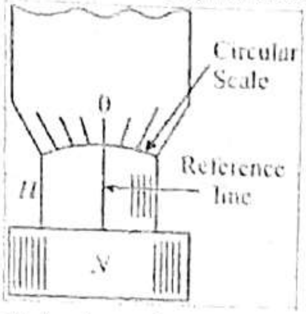
The least count is generally 0.001 cm.

Determination of Zero Error and Zero Correction

For this purpose, the screw is rotated forward till plane face B of the screw just touches the fixed plane face A of the stud and edge of cap comes on zero mark of linear scale. Screw gauge is held keeping the linear scale vertical with its zero downwards. One of the following three situations will arise.

(*i*) **Zero mark of circular scale comes on the reference line**

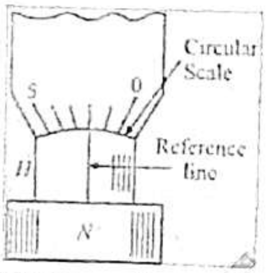
In this case, zero error and zero correction, both are nil. Actual thickness = Observed (measured) thickness.



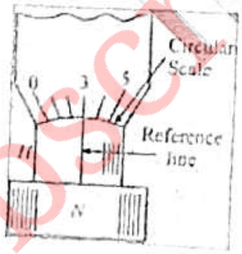
(*ii*) **Zero mark of circular scale remains on right of reference line and does not cross it**

Here 2nd division on circular scale comes on reference line. Zero reading is already 0.02 mm. It makes zero error = + 0.02 mm and zero correction = – 0.02 mm.

Actual thickness will be 0.02 mm less than the observed (measured) thickness.



(*iii*) **Zero mark of circular scale goes to left on reference line after crossing it.** Here zero of circular scale has advanced from reference line by 3 divisions on circular scale. A backward rotation by 0.03 mm will make reading zero. It makes zero error = –0.03 mm & zero correction = + 0.03 mm.



Actual thickness will be 0.03 mm more than the observed (measured) thickness.

Measurement Using Screw Gauge

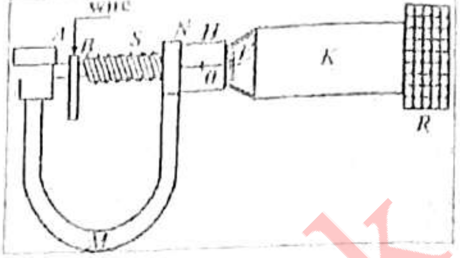
To measure diameter of a given wire using a screw gauge:

1. Determine of least count of screw gauge
2. If with the wire between plane faces A and B, the edge of the cap lies ahead of N^{th} division of linear scale, then, linear scale reading (L.S.R.) = N

If nth division of circular scale lies over reference line, then, circular scale reading (C.S.R.) = $n \times$ (L.C.)

(L.C. is least count of screw gauge)

Total reading (T.R.) = L.S.R. + C.S.R. = $N + n \times$ (L.C.)



Train Your Brain

Example 35: In four complete revolutions of the cap, the distance traveled on the pitch scale is 2 mm. If there are fifty divisions on the circular scale, then

(*i*) Calculate the pitch of the screw gauge.

(*ii*) Calculate the least count of the screw gauge.

Sol. Pitch of screw = Linear distance traveled in one revolution $P = \frac{2\text{ mm}}{4} = 0.5\text{ mm} = 0.05\text{ cm}$

$$\text{Least count} = \frac{\text{Pitch}}{\text{no.of divisions in circular scale}}$$

$$= \frac{0.05}{50} = 0.001\text{ cm}$$

Example 36: The pitch of a screw gauge is 0.5 mm and there are 50 divisions on the circular scale. In measuring the thickness of a metal plate, there are five divisions on the pitch scale (or main scale) and thirty fourth division coincides with the reference line. Calculate the thickness of the metal plate.

Sol. Pitch of screw = 0.5 mm., LC = $\frac{0.5}{50}$ = 0.01 mm.

Thickness = $(5 \times 0.5 + 34 \times 0.01)$ mm

$= (2.5 + 0.34) = 2.84$ mm

Concept Application

30. In a screw gauge, five complete rotations of the screw causes it to move a linear distance of 0.25 cm. There are 100 circular divisions. Four main scale divisions and 30 circular scale divisions is obtained as the thickness of the wire measured by this instrument. Assuming negligible zero error, find the thickness of wire (in cm).

31. The thickness of a marker measured using a screw gauge whose LC = 0.001 cm, comes out to be 0.802 cm. The percentage error in the measurement would be

(*a*) 0.125%
(*b*) 2.43%
(*c*) 4.12%
(*d*) 2.14%

Aarambh (Solved Examples)

1. $\int \frac{x\,dx}{\sqrt{2ax-x^2}} = a^n \sin^{-1}\left[\frac{x}{a}-1\right]$. The value of n is

(*a*) 0 (*b*) −1

(*c*) 1 (*d*) None of these

Use dimensional analysis to solve the problem.

Sol. $\int \frac{x\,dx}{\sqrt{2ax-x^2}} = a^n \sin^{-1}\left[\frac{x}{a}-1\right]$

Denominator $2ax - x^2$ must have the dimension of $[x]^2$

($\because$ we can add or subtract only if quantities have same dimension

$\therefore \left[\sqrt{2ax-x^2}\right] = [x]$

Also, dx has the dimension of $[x]$

$\therefore \frac{x\ dx}{\sqrt{2ax-x^2}}$ is having dimension L

Equating the dimension of L.H.S. and R.H.S. we have

$[a^n] = M^0L^1T^0$

$\{\because \sin^{-1}\left(\frac{x}{a}-1\right)$ must be dimensionless$\}$

Therefore, option (*c*) is the correct answer.

2. In the formula: $P = \frac{nRT}{V-b} e^{-\frac{a}{RTV}}$ find the dimensions of '*b*' and '*a*' respectively, where P = pressure, n = no. of moles, T = temperature, V = volume and R = universal gas constant.

(*a*) $[L^3], [MLT^{-2}]$ (*b*) $[M^2L], [ML^5T^{-2}]$

(*c*) $[L^3], [ML^5T^{-2}mol^{-1}]$ (*d*) $[M^2L], [MLT^{-2}]$

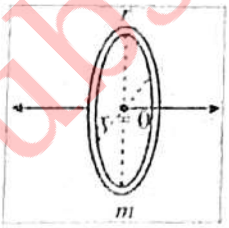
Sol. $[b] = [V] = [L^3]$

$$[a] = [RTV] = \frac{[PV]}{[n]}[V] = \frac{ML^2T^{-2}L^3}{mol} \quad (\because nRT = PV)$$

$= ML^5T^{-2}mol^{-1}$.

Therefore, option (*c*) is the correct answer.

3. A particle is performing SHM along the axis of a fixed ring. Due to gravitational force, its displacement at time t is given by $x = a \sin\omega t$.



In this equation ω is found to depend on radius of the ring (r), mass of the ring (m) and gravitational constant (G). Using dimensional analysis, find the expression of ω in terms of m, r and G.

(*a*) $\sqrt{\frac{Gm}{r}}$ (*b*) $\sqrt{\frac{Gm}{r^3}}$ (*c*) $\sqrt{\frac{G}{mr^2}}$ (*d*) $\sqrt{\frac{Gr^3}{m}}$

Sol. Let $\omega = Km^ar^bG^c$ where K is a dimensionless constant

Writing the dimension of both the sides and equating them we have,

$T^{-1} = M^aL^b(M^{-1}L^3T^{-2})^c$

$= M^{a-c}L^{b+3c}T^{-2c}$

Equating the exponents,

$-2c = -1$ or $c = 1/2$;

$b + 3c = 0$ or $-3c = b = -\frac{3}{2}$;

$a - c = 0, c = a = +\frac{1}{2}$

Thus the required equation is $\omega = K\sqrt{\frac{Gm}{r^3}}$

Therefore, option (*b*) is the correct answer.

4. Using screw gauge, the observation of the diameter of a wire are 1.324, 1.326, 1.334, 1.336 cm respectively. The value of diameter of wire along with its percentage uncertainty will be

(*a*) 1.33 ± 1% (*b*) 1.30 ± 0.4%

(*c*) 1.30 ± 1% (*d*) 1.33 ± 0.4%

Sol. Average diameter:

$$\bar{D} = \frac{\Sigma(D)}{N} = \frac{1.324+1.326+1.334+1.336}{4} = 1.330$$

$\Delta D_1 = 1.324 - 1.330 = -0.006$

$\Delta D_2 = 1.326 - 1.330 = -0.004$

$\Delta D_3 = 1.334 - 1.330 = 0.004$

$\Delta D_4 = 1.336 - 1.330 = 0.006$

Mean absolute error:

$$\overline{\Delta D} = \frac{|\Delta D_1| + |\Delta D_2| + |\Delta D_3| + |\Delta D_4|}{4}$$

$$= \frac{0.006+0.004+0.004+0.006}{4} = \frac{0.020}{4} = 0.005\,\text{cm}$$

$$\text{Relative error} = \frac{\Delta\bar{D}}{D} = \frac{0.005}{1.330} = 0.004$$

$$\%\ \text{error} = \frac{\Delta\bar{D}}{D} \times 100 = 0.4\%$$

Therefore, option (*d*) is the correct answer.

5. If a tuning fork of frequency (f_0) 340 Hz and tolerance ± 1% is used in resonance column method [$v = 2f_0(\ell_2 - \ell_1)$], the first and the second resonance are measured at $\ell_1 = 24.0$ cm and $\ell_2 = 74.0$ cm. Find maximum permissible error in speed of sound.

(*a*) 1.4% (*b*) 2.8% (*c*) 0.5% (*d*) 0.7%

Sol. $v = 2f_0(\ell_2 - \ell_1)$

$$\Rightarrow \left(\frac{\Delta v}{v}\right)_{max} = \frac{\Delta f_0}{f_0} + \frac{\Delta\ell_1 + \Delta\ell_2}{\ell_2 - \ell_1} = \frac{1}{100} + \frac{0.1+0.1}{74-24} = 1.4\%$$

Therefore, option (*a*) is the correct answer.

6. Side of a cube is measured with the help of vernier calliper. Main scale reading is 10 mm and vernier scale reading is 1. It is known that 9 M.S.D. = 10 V.S.D. Mass of the cube is 2.735 g. Find density of the cube upto appropriate significant figure

(a) 0.0027 g/mm^3 (b) 0.00265 g/mm^3
(c) 0.003 g/mm^3 (d) 0.002655 g/mm^3

Sol. Least count = 1 M.S.D. – 1 V.S.D. = 1 M.S.D. – $\frac{9}{10}$ M.S.D.

$$= \frac{1}{10}\text{ M.S.D.} = \frac{1}{10} \times 1\text{ mm}$$

Least count = 0.1 mm

Length of side of cube = M.S.R. + V.S.R. × least count

$$= 10 + 1 \times 0.1 = 10.1\text{ mm}$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{2.735}{(10.1)^3} = 0.0026546\ \text{g/mm}^3$$

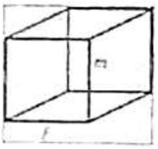
Using significant figures the correct answer would be 0.00265 (with 3 significant figures).

Therefore, option (b) is the correct answer.

7. For a cubical block, error in measurement of sides is + 1% and error in measurement of mass is + 2%, then maximum possible error in density is

(a) 1% (b) 5% (c) 3% (d) 7%

Sol.



$$\text{Density, } \rho = \frac{m}{V} = \frac{m}{\ell^3}$$

$$\text{Given: } \frac{\Delta m}{m} = \pm 2\% = \pm 2\times 10^{-2},\ \frac{\Delta \ell}{\ell} = \pm 1\% = \pm 1\times 10^{-2}$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta \ell}{\ell} = 2\times 10^{-2} + 3\times 10^{-2} = 5\times 10^{-2} = 5\%$$

Therefore, option (b) is the correct answer.

8. The length of a rectangular plate is measured by a meter scale and is found to be 10.0 cm. Its width is measured by vernier callipers as 1.00 cm. The least count of the meter scale and vernier callipers are 0.1 cm and 0.01 cm respectively (Obvious from readings). Maximum permissible error in area measurement is

(a) +0.2 cm^2 (b) +0.1 cm^2 (c) +0.3 cm^2 (d) Zero

Sol. $A = \ell b = 10.0 \times 1.00 = 10.00$

$$\text{Now, } \frac{\Delta A}{A} = \frac{\Delta \ell}{\ell} + \frac{\Delta b}{b}$$

$$\frac{\Delta A}{10.00} = \frac{0.1}{10.0} + \frac{0.01}{1.00}$$

$$\Rightarrow \Delta A = 10.00\left(\frac{1}{100} + \frac{1}{100}\right) = 10.00\left(\frac{2}{100}\right) = \pm 0.2\,\text{cm}^2$$

Therefore, option (a) is the correct answer.

9. To estimate 'g' (from $g = 4\pi^2 \frac{L}{T^2}$), error in measurement of L is + 2% and error in measurement of T is + 3%. The error in estimated 'g' will be

(a) +8% (b) +6% (c) +3% (d) +5%

Sol. $g = 4\pi \frac{\ell}{T^2}$

$$\frac{\Delta \ell}{\ell} = 2\% = \pm 2\times 10^{-2}$$

$$\frac{\Delta T}{T} = +3\% = \pm 3\times 10^{-2}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta T}{T} = 2\times 10^{-2} + 2\times 3\times 10^{-2}$$

$$= 8\times 10^{-2} = \pm 8\%$$

Therefore, option (a) is the correct answer.

10. The mass of a ball is 1.76 kg. The mass of 25 such balls is

(a) 0.44 × 10^3 kg (b) 44.0 kg
(c) 44 kg (d) 44.00 kg

Sol. $m = 1.76$ kg, $M = 25\,m = 25 \times 1.76 = 44.0$ kg

Mass of one unit has three significant figures and it is just multiplied by a pure number (magnified). So result should also have three significant figures.

Therefore, option (b) is the correct answer.

11. To measure the diameter of a wire, a screw gauge is used. In a complete rotation, spindle of the screw gauge advances by 1/2 mm and its circular scale has 50 division. The main scale is graduated to 1/2mm. If the wire is put between the jaws, 4 main scale divisions are clearly visible and 10 divisions of circular scale co-inside with the reference line. The resistance of the wire is measured to be (10Ω ± 1%). Length of the wire is measured to be 10 cm using a scale of least count 1mm. Maximum permissible error in resistivity measurement is

(a) 4% (b) 2% (c) 2.95% (d) 1.03%

Sol. $$\text{L.C. of screw gauge} = \frac{1/2\text{ mm}}{50} = 0.01\text{ mm}$$

Diameter of wire as measured by the screw gauge

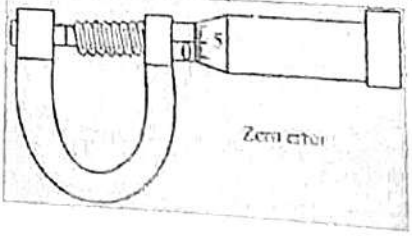
$$= 4 \times \frac{1}{2} + 10 \times 0.01 = 2.1\text{ mm}$$

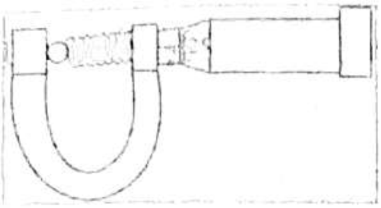
$$\text{Since, } \rho = \frac{R\pi \frac{d^2}{4}}{\ell} \Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{\max} = \frac{\Delta R}{R} + \frac{2\Delta d}{d} + \frac{\Delta \ell}{\ell}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = \frac{1}{100} + \frac{2\times 0.01}{2.1} + \frac{1}{100} = 2.95\%$$

Therefore, option (c) is the correct answer.

12. The number of circular divisions on the shown screw gauge is 50. It moves 0.5 mm on main scale for one complete rotation. Main scale reading is 2. The diameter of the ball is





(*a*) 2.25 mm (*b*) 2.20 mm (*c*) 1.20 mm (*d*) 1.25 mm

Sol. Least count $= \frac{0.5}{50} = 0.01$

Zero error $= 5 \times 0.01 = 0.05$ mm

Actual measurement $= 2 \times 0.5 + 25 \times \frac{0.5}{50} - 0.05$

$= 1$ mm $+ 0.25$ mm $- 0.05$ mm $= 1.20$ mm.

Therefore, option (*c*) is the correct answer.

13. A student performs an experiment for determination of $g\left(=\frac{4\pi^2 L}{T^2}\right)$, $L = 1\text{m}$ and he commits an error of ΔL. In measuring n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1 sec. For which of the following data, the measurement of g will be most accurate?

(*a*) $\Delta L = 0.5$, $\Delta T = 0.1$, $n = 20$
(*b*) $\Delta L = 0.5$, $\Delta T = 0.1$, $n = 50$
(*c*) $\Delta L = 0.5$, $\Delta T = 0.01$, $n = 50$
(*d*) $\Delta L = 0.1$, $\Delta T = 0.05$, $n = 50$

Sol. $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$

In option (*d*) error in g is minimum and number of repetition of measurement are maximum. In this case the error in g is minimum.

Therefore, option (*d*) is the correct answer.

14. Student I, II and III perform an experiment for measuring the acceleration due to gravity (*g*) using a simple pendulum. They use different lengths of the pendulum and record time for different number of oscillations. The observations are shown in the table.

Least count for length = 0.1 cm

Least count for time = 0.1 s

Student	Length of the pendulum	Number of oscillations (n)	Total time for n oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_{I}, E_{II} and E_{III} are the percentage error in g, i.e. $\left(\frac{\Delta g}{g} \times 100\right)$

(*a*) $E_{\text{I}} = 0$
(*b*) E_{I} is minimum
(*c*) $E_{\text{I}} = E_{\text{II}}$
(*d*) E_{III} is maximum

Sol. The least count of length $\Delta \ell = 0.1$ cm

The least count of length $\Delta t = 0.1$ s

% error of $g = \frac{\Delta g}{g} \times 100$

Now, $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g = \frac{4\pi^2 \ell}{T^2}$ where $T = \frac{1}{n}$

So, $g = \frac{4\pi^2 \ell}{t^2} n^2 \qquad \left(T = \frac{t}{n}\right)$

$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta t}{t}$

For student I, $\left(100 \times \frac{\Delta g}{g}\right)_{\text{I}} = \left(\frac{0.1}{64.0} + \frac{2 \times 0.1}{128.0}\right) \times 100$

$$E_I = \frac{0.2}{64.0} \times 100 = \frac{20}{64}$$

For student II,

$$\left(100 \times \frac{\Delta g}{g}\right)_{\text{II}} = \left(\frac{0.1}{64.0} + 2 \times \frac{0.1}{64.0}\right) \times 100$$

$$E_{\text{II}} = \frac{0.3}{64.0} \times 100 = \frac{30}{64}$$

For student III,

$$\left(100 \times \frac{\Delta g}{g}\right)_{\text{III}} = \left(\frac{0.1}{20.0} + \frac{0.1}{18.0}\right) \times 100 = \frac{19}{18}$$

$$E_{\text{III}} = \left(\frac{0.1}{20.0} + \frac{0.1}{18.0}\right) \times 100 = \frac{19}{18} \qquad \Rightarrow E_{\text{I}} \text{ is least.}$$

Therefore, option (*b*) is the correct answer.

15. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(*a*) 0.9% (*b*) 2.4% (*c*) 3.1% (*d*) 4.2%

Sol. Least count $= \frac{0.5}{50} = 0.01\,\text{mm}$

Diameter of ball $D = 2.5$ mm + (20)(0.01)

$\Rightarrow D = 2.7$ mm

$$\rho = \frac{M}{\text{Vol}} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3} \Rightarrow \left(\frac{\Delta\rho}{\rho}\right)_{\max} = \frac{\Delta M}{M} + 3\frac{\Delta D}{D}$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = 2\% + 3\left(\frac{0.01}{2.7}\right) \times 100\%$$

$$\left(\frac{\Delta\rho}{\rho}\right)_{\max} = 3.1\%$$

Therefore, option (*c*) is the correct answer.

School Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. Which among the following is the supplementary unit_____
   (a) Mass (b) Time
   (c) Solid angle (d) Luminosity

2. The number of significant digits in 1559.00 is_____
   (a) 6 (b) 5 (c) 3 (d) 4

3. Joule second is the unit of
   (a) Force (b) Angular momentum
   (c) Energy (d) Power

4. What is the number 75.66852 rounded off to 5 significant digits?
   (a) 75.67 (b) 75.669
   (c) 75.668 (d) 75.667

5. The division of energy by time is X. The dimensional formula of X is same as that of
   (a) power (b) electric field
   (c) momentum (d) torque

6. 1 kWh =
   (a) 1000 W (b) 36×10^5 J
   (c) 1000 J (d) 3600 J

7. Density of wood is 0.5 gm/cc in the CGS system of units. The corresponding value in MKS units is:
   (a) 500 (b) 5
   (c) 0.5 (d) 5000

8. Which of the following is a derived unit?
   (a) Unit of mass (b) Unit of area
   (c) Unit of time (d) Unit of current

9. Newton/meter^2 is the unit of
   (a) Energy (b) Momentum
   (c) Force (d) Pressure

10. Dimensional formula of heat energy is
    (a) ML^2T^{-2} (b) MLT^{-1}
    (c) $M^0L^0T^{-2}$ (d) None of these

11. **Assertion:** If the units of force and length are doubled, the unit of energy will become four times.

    Reason: The unit of energy is independent of the units of force and length.
    (a) Assertion and reason both are true and reason is correct explanation for assertion.
    (b) Assertion and reason both are true but reason is not correct explanation for assertion.
    (c) Assertion is true but reason is false.
    (d) Assertion is false but reason is true.

12. **Assertion:** Dimensional constants are the quantities whose values are constant

    Reason: Dimensional constants are dimensionless.
    (a) Assertion and reason both are true and reason is correct explanation for assertion.
    (b) Assertion and reason both are true but reason is not correct explanation for assertion
    (c) Assertion is true but reason is false.
    (d) Assertion is false but reason is true.

VERY SHORT ANSWER TYPE QUESTIONS

13. What is the number of significant figures in 0.06070?
14. If $x = a + bt + ct^2$, where x is in meter and t in seconds, what is the unit of c?
15. Will the dimensions of a physical quantity be the same, whatever be the units in which it is measured? Why?
16. What do you mean by the term measurement?
17. Define dimensions of a physical quantity.
18. S.I units are coherent. Explain.
19. What are the limitations of dimensional analysis? (Any two)

SHORT ANSWER TYPE QUESTIONS

20. 5.74 g of a substance occupies 1.2 cm^3. Express its density to correct significant figures.
21. Derive the dimensional formula of:
    (a) Angular velocity
    (b) Angular momentum

LONG ANSWER TYPE QUESTIONS

22. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm, respectively. Give the area and volume of the sheet to correct significant figures.
23. The frequency of vibration of a string depends on, (i) tension in the string (ii) mass per unit length of string, (iii) vibrating length of the string. Establish dimensionally the relation for frequency.

CASE STUDY BASED QUESTIONS

24. The nature of physical quantity is described by its dimension. All the physical quantities can be expressed in terms of some combination of seven fundamental units. The dimensions of a physical quantity are thus the powers to which the base quantities are raised to represent that quantity.
    If a given physical quantity depends on a^{th} power of mass, b^{th} power of length, c^{th} power of time etc., then its dimensions are expressed as $[M^a L^b T^c]$.

(i) The dimension of planck's constant equal to that of

(a) Energy
(b) Momentum
(c) angular momentum
(d) power

(ii) If force (F), velocity (V) and time (T) are taken as fundamental units, then dimension of mass are

(a) $[FVT^{-2}]$
(b) $[FV^{-1}T^{-1}]$
(c) $[FV^{-1}T]$
(d) $[FVT^{-1}]$

(iii) Which of the following sets have different dimensions

(a) Pressure and Young's modulus
(b) Emf and potential difference
(c) Heat and work done
(d) Dipole moment and electric flux

(iv) The dimension of a × b in the relations:-

$P = \frac{b - x^2}{at}$, where P is power, x is distance and t is time.

(a) $[M^{-1}L^{2}T^{2}]$
(b) $[M^{-1}L^{4}T^{2}]$
(c) $[M^{-1}L^{2}T^{-2}]$
(d) $[ML^{2}T^{-1}]$

(v) Find the value of 100 J on a system which has 20 cm, 250 g and half minute as fundamental units of length, mass and time.

(a) 9×10^{-7} new units
(b) 2.16×10^{7} new units
(c) 9×10^{6} new units
(d) 100 new units

25. The Vander Waal's equation is $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

Where P is pressure, V is volume, T is absolute temperature of given sample of a gas, R is called molar gas constant, a and b are Vander waal's constant

Now answer the following

(i) The dimensional formula for b is same as for

(a) V
(b) PV^2
(c) RT
(d) P

(ii) The dimensional formula for a is same as for

(a) V^2
(b) P
(c) PV^2
(d) RT

(iii) The dimensional formula of $\frac{ab}{RT}$ is

(a) $[ML^{5}T^{-2}]$
(b) $[M^{0}L^{3}T^{0}]$
(c) $[ML^{-1}T^{-2}]$
(d) $[M^{0}L^{6}T^{0}]$

(iv) The dimensional formula for RT is same as for

(a) Energy
(b) force
(c) Latent heat
(d) Specific heat

(v) The dimensional formula for RT is not same as that for

(a) $\frac{ab}{V^2}$
(b) Pb
(c) $\frac{a}{V^2}$
(d) PV

Prarambh (Topicwise)

UNITS, SYSTEM OF UNITS

1. Which of the following is not the unit of time?

(a) Solar day
(b) Parallactic second
(c) Leap year
(d) Lunar month

2. A unitless quantity

(a) Never has a non zero dimension
(b) Always has a non zero dimension
(c) May have a non zero dimension
(d) Does not exit

3. Which of the following is not the name of a physical quantity?

(a) Kilogram
(b) Impulse
(c) Energy
(d) Density

4. Parsec is a unit of

(a) Time
(b) Angle
(c) Distance
(d) Velocity

5. Which of the following system of units is not based on the unit of mass, length and time alone?

(a) FPS
(b) SI
(c) CGS
(d) MKS

6. In the S.I. system the unit of energy is:

(a) Erg
(b) Calorie
(c) Joule
(d) Electron volt

7. The SI unit of the universal gravitational constant G is

(a) N m kg^{-2}
(b) N m^2 kg^{-2}
(c) N m^2 kg^{-1}
(d) N m kg^{-1}

8. Surface tension has unit of

(a) Joule m^2
(b) Joule m^{-2}
(c) Joule m^{-1}
(d) Joule m^3

9. The specific resistance has the unit of:

(a) ohm/m
(b) ohm/m^2
(c) ohm m^2
(d) ohm m

10. The unit of magnetic moment is:

(a) Amp m^2
(b) Amp m^{-2}
(c) Amp m
(d) Amp m^{-1}

11. The SI unit of the universal gas constant R is

(a) Erg K^{-1} mol^{-1}
(b) Watt K^{-1} mol^{-1}
(c) Newton K^{-1} mol^{-1}
(d) Joule K^{-1} mol^{-1}

12. The SI unit of Stefan's constant is

(a) W s^{-1} m^{-2} K^{-4}
(b) J s m^{-1} K^{-1}
(c) J s^{-1} m^{-2} K^{-1}
(d) W m^{-2} K^{-4}

DIMENSIONS OF PHYSICAL QUANTITIES

13. In SI units the angular acceleration has unit of
(a) N m kg^{-1} (b) m s^{-1} (c) rad s^{-2} (d) N kg^{-1}

14. The angular frequency is measured in rad s^{-1}. Its exponent in length are
(a) −2 (b) −1 (c) 0 (d) 2

15. $[MLT^{-1}]$ are the dimensions of
(a) Power (b) Momentum
(c) Force (d) Couple

16. What are the dimensions of Boltzmann's constant?
(a) $MLT^{-2}K^{-1}$ (b) $ML^2T^{-2}K^{-1}$
(c) M^0LT^{-2} (d) $M^0L^2T^{-2}K^{-1}$

17. The pair of physical quantities having the same dimensional formula among the following is
(a) Angular momentum and torque
(b) Torque and energy
(c) Force and power
(d) Power and angular momentum

18. Which one of the following has the dimensions of $ML^{-1}T^{-2}$?
(a) Torque (b) Surface tension
(c) Viscosity (d) Stress

19. The dimensions of work done per unit mass per unit relative density would be equivalent to the dimensions of
(a) (Acceleration)^2 (b) (Velocity)^2
(c) (Force)^2 (d) (Torque)^2

20. Which of the following is dimensional formula of intensity?
(a) MT^{-3} (b) $M^{-1}L^2T^{-2}$ (c) $ML^{-2}T^{-1}$ (d) None

21. The dimensions of $\left[\frac{h}{G}\right]$, where h = Planck's constant and G = gravitational constant, is
(a) $ML^{-1}T^2$ (b) $M^{-1}L^2T^2$ (c) $M^2L^{-1}T$ (d) $M^3L^0T^{-1}$

22. A dimensionless quantity:
(a) Never has a unit (b) Always has a unit
(c) May have a unit (d) Does not exit

DIMENSIONAL ANALYSIS AND ITS APPLICATION

Principle of Homogeneity

23. Force F is given in terms of time t and distance x by $F = A \sin C\,t + B \cos D\,x$. Then the dimensions of A/B and C/D are given by
(a) $MLT^{-2}, M^0L^0T^{-1}$ (b) $MLT^{-2}, M^0L^{-1}T^0$
(c) $M^0L^0T^0, M^0L^1T^{-1}$ (d) $M^0L^1T^{-1}, M^0L^0T^0$

24. The velocity of sound in a gas is given by $v = \sqrt{\gamma k_b \frac{T}{m}}$. Velocity v is measured in m/s, γ is a dimensionless constant, T is temperature in kelvin (K), and m is mass in kg. What are the units for the Boltzmann constant, k_b?
(a) kgm^{2}s^{-2}K^{-1} (b) kgm^{2}s^{2}K
(c) kgms^{-1}K^{-2} (d) kgm^{2}s^{-2}K

25. A wave is represented by $y = a \sin (At - Bx + C)$ where A, B, C are constants, t is in seconds, x and y are in metre. The dimensions of A, B and C are
(a) $T^{-1}, L, M^0L^0T^0$ (b) $T^{-1}, L^{-1}, M^0L^0T^0$
(c) T, L, M (d) T^{-1}, L^{-1}, M^{-1}

26. If $v = \sqrt{\frac{\gamma P}{\rho}}$, then the dimensions of γ are (P is pressure, ρ is density and v is speed of sound)
(a) $M^0L^0T^0$ (b) $M^0L^0T^{-1}$
(c) $M^1L^0T^0$ (d) $M^0L^1T^0$

27. Consider the equation $\frac{d}{dt}\left[\int \vec{F} \cdot d\vec{s}\right] = A\left[\vec{F} \cdot \vec{P}\right]$. Then dimensions of A will be ($\vec{F}$ = force, $d\vec{s}$ = small displacement, t = time and $\vec{P}$ = linear momentum)
(a) $M^0L^0T^0$ (b) $M^1L^0T^0$
(c) $M^{-1}L^0T^0$ (d) $M^0L^0T^{-1}$

28. If $F = \frac{A}{\sqrt{m}} + B$ where F = Force, m = Mass.
Then dimensions of $A \times B$ is,
(a) $M^{5/2}L^2T^{-4}$ (b) $M^{2.5}L^2T^{-1}$
(c) $M^2L^{2/3}T^{-1}$ (d) $M^{-1}L^{2/3}T^{-2}$

29. If $v = At^3 + \frac{B}{m}$, where m = mass, v = velocity and t = time. Then dimensions of A in the given equation would be
(a) LT^{-2} (b) L^2T^{-3} (c) L^3T^{-2} (d) LT^{-4}

Deducing Relation Among Various Physical Quantities

30. The velocity of water waves may depend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. The method of dimensions gives the relation between these quantities as (where k is a dimensionless constant)
(a) $v^2 = k\lambda^{-1} g^{-1} \rho^{-1}$ (b) $v^2 = k g \lambda$
(c) $v^2 = k g \lambda \rho$ (d) $v^2 = k \lambda^3 g^{-1} \rho^{-1}$

31. Force applied by water stream depends on density of water (ρ), velocity of the stream (v) and cross-sectional area of the stream (A). The expression of the force should be
(a) $\rho A v$ (b) $\rho A v^2$
(c) $\rho^2 A v$ (d) $\rho A^2 v$

32. If velocity (v), frequency (f) and mass (m) are taken as fundamental quantities, how energy (E) may be described using above quantity.
(a) Kvf^2m (b) Kv^2f^0m
(c) Kvf^2m^0 (d) $Kv^{1/2}f^{-1}m^2$

33. If the power, P delivered by a motor is dependent on force F, velocity v and density of material ρ. Then power may be proportional to
(a) $F^2v\rho^{-1}$ (b) $Fv^2\rho$
(c) $Fv\rho^0$ (d) None of these

34. The velocity of a freely falling body changes as $g^p h^q$ where g = acceleration due to gravity and h is height. The value of p and q are

(*a*) $1, \frac{1}{2}$ (*b*) $\frac{1}{2}, \frac{1}{2}$

(*c*) $\frac{1}{2}, 1$ (*d*) 1, 1

onversion from One System of Units to Other

35. One watt-hour is equivalent to

(*a*) 6.3×10^3 Joule (*b*) 6.3×10^{-7} Joule

(*c*) 3.6×10^3 Joule (*d*) 3.6×10^{-3} Joule

36. The pressure of 10^6 dyne/cm^2 is equivalent to

(*a*) 10^5 N/m^2 (*b*) 10^6 N/m^2

(*c*) 10^7 N/m^2 (*d*) 10^8 N/m^2

37. Consider $\rho = 2$ g/cm^3. Convert it into MKS system

(*a*) $2 \times 10^{-3} \frac{\text{kg}}{\text{m}^3}$ (*b*) $2 \times 10^3 \frac{\text{kg}}{\text{m}^3}$

(*c*) $4 \times 10^3 \frac{\text{kg}}{\text{m}^3}$ (*d*) $2 \times 10^6 \frac{\text{kg}}{\text{m}^3}$

38. The density of mercury is 13600 kg m^{-3}. Its value of CGS system will be

(*a*) 13.6 g cm^{-3} (*b*) 1360 g cm^{-3}

(*c*) 136 g cm^{-3} (*d*) 1.36 g cm^{-3}

39. Force in CGS system is 20. Its value in SI units will be

(*a*) 20×10^5 (*b*) 20×10^{-5}

(*c*) 200 (*d*) 2×10^{-3}

40. If in a system of measurements unit of mass is α kg, unit of length is β m and that of time is γ sec. Find the value of 100 joule in this system.

(*a*) $100\, \alpha^{-1}\beta^{-2}\gamma^2$ (*b*) $100\, \alpha^{-2}\beta^{-1}\gamma^{-2}$

(*c*) $100\, \alpha\beta^{-2}\gamma$ (*d*) $1000\, \alpha^{-2}\beta^2\gamma^{-1}$

ERRORS IN MEASUREMENT

41. Which of the following measurements is most precise?

(*a*) 9×10^{-2} m (*b*) 90×10^{-3} m

(*c*) 900×10^{-4} m (*d*) 0.090 m

42. A system takes 70.40 seconds to complete 20 oscillations. The time period of the system is:

(*a*) 3.52 s (*b*) 35.2×10 s

(*c*) 3.520 s (*d*) 3.5200 s

43. The percentage error in the measurement of mass and speed are 1% and 2% respectively. What is the percentage error in kinetic energy?

(*a*) 5% (*b*) 2.5% (*c*) 3% (*d*) 1.5%

44. Number 15462 when rounded off to three significant digits will be

(*a*) 15500 (*b*) 155

(*c*) 1546 (*d*) 150

45. Value of expression $\frac{25.2 \times 1374}{33.3}$ will be
(All the digits in the expression are significant)

(*a*) 1040 (*b*) 1039 (*c*) 1038 (*d*) 1041

46. Value of 24.36 + 0.0623 + 256.2 will be (considering rules of significant digits)

(*a*) 280.6 (*b*) 280.8 (*c*) 280.7 (*d*) 280.6224

47. The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimation of momentum obtained by measuring mass and speed?

(*a*) 11% (*b*) 8% (*c*) 5% (*d*) 1%

48. The random error in the arithmetic mean of 100 observations is x. Then random error in the arithmetic mean of 400 observations would be

(*a*) $4x$ (*b*) $\frac{1}{4}x$ (*c*) $2x$ (*d*) $\frac{1}{2}x$

49. What is the number of significant figures in 0.310×10^3

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 6

50. Error in the measurement of radius of a sphere is 1%. The error in the calculated value of its volume is

(*a*) 1% (*b*) 3% (*c*) 5% (*d*) 7%

51. The mean time period of second's pendulum is 2.00 s and mean absolute error in the time period is 0.05 s. To express maximum estimate of error, the time period should be written as

(*a*) (2.00 ± 0.01)s (*b*) (2.00 +0.025)s

(*c*) (2.00 ± 0.05)s (*d*) (2.00 ± 0.10)s

52. The unit of percentage error is

(*a*) Same as that of physical quantity

(*b*) Different from that of physical quantity

(*c*) Percentage error is unit less

(*d*) Errors have got their own units which are different from that of physical quantity measured

53. The decimal equivalent of 1/20 upto three significant figures is

(*a*) 0.0500 (*b*) 0.05000

(*c*) 0.0050 (*d*) 5.0×10^{-2}

54. A thin copper wire of length l metre increases in length by 2% when heated through 10 °C. What is the percentage increase in area when a square copper sheet of length l metre is heated through 10 °C?

(*a*) 4% (*b*) 8% (*c*) 16% (*d*) 32 %

55. The length and breadth of a rectangle are 20 ± 0.2 cm and 10 ± 0.1 cm. The percentage error in area would be

(*a*) 1% (*b*) 2% (*c*) 3% (*d*) 4%

56. The mass rate of flow of liquid through a pipe is given by $\frac{dm}{dt} = \rho A v$, where ρ = density, A = area of cross-section and v = velocity. The readings of area, $A = 10 \pm 0.1$ m^2 and $v = 30 \pm 0.3$ m/sec. Find the percentage error in measurement of mass flow rate.

(*a*) 5% (*b*) 2% (*c*) 8% (*d*) 3.5%

MEASURING INSTRUMENTS

57. In a vernier callipers, ten divisions of the vernier scale is equal to the length of nine division on the main scale. If the smallest division on the main scale is half millimeter, then the vernier constant is:
(*a*) 0.5 mm
(*b*) 0.1 mm
(*c*) 0.05 mm
(*d*) 0.005 mm

58. A vernier callipers has 20 divisions on the vernier scale, which coincide with 19 divisions on the main scale. The least count of the instrument is 0.1 mm. The main scale divisions are of
(*a*) 0.5 mm
(*b*) 1 mm
(*c*) 2 mm
(*d*) 1/4 mm

59. A vernier callipers having 1 main scale division = 0.1 cm is designed to have a least count of 0.02 cm. If n be the number of divisions on vernier scale and m be the length of vernier scale, then
(*a*) $n = 10, m = 0.5$ cm
(*b*) $n = 9, m = 0.4$ cm
(*c*) $n = 10, m = 0.8$ cm
(*d*) $n = 10, m = 0.2$ cm

60. The pitch of a screw gauge is 0.05 cm. In how many revolutions of hollow cylinder the screw will advance 0.35 cm in the straight line?
(*a*) 7
(*b*) 10
(*c*) 15
(*d*) 14

61. A student in the laboratory measures thickness of a wire using screw gauge. The readings are 1.22 mm, 1.23 mm, 1.19 mm, 1.20 mm. The percentage error in measurement.
(*a*) 2.20%
(*b*) 1.24%
(*c*) 2.85%
(*d*) 3.52%

Prabal (JEE Main Level)

1. The unit of impulse is the same as that of
(*a*) Moment of force
(*b*) Linear momentum
(*c*) Rate of change of linear momentum
(*d*) Force

2. Which of the following is not the unit of energy?
(*a*) Watt-hour
(*b*) Electron-volt
(*c*) N × m
(*d*) kg × m/sec^2

3. If a and b are two physical quantities having different dimensions then which of the following can denote a new physical quantity?
(*a*) $a + b$
(*b*) $a - b$
(*c*) a/b
(*d*) $e^{a/b}$

4. The time dependence of a physical quantity $P = P_0 \exp(-\alpha t^2)$ where α is a constant and t is time The constant α
(*a*) will be dimensionless
(*b*) will have dimensions of T^{-2}
(*c*) will have dimensions as that of P
(*d*) will have dimensions equal to the dimension of P multiplied by T^{-2}

5. Which pair of following quantities has dimensions different from each other?
(*a*) Impulse and linear momentum
(*b*) Plank's constant and angular momentum
(*c*) Moment of inertia and moment of force
(*d*) Young's modulus and pressure

6. The product of energy and time is called action. The dimensional formula for action is same as that for
(*a*) Power
(*b*) Angular energy
(*c*) Force × velocity
(*d*) Impulse × distance

7. What is the physical quantity whose dimensions are $[\mathrm{M\,L^2\,T^{-2}}]$?
(*a*) Kinetic energy
(*b*) Pressure
(*c*) Momentum
(*d*) Power

8. If E, M, J and G denote energy, mass, angular momentum and gravitational constant respectively, then $\dfrac{EJ^2}{M^5G^2}$ has the dimensions of
(*a*) Length
(*b*) Angle
(*c*) Mass
(*d*) Time

9. The position of a particle at time 't' is given by the relation $x(t) = \dfrac{V_0}{\alpha}[1 - e^{-\alpha t}]$ where V_0 is a constant and $\alpha > 0$. The dimensions of V_0 and α are respectively.
(*a*) $\mathrm{M^0L^1T^0}$ and $\mathrm{T^{-1}}$
(*b*) $\mathrm{M^0L^1T^0}$ and $\mathrm{T^{-2}}$
(*c*) $\mathrm{M^0L^1T^{-1}}$ and $\mathrm{T^{-1}}$
(*d*) $\mathrm{M^0L^1T^{-1}}$ and $\mathrm{T^{-2}}$

10. If force (F) is given by $F = Pt^{-1} + \alpha t$, where t is time. The unit of P is same as that of
(*a*) Velocity
(*b*) Displacement
(*c*) Acceleration
(*d*) Momentum

11. When a wave traverses a medium, the displacement of a particle located at x at time t is given by $y = a\sin(bt - cx)$ where a, b and c are constants of the wave. The dimensions of b are the same as those of
(*a*) Wave velocity
(*b*) Amplitude
(*c*) Wavelength
(*d*) Wave frequency

12. In a book, the answer for a particular question is expressed as $b = \dfrac{ma}{k}\left[\sqrt{1 + \dfrac{2kl}{ma}}\right]$ here m represents mass, a represents accelerations, l represents length. The unit of b should be
(*a*) m/s
(*b*) m/s^2
(*c*) meter
(*d*) /sec

13. $\alpha = \frac{F}{v^2}\sin(\beta t)$ (here v = velocity, F = force, t = time) Find the dimensions of α and β

(a) $\alpha = [M^1L^1T^0]$, $\beta = [T^{-1}]$
(b) $\alpha = [M^1L^{-1}T^{-1}]$, $\beta = [T^1]$
(c) $\alpha = [M^1L^1T^{-1}]$, $\beta = [T^{-1}]$
(d) $\alpha = [M^1L^{-1}T^0]$, $\beta = [T^{-1}]$

14. If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be

(a) FT^2 (b) $F^{-1}A^2T^{-1}$ (c) FA^2T (d) AT^2

15. If the unit of length is micrometer and the unit of time is microsecond, the unit of velocity will be

(a) 100 m/s (b) 10 m/s (c) 10^{-6} m/s (d) 1 m/s

16. In a certain system of units, unit of time is 5 s, unit of mass is 20 kg and unit of length is 10m. In this system, one unit of power will be equal to

(a) 16 watts (b) 1/16 watts
(c) 25 watts (d) None of these

17. If the unit of force is 1 kN, the unit of length is 1 km and the unit of time is 100 s, what will be the unit of mass?

(a) 1000 kg (b) 10 kg (c) 10000 kg (d) 100 kg

18. The units of length, velocity and force are doubled. Which of the following is the correct change in the other units?

(a) Unit of time is doubled
(b) Unit of mass is doubled
(c) Unit of momentum is doubled
(d) Unit of energy is doubled

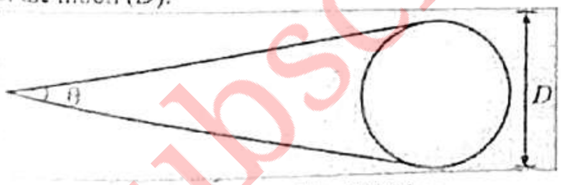
19. If the units of force and that of length are doubled, the unit of energy will become

(a) 1/4 times (b) 1/2 times
(c) 2 times (d) 4 times

20. If the units of mass and length are doubled then the unit of kinetic energy will become

(a) 2 times (b) 4 times (c) 8 times (d) 16 times

21. The angle subtended by the moon's diameter at a point on the earth is about 0.50°. Use this and the fact that the moon is about 384000 km away to find the approximate diameter of the moon (D).



(a) 192000 km (b) 3350 km
(c) 1600 km (d) 1920 km

22. The least count of a stop watch is 0.2 second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. The percentage error in the time period is

(a) 16% (b) 0.8 % (c) 1.8 % (d) 8 %

23. The dimensions of a cuboidal block measured with a vernier callipers having least count of 0.1 mm is 5 mm × 10 mm × 5 mm. The maximum percentage error in measurement of volume of the block is

(a) 5 % (b) 10 % (c) 15 % (d) 20 %

24. In an experiment the quantities x, y and z are measured. Then t is calculated from the data as $t = \frac{xy^2}{z^3}$. If percentage errors in x, y and z are respectively 1%, 3%, 2%, then percentage error in t is:

(a) 10 % (b) 4 % (c) 7 % (d) 13 %

25. The external and internal diameters of a hollow cylinder are measured to be (4.23 ± 0.01) cm and (3.89 ± 0.01) cm. The thickness of the wall of the cylinder is

(a) (0.34 ± 0.02) cm (b) (0.17 ± 0.02) cm
(c) (0.17 ± 0.01) cm (d) (0.34 ± 0.01) cm

26. The mass of a ball is 1.76 kg. The mass of 25 such balls is

(a) 0.44×10^3 kg (b) 44.0 kg
(c) 44 kg (d) 44.00 kg

27. Two resistors R_1 (24 ± 0.5) Ω and R_2 (8 ± 0.3) Ω are joined in series. The equivalent resistance is

(a) 32 ± 0.33 Ω (b) 32 ± 0.8 Ω
(c) 32 ± 0.2 Ω (d) 32 ± 0.5 Ω

28. The pitch of a screw gauge is 0.5 mm and there are 100 divisions on its circular scale. The instrument reads +2 divisions when nothing is put in-between its jaws. In measuring the diameter of a wire, there are 8 divisions on the main scale and 83rd division coincides with the reference line. Then the diameter of the wire is

(a) 4.05 mm (b) 4.405 mm
(c) 3.05 mm (d) 1.25 mm

29. The pitch of a screw gauge having 50 divisions on its circular scale is 1 mm. When the two jaws of the screw gauge are in contact with each other, the zero of the circular scale lies 6 divisions below the line of graduation. When a wire is placed between the jaws, 3 linear scale divisions are clearly visible while 31st division on the circular scale coincides with the reference line. The diameter of the wire is

(a) 3.62 mm (b) 3.50 mm
(c) 3.5 mm (d) 3.74 mm

30. The smallest division on the main scale of a vernier callipers is 1 mm, and 10 vernier divisions coincide with 9 main scale divisions. While measuring the diameter of a sphere, the zero mark of the vernier scale lies between 2.0 and 2.1 cm and the fifth division of the vernier scale coincides with a mark on main scale. Then diameter of the sphere is

(a) 2.05 cm (b) 3.05 cm
(c) 2.50 cm (d) None of these

31. For a satellite orbiting around the Earth, its orbital velocity (v_0) is found to depend on mass of Earth M, radius of earth R and universal gravitational constant G. The orbital velocity is proportional to

(a) $G^{-1}M^1R^{-1}$ (b) $G^1M^1R^{-1}$
(c) $G^{1/2}M^{1/2}R^{-1/2}$ (d) None of these

32. The vernier constant of a vernier callipers is 0.1 mm and it has zero error of – 0.05 cm. While measuring the diameter of a sphere, the main scale reading is 1.7 cm and coinciding vernier division is 5. The correct diameter will be $n \times 10^{-2}$ cm. Find n.

(a) 180 (b) 220 (c) 160 (d) 200

33. In a particular system, the unit of length, mass and time are chosen to be 10 cm, 10g and 0.1s respectively. The unit of force in this system will be equal to

(*a*) 1/10 N (*b*) 1 N (*c*) 10 N (*d*) 100 N

34. In sub-atomic physics, one often associates a characteristic wavelength λ with a particle of mass *m*. If $\hbar = \frac{h}{2\pi}$ (*h* being Planck's constant) and *c* is the speed of light, which of the following expression is most likely to be correct one ? (Use formula $E = hf$)

(*a*) $\lambda = \frac{\hbar c}{m}$ (*b*) $\lambda = \frac{\hbar}{mc^2}$

(*c*) $\lambda = \frac{m\hbar}{c}$ (*d*) $\lambda = \frac{\hbar}{mc}$

35. If the mass, time and energy are taken as fundamental physical quantities then dimensional formula of length is

(*a*) $[M^{1/2}T^1E^{-1/2}]$ (*b*) $[M^{-1/2}T^1E^{1/2}]$

(*c*) $[M^{-1}T^2E]$ (*d*) None of these

36. Given that $\ln(\alpha/p\beta) = \alpha z/k_B\theta$ where *p* is pressure, *z* is distance, k_B is Boltzmann constant and θ is temperature. The dimensions of β are (Useful formula: Energy = k_B × temperature)

(*a*) $L^0M^0T^0$ (*b*) $L^1M^{-1}T^2$

(*c*) $L^2M^0T^0$ (*d*) $L^{-1}M^1T^{-2}$

37. The dependence of *g* on geographical latitude at sea level is given by $g = g_0(1 + \beta\sin^2\phi)$ where ϕ is the latitude angle and β is a dimensionless constant. If Δ*g* is the error in the measurement of *g*, then the error in measurement of latitude angle is

(*a*) zero (*b*) $\Delta\phi = \frac{\Delta g}{g_0\beta\sin(2\phi)}$

(*c*) $\Delta\phi = \frac{\Delta g}{g_0\beta\cos(2\phi)}$ (*d*) $\Delta\phi = \frac{\Delta g}{g_0}$

38. Let $y = l^2 - \frac{l^3}{z}$ where $l = 2.0 \pm 0.1$, $z = 1.0 \pm 0.1$, then the value of *y* is given by

(*a*) -4 ± 1.6 (*b*) -4 ± 2.4

(*c*) -4 ± 0.8 (*d*) None of these

39. The time periods of a pendulum are measured to be $T_1 = 8.01$ s and $T_2 = 8.41$ s by a student who used stop watch having least count = 0.01 sec, then the best reported measurement of time period (in sec) is

(*a*) 8.2 ± 0.2 (*b*) 8.41 ± 0.2

(*c*) 8.21 ± 0.01 (*d*) 8.41 ± 0.01

40. Assume pressure (*P*), length (*L*) and velocity (*V*) as fundamental quantities. The dimensions of coefficient of viscosity (η) are

(*a*) $[PL^{-1}V]$ (*b*) $[PLV^{-1}]$

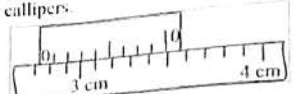
(*c*) $[P^{-2}LV^{-1}]$ (*d*) $[PL^{-1}V^{-2}]$

41. A gas bubble from an explosion under water oscillates with a period *T* proportional to $p^a d^b E^c$, where *p* is static pressure, *d* is the density of water, *E* is the total energy of the explosion. The values of *c*, *b*, *a* respectively will be

(*a*) $-\frac{5}{6}, \frac{1}{2}, \frac{1}{3}$ (*b*) $-\frac{1}{3}, \frac{1}{3}, \frac{1}{4}$

(*c*) $\frac{1}{3}, \frac{1}{2}, -\frac{5}{6}$ (*d*) $-\frac{5}{6}, \frac{1}{3}, \frac{1}{2}$

42. The diagram shows part of the vernier scale of a pair of vernier callipers.



Which reading is correct ?

(*a*) 2.74 cm (*b*) 3.10 cm (*c*) 3.26 cm (*d*) 3.64 cm

INTEGER TYPE QUESTIONS

43. Number of significant figures in 0.007 m² are ____.

44. Number of significant figures in 2.64 × 10²⁴ are ____.

45. Number of significant figures in 6.032 N are ____.

46. The velocity of sound (*v*) in a gas depends on its pressure (*p*) and density (*D*). If relation between velocity, pressure and density is given by $v = Kp^aD^b$, then $(a + b)$ is ____.

47. The density of a cube is calculate D by measuring its mass and the length of its sides. If the maximum errors in the measurement of mass and length are 3% and 2% respectively, then the maximum error in the value of density is____%.

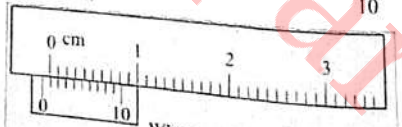
48. The time period of oscillation of a body is given by

$$T = 2\pi\sqrt{\frac{mgA}{K}}$$

K represents kinetic energy, *m* represents mass, *g* represents acceleration due to gravity and *A* is unknown. If $[A] = M^xL^yT^z$, then what is the value of $x + y + z$?

49. The radius of a sphere is measured to be 5.3 ± 0.1 cm Calculate percentage error in volume. Round off to nearest integer.

50. The main scale of a Vernier callipers reads in millimeter and its vernier scale is divided into 10 divisions which coincide with 9 divisions of the main scale. The length of the object for the situation shown below is found to be $\frac{12x}{10}$ mm. Find the value of *x*.



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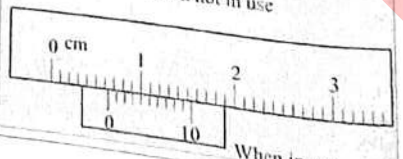


When in use

Parikshit (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. Choose the correct statement(s).
   (*a*) All quantities may be represented dimensionally in terms of the base quantities.
   (*b*) A base quantity cannot be represented dimensionally in terms of the rest of the base quantities.
   (*c*) The dimension of a base quantity in other base quantities is always zero.
   (*d*) The dimension of a derived quantity is never zero in any base quantity.

2. The dimensions $[ML^{-1}T^{-2}]$ may correspond to
   (*a*) Work done by a force
   (*b*) Linear momentum
   (*c*) Pressure
   (*d*) Energy per unit volume

3. A student curiously picks up Resnick and Halliday and tries to understand the answers given at the end of the book using his new found knowledge of physics. He marks four answers. In which of the following quantity A has the same units as that of angular momentum ?

   Useful formula, $\vec{L} = \vec{r} \times \vec{p}$, $\omega = \dfrac{2\pi}{T}$, $\vec{\tau} = \vec{r} \times \vec{F}$, E represent energy, l represents length c represents velocity of light, f represents frequency and t represents time.

   (*a*) $\frac{1}{2}mv^2 = Af$ (*b*) $\Delta p = \dfrac{2A\omega}{v}\sqrt{1-\dfrac{v^2}{c^2}}$

   (*c*) $\sin(kl) = k\sqrt{\dfrac{A^2}{2mE}}$ (*d*) $t = Al$

4. The quantity/quantities that does/do not have mass in its/their dimensions (when we take standard 7 quantities as fundamental) is/are
   (*a*) Specific heat (*b*) Latent heat
   (*c*) Luminous intensity (*d*) Mole

5. The power output for a hovering helicopter depends on its linear size, the density of air and g × density of the helicopter as
   $P \propto (\text{linear size})^x (\text{density of air})^y (g \times \text{density of helicopter})^z$
   where g is acceleration due to gravity.
   [Given [Power] = $[ML^2T^{-3}]$, [Linear size] = [L], [Density] = $[ML^{-3}]$, [g × density] = $[ML^{-2}T^{-2}]$]
   (*a*) The value of y is $-1/2$.
   (*b*) The ratio of power output of engines of two hovering helicopters when linear size of one helicopter is one fourth of the other keeping all other parameters same, is 64.
   (*c*) The ratio of the power output to hover a helicopter on the earth and on an imaginary planet, where $g_{planet} = g/4$ and density of air on imaginary planet is same as that of earth, is 8.
   (*d*) If helicopter is to hover at higher altitudes then we need less powerful engine

6. A student taking a quiz finds on a reference sheet the two equations $v = 1/T$ and $v = \sqrt{T/\mu}$

   (μ = mass/length, torque = $\vec{r} \times \vec{F}$ and rest of symbols have usual notations.)

   He has forgotten what T represents in each equation. Use dimensional analysis to determine the units of T in each equation.
   (*a*) In first equation T represents tension.
   (*b*) In first equation T represents time.
   (*c*) In second equation T represents torque.
   (*d*) In second equation T represents tension.

7. A chunk of unknown rock has mass 38.254 ± 0.003 grams and has a volume of 15.0 cm^3.
   (*a*) The density of the rock is 2.55 g/cm^3.
   (*b*) The absolute error in density is 0.02 g/cm^3.
   (*c*) The relative error in density is 0.007.
   (*d*) The number of significant figure 3 in density are 3.

8. Using a screw gauge the diameter of a wire is found to be 5.00 mm. The length of wire is measured by using a scale and is found to be 50.0 cm. If mass of wire is measured as 25 g, then mark the correct statement(s) (Take $\pi = 3.14$).
   (*a*) The density has to be computed upto 2 significant digits.
   (*b*) The least count of scale used to measure length of wire is 1 mm.
   (*c*) The density of wire is 2.6 g/cm^3
   (*d*) The least count of screw gauge is 0.01 mm

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 9 to 11): Let us consider a particle P which is moving straight on the X-axis. We also know that the rate of change of its position is given by $\dfrac{dx}{dt}$; where x is its separation from the origin and t is time. This term $\dfrac{dx}{dt}$ is called the velocity of particle (v). Further the second derivation of x with respect to time is called acceleration (a) or rate of change of velocity and is represented by $\dfrac{d^2x}{dt^2}$ or $\dfrac{dv}{dt}$. If the acceleration of this particle is found to depend upon time as follows $a = At + Bt^2 + \dfrac{Ct}{D+t^2}$ then

9. The dimensions of A are
   (*a*) LT^{-2} (*b*) LT^{-3} (*c*) LT^3 (*d*) L^2T^3

10. The dimensions of B are
    (*a*) LT^{-4} (*b*) L^2T^{-3} (*c*) LT^4 (*d*) LT^{-2}

11. The dimensions of C are
    (*a*) L^2T^{-2} (*b*) LT^{-2} (*c*) LT^{-1} (*d*) T^2

Comprehension (Q. 12 to 14): According to Coulomb's law of electrostatics there is a force between two charged particles q_1 & q_2 separated by a distance r such that $F \propto q_1$, $F \propto q_2$ & $F \propto \frac{1}{r^2}$; combining all three we get $F \propto \frac{q_1q_2}{r^2}$ or $F = \frac{kq_1q_2}{r^2}$, where k is a constant which depends on the medium and is given by $1/4\pi\varepsilon_0\varepsilon_r$ where ε_0 is absolute permittivity & ε_r is relative permittivity.

But in case of protons of a nucleus there exists another force called nuclear force which is much higher in magnitude in comparison to electrostatic force and is given by $F = \frac{Ce^{-kr}}{r^2}$.

12. What are the dimensions of C?
(a) $M^2L^3T^{-1}$ (b) ML^3T^{-3} (c) ML^3T^{-2} (d) ML^2T^{-3}

13. What are the dimensions of k?
(a) L (b) L^2 (c) L^{-3} (d) L^{-1}

14. What are the SI units of C?
(a) Nm^{-2} (b) Nm^2 (c) Nm^{-3} (d) Nm

Comprehension (Q. 15 to 17): When numbers having uncertainties or errors are used to compute other numbers, these will be uncertain too. It is especially important to understand it when a number obtained from measurements is to be compared with a value obtained from theoretical prediction. Assume a student wants to verify the value of π as the ratio of circumference to diameter of a circle. The correct value of ten digits is 3.141592654. He draws a circle and measures its diameter and circumference to its nearest millimeter obtaining the values 135 mm and 424 mm, respectively. Using a calculator he finds $\pi = 3.140740741$.

15. Why does measured value not match with calculated value?
(a) Due to systematic error
(b) Due to error in calculation
(c) Due to random error
(d) Lack of precision in measuring

16. What is the value of π in the passage measured by the student?
(a) 3.140 (b) 3.141 (c) 3.1407 (d) 3.14

17. If diameter and circumference both have an error of 1%, what is the error in value of π?
(a) 2% (b) 1% (c) 0.5% (d) 0%

Comprehension (Q. 18 to 19): Problems in your book ask you to ignore air resistance; several even say that you "can" ignore air resistance. Is it true? Certainly if we say that you may ignore air resistance, then you may. However, the question remains: When is it physically correct to ignore air resistance?
Dimensional analysis is a method for obtaining approximate answers to physics problems by consideration only (or primarily) of the dimensions or units of the quantities involved. Air resistance comes, primarily, because there is air in the way of the ball, and the ball must push the air out of the way. In detail this depends on many things, but the dominant dependence is on the speed v of the ball (because it affects how quickly the air must be moved out of the way), the cross-sectional area A of the ball (because it affects how much air must be moved out of the way), and the density ρ ("rho") of the air (because it affects how much mass is associated with that much air). You can assume the dimensionless constant of proportionality to be 1.

18. If a spherical ball of constant density is dropped from a large height, its terminal velocity (velocity after a long time) will depend on a power of its radius. This power is
(a) 1 (b) 2
(c) 0.5 (d) –0.5

19. What is the rate at which energy is dissipated due to air resistance?
(a) ρAv^3 (b) ρAv^2
(c) ρA^2v (d) ρA^2v^2

Comprehension (Q. 20 to 22): Max Planck noted in 1899 the existence of a system of units based on the three fundamental constants G, c, and h. These constants are dimensionally independent in the sense that no combination is dimensionless and a length, a time, and a mass may be constructed from them. Specifically, with $\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}$ SI units in preference to h, the Planck scale is represented by ℓ_P, T_P, M_P.

20. Which of the following formula can represent length in terms of $\hbar$, c and G
(a) $\sqrt{\frac{\hbar G}{c^3}}$ (b) $\sqrt{\frac{\hbar c}{G}}$
(c) $\sqrt{\frac{Gc}{\hbar}}$ (d) $\sqrt{G\hbar c}$

21. Which among the following would be the order of numerical value of a unit of mass defined in terms of $\hbar$, c and G
(a) 10^{-8} kg (b) 10^{-10} kg
(c) 10^{-5} kg (d) 10^{-27} kg

22. The dimensional formula for acceleration in terms of these quantities would be
(a) $\hbar^{1/2}c^{3/2}G^{-1/2}$ (b) $\hbar^{3/2}c^{5/2}G^{1/2}$
(c) $\hbar^{-1/2}c^{7/2}G^{-1/2}$ (d) $\hbar^{5/2}c^{3/2}G^{1/2}$

Comprehension (Q. 23 to 25): The Van-der Waals equation is $\left(P+\frac{a}{V^2}\right)(V-b)=nRT$, where P is pressure, V is volume, T is the temperature of the given sample of gas, R is called molar gas constant and a and b are called Van-der Waals constants

23. The dimensional formula for a is same as that for
(a) V^2 (b) P (c) PV^2 (d) RT

24. Which of the following does not possess the same dimensional formula as that for nRT?
(a) PV (b) Pb (c) a/V^2 (d) ab/V^2

25. The dimensional formula of nRT is same as that of
(a) Energy (b) Force
(c) Specific heat (d) Latent heat

[illegible] TYPE QUESTIONS

[illegible]

Column-I	Column-II
A. [illegible]	p. [illegible]
B. [illegible]	q. [illegible]

[illegible]	$\frac{\text{newton}}{\text{metre}^2}$
[illegible]	$\frac{\text{newton metre}}{\text{second}}$
[illegible]	newton metre

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

29. Suppose [illegible] students are trying to make a new measurement system so that they can use it like a code measurement system and others do not understand it. Instead of taking [illegible] as basic unit they took unit of mass as α kg, the unit of length as β m and unit of time as γ second. They called power in new system as SHAKTI, then match the two columns.

Column-I	Column-II
A. 1 N in new system	p. $\alpha^{-1}\beta^{-2}\gamma^2$
B. 1 J in new system	q. $\alpha^{-1}\beta^{-1}\gamma^2$
C. 1 Pascal (SI unit of pressure) in new system	r. $\alpha^{-1}\beta\gamma^2$
D. 1 SHAKTI in watt	s. $\alpha^2\beta^2\gamma^{-3}$

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

30. Match the following

Column-I	Column-II
A. Latent heat of fusion	p. $M^0L^0T^0$
B. Coefficient of friction	q. ML^2
C. Avogadro constant	r. ML^0T^{-3}
D. Intensity of wave	s. L^2T^{-2}
E. Moment of inertia	t. mol^{-1}

(a) [illegible]
(b) [illegible]
(c) [illegible]
(d) [illegible]

NUMERICAL TYPE QUESTIONS

Answer should be rounded off upto two decimal places.

31. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47th division on the circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm²) of the wire.

32. The length of the string of a simple pendulum is measured with a metre scale to be 90.01 cm. The radius of the bob plus the length of the hook is measured to be 2.132 cm using a screw gauge. What is the effective length (in cm) of the pendulum? (This effective length is defined as the distance between the point of suspension and the center of the bob).

33. In an experiment of simple pendulum, time period measured was 50 s for 25 oscillations when the length of the simple pendulum was taken as 100 cm. If the least count of stop watch is 0.1 s and that of meter scale is 0.01 cm, then the maximum possible percentage error in the calculated value of g is______%.

34. In an optical bench experiment to determine the focal length of the concave mirror, the least count of the bench is 0.1 cm. The object needle reading is 10.7 cm and the image needle reading is 5.7 cm. The mirror is positioned at 15.7 cm. Find the maximum possible percentage error (in %) the focal length of the mirror with this set of readings.

INTEGER TYPE QUESTIONS

35. A gas bubble, from an explosion under water, oscillates with a period proportional to $P^a d^b E^c$, where P is the static pressure, d is the density and E is the total energy of the explosion. Find the value of $a + b + c$.

36. An artificial satellite is revolving around a planet of mass M and radius R, in a circular orbit of radius r. From Kepler's third law for the period of a satellite around a common central body, the cube of the radius of the orbit r is proportional to square of time period of revolution T. This means that $r^3 = kT^2$. Here k depends on mass of planet M, its radius R and universal gravitational constant G. If $k \propto G^x$, find x.

37. A quantity A appears in an equation.

$$\text{Pressure} = \frac{Ae^{-Af/kT}}{B}$$

T represent temperature, 'f' frequency & 'k' boltzmann constant. The dimensions of B are given as, $[B] = [M^a L^b T^c]$. Then find the value of $a + b + c$?

38. If a composite physical quantity in terms of moment of inertia I, force F, velocity v, work w and length L is defined as $Q = \frac{IFv^2}{wL^3}$. Then the dimensions of Q in mass is_____.

39. The loss of pressure when a fluid flows through a pipe is given by $P = k\,\rho^a\, l\, V^b\, d^c\, \mu$ where d and l are diameter and length of the pipe respectively, ρ and μ are the mass density and coefficient of viscosity of the fluid, V is the mean velocity of flow through the pipe and k is a numerical constant. Find the values of $a + 2b - 3c$.

40. Specific heat of hydrogen at constant pressure, C_p = 29 Joule Kelvin^{-1} mol^{-1}. Unit of length is changed to 50 cm, unit of time is changed to 2 sec, unit of temperature is changed to 2K keeping unit of mass and unit of amount of substance same. Find the value of specific heat of hydrogen in new system of units.

SINGLE CORRECT TYPE QUESTIONS

41. An unknown quantity 'α' is expressed as $\alpha = \frac{2ma}{\beta}\log\left(1+\frac{2\beta l}{ma}\right)$, where m = mass, a = acceleration, l = length. The unit of α should be

(a) s^{-1} (b) m/s (c) m/s^2 (d) meter

42. The volume V of water passing through a uniform tube during t seconds is related to the cross sectional area A of the tube and velocity v of water by relation

$V = A^\alpha v^\beta t^\gamma$

Which one of the following will be true ?

(a) $\alpha > \beta = \gamma$ (b) $\alpha = \beta = \gamma$

(c) $\beta = \gamma$ (d) $\alpha \neq \beta \neq \gamma$

43. A projectile is to be projected towards enemy territory at the same horizontal level. The initial velocity of the projectile is known to be 100 ± 1 m/s. Initial angle of the projectile is known to be 45° ± 1° with horizontal. What is the possible range of the projectile?

(a) $990\ m \le R \le 1010\ m$ (b) $980\ m \le R \le 1020\ m$

(c) $970\ m \le R \le 1030\ m$ (d) $930\ m \le R \le 970\ m$

44. The dependence of g on geographical latitude at sea level is given by $g = g_0(1 + \beta\sin^2\phi)$ where ϕ is the latitude angle and β is a dimensionless constant. If Δg is the error in the measurement of g then the error in measurement of latitude angle is

(a) zero (b) $\Delta\phi = \frac{\Delta g}{g_0\beta\sin(2\phi)}$

(c) $\Delta\phi = \frac{\Delta g}{g_0\beta\cos(2\phi)}$ (d) $\Delta\phi = \frac{\Delta g}{g_0}$

45. Given that ln $(\alpha/p\beta) = \alpha z/K_B\theta$ where p is pressure, z is distance, K_B is Boltzmann constant and θ is temperature. The dimensions of β are (useful formula, Energy = K_B × temperature)

(a) $L^0M^0T^0$ (b) $L^1M^{-1}T^2$

(c) $L^2M^0T^0$ (d) $L^{-1}M^1T^{-2}$

46. If measurement of x is equal to $1.00 \times 10^2 \pm 6$, then find the value of $\sqrt{x}$.

(a) 10.0 ± 0.3 (b) 10.0 ± 0.5

(c) 10 ± 6 (d) 10 ± 2

PYQ's (Past Year Questions)

UNITS, SYSTEM OF UNITS

1. Electric field in a certain region is given by $\vec{E} = \left(\frac{A}{x^2}\hat{i} + \frac{B}{y^3}\hat{j}\right)$. The SI unit of A and B are: **[30 Jan, 2023 (Shift-I)]**
   (*a*) $Nm^3C^{-1}; Nm^2C^{-1}$
   (*b*) $Nm^2C^{-1}; Nm^3C^{-1}$
   (*c*) $Nm^3C; Nm^2C$
   (*d*) $Nm^2C; Nm^3C$

2. Match List-I with List-II. **[27 Aug, 2021 (Shift-II)]**

List-I		List-II	
A.	R_H (Rydberg constant)	I.	kg m^{-1} s^{-1}
B.	h(Planck's constant)	II.	kg m^2 s^{-1}
C.	μ_B (Magnetic field energy density)	III.	m^{-1}
D.	η (coefficient of viscocity)	IV.	kg m^{-1} s^{-2}

   (*a*) A-II, B-III, C-IV, D-I (*b*) A-III, B-II, C-I, D-IV
   (*c*) A-III, B-II, C-IV, D-I (*d*) A-IV, B-II, C-I, D-III

3. If E and H represents the intensity of electric field and magnetising field respectively, then the unit of E/H will be: **[27 Aug, 2021 (Shift-I)]**
   (*a*) Joule (*b*) Newton
   (*c*) Ohm (*d*) Mho

4. A physical quantity $\vec{S}$ is defined as $\vec{S} = (\vec{E} \times \vec{B})/\mu_0$, where $\vec{E}$ is electric field, $\vec{B}$ is magnetic field and μ_0 is the permeability of free space. The dimensions of $\vec{S}$ are the same as the dimensions of which of the following quantity (ies)? **[JEE Adv, 2021]**
   (*a*) $\frac{\text{Energy}}{\text{Charge} \times \text{Current}}$ (*b*) $\frac{\text{Force}}{\text{Length} \times \text{Time}}$
   (*c*) $\frac{\text{Energy}}{\text{Volume}}$ (*d*) $\frac{\text{Power}}{\text{Area}}$

5. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass 50 g, the numerical value of density of the material is **[10 Jan, 2019 (Shift-I)]**
   (*a*) 40 (*b*) 16 (*c*) 640 (*d*) 410

DIMENSION, FINDING DIMENSIONAL FORMULA

6. The dimensional formula of latent heat is: **[09 April, 2024 (Shift-I)]**
   (*a*) $[M^0LT^{-2}]$ (*b*) $[MLT^{-2}]$
   (*c*) $[M^0L^2T^{-2}]$ (*d*) $[ML^2T^{-2}]$

7. If ε_0 is the permittivity of free space and E is the electric field, then $\varepsilon_0 E^2$ has the dimensions: **[08 April, 2024 (Shift-II)]**
   (*a*) $[M^0L^{-2}TA]$ (*b*) $[ML^{-1}T^{-2}]$
   (*c*) $[M^{-1}L^{-3}T^4A^2]$ (*d*) $[ML^2T^{-2}]$

8. Given below are two statements:

   Statement (I): Dimensions of specific heat is $[L^2T^{-2}K^{-1}]$

   Statement (II): Dimensions of gas constant is $[ML^2T^{-1}K^{-1}]$

   [06 April, 2024 (Shift-II)]
   (*a*) Statement (I) is incorrect but statement (II) is correct
   (*b*) Both statement (I) and statement (II) are incorrect
   (*c*) Statement (I) is correct but statement (II) is incorrect
   (*d*) Both statement (I) and statement (II) are correct

9. Match List-I with List-II

	List-I		List-II
A.	Torque	I.	$[M^1L^1T^{-2}A^{-2}]$
B.	Magnetic field	II.	$[L^2A^1]$
C.	Magnetic moment	III.	$[M^1T^{-2}A^{-1}]$
D.	Permeability of free space	IV.	$[M^1L^2T^{-2}]$

   Choose the correct answer from the options given below: **[06 April, 2024 (Shift-I)]**
   (*a*) A-I, B-III, C-II, D-IV (*b*) A-IV, B-III, C-II, D-I
   (*c*) A-III, B-I, C-II, D-IV (*d*) A-IV, B-II, C-III, D-I

10. What is the dimensional formula of ab^{-1} in the equation $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where letters have their usual meaning. **[05 April, 2024 (Shift-II)]**
    (*a*) $[M^0L^3T^{-2}]$ (*b*) $[ML^2T^{-2}]$
    (*c*) $[M^{-1}L^5T^3]$ (*d*) $[M^6L^7T^4]$

11. If G be the gravitational constant and u be the energy density then which of the following quantity has the dimension as that of $\sqrt{uG}$: **[05 April, 2024 (Shift-I)]**
    (*a*) Pressure gradient per unit mass
    (*b*) Force per unit mass
    (*c*) Gravitational potential
    (*d*) Energy per unit mass

12. The dimensional formula of angular impulse is: **[1 Feb, 2024 (Shift-I)]**
    (*a*) $[ML^{-2}T^{-1}]$ (*b*) $[ML^2T^{-2}]$
    (*c*) $[MLT^{-1}]$ (*d*) $[ML^2T^{-1}]$

13. Given below are two statements: [27 Jan, 2024 (Shift-I)]

Statement-I: Planck's constant and angular momentum have same dimensions.

Statement-II: Linear momentum and moment of force have same dimensions.

In the light of the above statements, choose the correct answer from the options given below:

(*a*) Statement-I is true but Statement-II is false

(*b*) Both Statement-I and Statement-II are false

(*c*) Both Statement-I and Statement-II are true

(*d*) Statement-I is false but Statement-II is true

14. Match List-I with List-II. [30 Jan, 2024 (Shift-I)]

	List-I		List-II
A.	Coefficient of viscosity	I.	$[ML^2T^{-2}]$
B.	Surface Tension	II.	$[ML^2T^{-1}]$
C.	Angular momentum	III.	$[ML^{-1}T^{-1}]$
D	Rotational kinetic energy	IV.	$[ML^0T^{-2}]$

(*a*) A-II, B-I, C-IV, D-III (*b*) A-I, B-II, C-III, D-IV

(*c*) A-III, B-IV, C-II, D-I (*d*) A-IV, B-III, C-II, D-I

15. Match List-I with List-II

	List-I		List-II
A.	Spring constant	I.	(T^{-1})
B.	Angular speed	II.	(MT^{-2})
C.	Angular momentum	III.	(ML^2)
D.	Moment of Inertia	IV.	(ML^2T^{-1})

Choose the correct answer from the options given below:
[12 April, 2023 (Shift-I)]

(*a*) A-II, B-I, C-IV, D-III (*b*) A-IV, B-I, C-III, D-II

(*c*) A-II, B-III, C-I, D-IV (*d*) A-I, B-III, C-II, D-IV

16. Match List-I with List-II

	List-I		List-II
A.	Planck's constant (*h*)	I.	$[M^1L^2T^{-2}]$
B.	Stopping potential (V_s)	II.	$[M^1L^1T^{-1}]$
C.	Work function (ϕ)	III.	$[M^1L^2T^{-1}]$
D.	Momentum (*p*)	IV.	$[M^1L^2T^{-3}A^{-1}]$

[24 Jan, 2023 (Shift-I)]

(*a*) A-III, B-I, C-II, D-IV (*b*) A-III, B-IV, C-I, D-II

(*c*) A-II, B-IV, C-III, D-I (*d*) A-I, B-III, C-IV, D-II

17. Which of the following physical quantities have the same dimensions? [25 July, 2022 (Shift-I)]

(*a*) Electric displacement ($\vec{D}$) and surface charge density

(*b*) Displacement current and electric field

(*c*) Current density and surface charge density

(*d*) Electric potential and energy

18. Match List-I with List-II. [27 July, 2021 (Shift-II)]

	List-I		List-II
A.	Capacitance, *C*	I.	$M^1L^1T^{-3}A^{-1}$
B.	Permittivity of free space, ε_0	II.	$M^{-1}L^{-3}T^4A^2$
C.	Permeability of free space, μ_0	III.	$M^{-1}L^{-2}T^4A^2$
D.	Electric field, E	IV.	$M^1L^1T^{-2}A^{-2}$

Choose the correct answer from the options given below:

(*a*) A-III, B-II, C-IV, D-I (*b*) A-III, B-IV, C-II, D-I

(*c*) A-IV, B-II, C-III, D-I (*d*) A-IV, B-III, C-II, D-I

19. If surface tension (*S*), Moment of inertia (*I*) and Planck's constant (*h*), were to be taken as the fundamental units, the dimensional formula for linear momentum would be
[8 April, 2019 (Shift-II)]

(*a*) $S^{3/2}I^{1/2}h^0$ (*b*) $S^{1/2}I^{1/2}h^0$

(*c*) $S^{1/2}I^{1/2}h^{-1}$ (*d*) $S^{1/2}I^{3/2}h^{-1}$

APPLICATION OF DIMENSIONAL ANALYSIS

20. Applying the principle of homogeneity of dimensions, determine which one is correct, where *T* is time period, *G* is gravitational constant, *M* is mass, *r* is radius of orbit. [04 April, 2024 (Shift-II)]

(*a*) $T^2 = \frac{4\pi^2 r}{GM^2}$ (*b*) $T^2 = 4\pi^2 r^3$

(*c*) $T^2 = \frac{4\pi^2 r^3}{GM}$ (*d*) $T^2 = \frac{4\pi^2 r^2}{GM}$

21. Consider two physical quantities *A* and *B* related to each other as $E = \frac{B - x^2}{At}$ where *E*, *x* and *t* have dimensions of energy, length and time respectively. The dimension of *AB* is [31 Jan, 2024 (Shift-II)]

(*a*) $L^{-2}M^1T^0$ (*b*) $L^2M^{-1}T^1$

(*c*) $L^{-2}M^{-1}T^1$ (*d*) $L^0M^{-1}T^1$

22. A force is represented by $F = ax^2 + bt^{1/2}$ Where *x* = distance and *t* = time. The dimensions of b^2/a are:
[31 Jan, 2024 (Shift-I)]

(*a*) $[ML^3T^{-3}]$ (*b*) $[MLT^{-2}]$

(*c*) $[ML^{-1}T^{-1}]$ (*d*) $[ML^2T^{-3}]$

23. If mass is written as $m = k\,c^P\,G^{-1/2}\,h^{1/2}$ then the value of *P* will be: (Constants have their usual meaning with *k* a dimensionless constant) [30 Jan, 2024 (Shift-II)]

(*a*) 1/2 (*b*) 1/3 (*c*) 2 (*d*) −1/3

24. The equation of state of a real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where *P*, *V* and *T* are pressure volume and temperature respectively and *R* is the universal gas constant. The dimensions of $\frac{a}{b^2}$ is similar to that of: [27 Jan, 2024 (Shift-II)]

(*a*) *PV* (*b*) *P* (*c*) *RT* (*d*) *R*

25. The frequency (v) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as: $v = r^a \rho^b s^c$. The values of a, b and c respectively are **[24 Jan, 2023 (Shift-II)]**

(a) $\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right)$ (d) $\left(-\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$

26. Young's modulus of elasticity Y is expressed in terms of three derived quantities, namely, the gravitational constant G, Planck's constant h and the speed of light c, as $Y = c^\alpha h^\beta G^\gamma$. Which of the following is the correct option? **[JEE Adv, 2023]**

(a) $\alpha = 7, \beta = -1, \gamma = -2$

(b) $\alpha = -7, \beta = -1, \gamma = -2$

(c) $\alpha = 7, \beta = -1, \gamma = 2$

(d) $\alpha = -7, \beta = 1, \gamma = -2$

27. In a particular system of units, a physical quantity can be expressed in terms of the electric charge e, electron mass m_e, Planck's constant h, and coulomb's constant $k = \frac{1}{4\pi\varepsilon_0}$, where ε_0 is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^\alpha [m_e]^\beta [h]^\gamma [k]^\delta$. The value of $\alpha + \beta + \gamma + \delta$ is ____________. **[JEE Adv, 2022]**

28. If time (t), velocity (v), and angular momentum (ℓ) are taken as the fundamental units. Then the dimension of mass (m) in terms of t, v, and ℓ is: **[20 July, 2021 (Shift-II)]**

(a) $[t^{-2} v^{-1} \ell^1]$ (b) $[t^{-1} v^1 \ell^{-2}]$

(c) $[t^{-1} v^{-2} \ell^1]$ (d) $[t^1 v^2 \ell^{-1}]$

29. In a typical combustion engine the work done by a gas molecule is given by $W = \alpha^2 \beta e^{-\frac{\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be: **[26 Feb, 2021 (Shift-I)]**

(a) $[MLT^{-1}]$ (b) $[M^0 L T^0]$ (c) $[M^2LT^{-2}]$ (d) $[MLT^{-2}]$

30. Dimensional formula for thermal conductivity is (here K denotes the temperature): **[04 Sep, 2020 (Shift-I)]**

(a) $MLT^{-3} K^{-1}$ (b) $MLT^{-2} K^{-2}$

(c) $MLT^{-2} K$ (d) $MLT^{-3} K$

31. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to: **[9 Jan, 2019 (Shift-II)]**

(a) $\sqrt{\frac{hc^5}{G}}$ (b) $\sqrt{\frac{c^3}{Gh}}$

(c) $\sqrt{\frac{Gh}{c^5}}$ (d) $\sqrt{\frac{Gh}{c^3}}$

32. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement(s) is/are correct? **[JEE Adv, 2019]**

(a) The dimension of force is L^{-3}.

(b) The dimension of energy of L^{-2}.

(c) The dimension of power is L^{-5}.

(d) The dimension of linear momentum is L^{-1}.

ERRORS IN MEASUREMENT

33. Young's modulus is determined by the equation given by $Y = 49000 \frac{m}{\ell} \frac{\text{dyne}}{\text{cm}^2}$ where m is the mass and ℓ is the extension of wire used in the experiment. Now error in Young modulus (Y) is estimated by taking data from m-ℓ plot in graph paper. The smallest scale divisions are 5 g and 0.02 cm along load axis and extension axis respectively. If the value of m and ℓ are 500 g and 2 cm respectively then percentage error of Y is: **[08 April, 2024 (Shift-I)]**

(a) 0.2% (b) 0.02% (c) 2% (d) 0.5%

34. In an expression $a \times 10^b$: **[08 April, 2024 (Shift-I)]**

(a) a is order of magnitude for $b \le 5$

(b) b is order of magnitude for $a \le 5$

(c) b is order of magnitude for $5 < a \le 10$

(d) b is order of magnitude for $a \ge 5$

35. To find the spring constant (k) of a spring experimentally, a student commits 2% positive error in the measurement of time and 1% negative error in measurement of mass. The percentage error in determining value of k is: **[06 April, 2024 (Shift-I)]**

(a) 3% (b) 1% (c) 4% (d) 5%

36. Time periods of oscillation of the same simple pendulum measured using four different measuring clocks were recorded as 4.62 s, 4.632s, 4.6s and 4.64s. The arithmetic mean of these reading in correct significant figure is. **[05 April, 2024 (Shift-I)]**

(a) 4.623 s (b) 4.62 s

(c) 4.6 s (d) 5 s

37. In an experiment to measure focal length (f) of convex lens, the least counts of the measuring scales for the position of object (u) and for the position of image (v) are Δu and Δv, respectively. The error in the measurement of the focal length of the convex lens will be: **[04 April, 2024 (Shift-I)]**

(a) $\frac{\Delta u}{u} + \frac{\Delta v}{v}$ (b) $f^2\left[\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}\right]$

(c) $2f\left[\frac{\Delta u}{u} + \frac{\Delta v}{v}\right]$ (d) $f\left[\frac{\Delta u}{u} + \frac{\Delta v}{u}\right]$

38. Match List-I with List-II.

	List-I (Number)		List-II (Significant figure)
(A)	1001	(I)	3
(B)	010.1	(II)	4
(C)	100.100	(III)	5
(D)	0.0010010	(IV)	6

Choose the correct answer from the options given below:
[1 Feb, 2024 (Shift-II)]

(*a*) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(*b*) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(*c*) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(*d*) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

39. The radius (r), length (l) and resistance (R) of a metal wire was measured in the laboratory as
$r = (0.35 \pm 0.05)$ cm, $R = (100 \pm 10)$ ohm, $l = (15 \pm 0.2)$ cm
The percentage error in resistivity of the material of the wire is: **[1 Feb, 2024 (Shift-I)]**
(*a*) 25.6% (*b*) 39.9% (*c*) 37.3% (*d*) 35.6%

40. The measured value of the length of a simple pendulum is 20 cm with 2 mm accuracy. The time for 50 oscillations was measured to be 40 seconds with 1 second resolution. From these measurements, the accuracy in the measurement of acceleration due to gravity is N%. The value of N is: **[31 Jan, 2024 (Shift-II)]**
(*a*) 4 (*b*) 8 (*c*) 6 (*d*) 5

41. If the percentage errors in measuring the length and the diameter of a wire are 0.1% each. The percentage error in measuring its resistance will be: **[31 Jan, 2024 (Shift-I)]**
(*a*) 0.2% (*b*) 0.3% (*c*) 0.1% (*d*) 0.144%

42. The resistance $R = \frac{V}{I}$ where $V = (200 \pm 5)$V and $I = (20 \pm 0.2)$A, the percentage error in the measurement of R is: **[29 Jan, 2024 (Shift-I)]**
(*a*) 3.5% (*b*) 7% (*c*) 3% (*d*) 5.5%

43. A physical quantity Q is found to depend on quantities a, b, c by the relation $Q = \frac{a^4b^3}{c^2}$ The percentage error in a, b and c are 3%, 4% and 5% respectively. Then, the percentage error in Q is: **[29 Jan, 2024 (Shift-II)]**
(*a*) 66% (*b*) 43% (*c*) 34% (*d*) 14%

44. Two resistances are given as $R_1 = (10 \pm 0.5)\Omega$ and $R_2 = (15 \pm 0.5)\Omega$. The percentage error in the measurement of equivalent resistance when they are connected in parallel is **[6 Apr, 2023 (Shift-I)]**
(*a*) 6.33 (*b*) 2.33 (*c*) 4.33 (*d*) 5.33

45. If $Z = \frac{A^2B^3}{C^4}$, then the relative error in Z will be: **[25 June, 2022 (Shift-I)]**
(*a*) $\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$ (*b*) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} - \frac{4\Delta C}{C}$
(*c*) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}$ (*d*) $\frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{\Delta C}{C}$

46. A student determined Young's Modulus of elasticity using the formula $Y = \frac{MgL^3}{4bd^3\delta}$. The value of g is taken to be 9.8 m/s^2, without any significant error, his observation are as follows: **[1 Sept, 2021 (Shift-II)]**

Physical Quantity	Least count of the Equipment used for measurement	Observed value
Mass (M)	1 g	2 kg
Length of bar (L)	1 mm	1 m
Breadth of bar (b)	0.1 mm	4 cm
Thickness of bar (d)	0.01 mm	0.4 cm
Depression (δ)	0.01 mm	5 mm

Then the fractional error in the measurement of Y is:
(*a*) 0.083 (*b*) 0.0155
(*c*) 0.0083 (*d*) 0.155

47. A simple pendulum is being used to determine the value of gravitational acceleration g at a certain place. The length of the pendulum is 25.0 cm and a stop watch with 1s resolution measures the time taken for 40 oscillations to be 50 s. The accuracy in g is: **[8 Jan, 2020 (Shift-II)]**
(*a*) 4.40% (*b*) 3.40%
(*c*) 2.40% (*d*) 5.40%

48. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to: **[8 April, 2019 (Shift-II)]**
(*a*) 0.7% (*b*) 3.5% (*c*) 6.8% (*d*) 0.2%

MEASURING INSTRUMENTS

49. One main scale division of a vernier calliper is equal to m units. If nth division of main scale coincides with $(n + 1)^{th}$ division of vernier scale, the least count of the vernier calliper is: **[09 April, 2024 (Shift-II)]**
(*a*) $\frac{n}{(n+1)}$ (*b*) $\frac{m}{(n+1)}$
(*c*) $\frac{1}{(n+1)}$ (*d*) $\frac{m}{n(n+1)}$

50. There are 100 divisions on the circular scale of a screw gauge of pitch 1 mm. With no measuring quantity in between the jaws, the zero of the circular scale lies 5 divisions below the reference line. The diameter of a wire is then measured using this screw gauge. It is found the 4 linear scale divisions are clearly visible while 60 divisions on circular scale coincide with the reference line. The diameter of the wire is: **[08 April, 2024 (Shift-II)]**
(*a*) 4.65 mm (*b*) 4.55 mm
(*c*) 4.60 mm (*d*) 3.35 mm

51. Least count of a vernier calliper is $\frac{1}{20N}$ cm. The value of one division on the main scale is 1 mm. Then the number of divisions of main scale that coincide with N divisions of vernier scale is: **[08 April, 2024 (Shift-II)]**

(*a*) $\left(\frac{2N-1}{20N}\right)$ (*b*) $\left(\frac{2N-1}{2}\right)$

(*c*) $(2N-1)$ (*d*) $\left(\frac{2N-1}{2N}\right)$

52. The diameter of a sphere is measured using a vernier caliper whose 9 divisions of main scale are equal to 10 divisions of vernier scale. The shortest division on the main scale is equal to 1 mm. The main scale reading is 2 cm and second division of vernier scale coincides with a division on main scale. If mass of the sphere is 8.635 g, the density of the sphere is: **[08 April, 2024 (Shift-I)]**

(*a*) 2.5 g/cm^3 (*b*) 1.7 g/cm^3

(*c*) 2.2 g/cm^3 (*d*) 2.0 g/cm^3

53. In a vernier calliper, when both jaws touch each other, zero of the vernier scale shifts towards left and its 4th division coincides exactly with a certain division on main scale. If 50 vernier scale divisions equal to 49 main scale divisions and zero error in the instrument is 0.04 mm then how many main scale divisions are there in 1 cm? **[06 April, 2024 (Shift-II)]**

(*a*) 40 (*b*) 5 (*c*) 20 (*d*) 10

54. In finding out refractive index of glass slab the following observations were made through travelling microscope 50 vernier scale division = 49 MSD; 20 divisions on main scale in each cm **[06 April, 2024 (Shift-II)]**

For mark on paper

MSR = 8.45 cm, VC = 26

For mark on paper seen through slab

MSR = 7.12 cm, VC = 41

For powder particle on the top surface of the glass slab

MSR = 4.05 cm, VC = 1

(MSR = Main Scale Reading, VC = Vernier Coincidence)

Refractive index of the glass slab is:

(*a*) 1.42 (*b*) 1.52 (*c*) 1.24 (*d*) 1.35

55. While measuring diameter of wire using screw gauge the following readings were noted. Main scale reading is 1 mm and circular scale reading is equal to 42 divisions. Pitch of screw gauge is 1 mm and it has 100 divisions on circular scale. The diameter of the wire is $\frac{x}{50}$ mm. The value of x is: **[06 April, 2024 (Shift-I)]**

(*a*) 142 (*b*) 71

(*c*) 42 (*d*) 21

56. A vernier callipers has 20 divisions on the vernier scale, which coincides with 19th division on the main scale. The least count of the instrument is 0.1 mm. One main scale division is equal to ____ mm. **[05 April, 2024 (Shift-II)]**

(*a*) 1 (*b*) 0.5

(*c*) 2 (*d*) 5

57. 10 divisions on the main scale of a Vernier calliper coincide with 11 divisions on the Vernier scale. If each division on the main scale is of 5 units, the least count of the instrument is: **[1 Feb, 2024 (Shift-I)]**

(*a*) $\frac{1}{2}$ (*b*) $\frac{10}{11}$ (*c*) $\frac{50}{11}$ (*d*) $\frac{5}{11}$

58. If 50 Vernier divisions are equal to 49 main scale divisions of a travelling microscope and one smallest reading of main scale is 0.5 mm, the Vernier constant of travelling microscope is: **[30 Jan, 2024 (Shift-II)]**

(*a*) 0.1 mm (*b*) 0.1 cm (*c*) 0.01 cm (*d*) 0.01 mm

59. Identify the physical quantity that cannot be measured using spherometer: **[27 Jan, 2024 (Shift-I)]**

(*a*) Radius of curvature of concave surface

(*b*) Specific rotation of liquids

(*c*) Thickness of thin plates

(*d*) Radius of curvature of convex surface

60. Given below are two statements: one is labelled as Assertion(A) and the other is labelled as Reason (R).

Assertion (A): In Vernier calliper if positive zero error exists, then while taking measurements, the reading taken will be more than the actual reading.

Reason (R): The zero error in Vernier Calliper might have happened due to manufacturing defect or due to rough handling.

In the light of the above statements, choose the correct answer from the options given below: **[27 JAN, 2024 (Shift-II)]**

(*a*) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(*b*) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(*c*) (A) is true but (R) is false

(*d*) (A) is false but (R) is true

61. Given below are two statements:

Statements-I: Astronomical unit (Au). Parsec (Pc) and Light year (ly) are units for measuring astronomical distances.

Statements-II: Au < Parsec (Pc) < ly

In the light of the above statements. Choose the most appropriate answer from the options given below: **[11 Apr, 2023 (Shift-I)]**

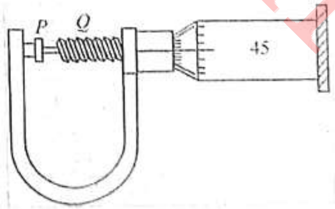
(*a*) Both Statements-I and Statements-II are correct

(*b*) Statements-I is correct but Statements-II is incorrect

(*c*) Both Statements-I and Statements-II are incorrect

(*d*) Statements-I is incorrect but statements-II is correct

62. In an experiment to find out the diameter of wire using screw gauge, the following observations were noted:



A. Screw moves 0.5 mm on main scale in one complete rotation.
B. Total divisions on circular scale 50
C. Main scale reading is 2.5 mm
D. 45^{th} division of circular scale is in the pitch line

Then the diameter of wire is **[29 July, 2022 (Shift-I)]**

(*a*) 2.92 mm (*b*) 2.54 mm
(*c*) 2.98 mm (*d*) 3.45 mm

63. The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that zero on the vernier scale lies between 8.5 cm and 8.6 cm, and vernier coincidence is 6, then the correct value of measurement is ______ (least count = 0.01 cm) **[17 March, 2021 (Shift-I)]**

(*a*) 8.58 cm (*b*) 8.54 cm
(*c*) 8.56 cm (*d*) 8.36 cm

64. **Assertion A:** If in five complete rotations of the circular scale, the distance travelled on main scale of the screw gauge is 5 mm and there are 50 total divisions on circular scale, then least count is 0.001 cm.

Reason R: Least Count $= \dfrac{\text{Pitch}}{\text{Total divisions on circular scle}}$

In the light of the above statement, choose the most appropriate answer from the options given below: **[27 July, 2021 (Shift-I)]**

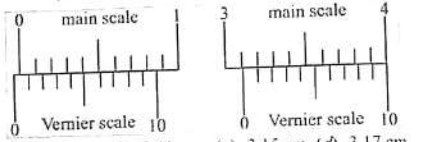
(*a*) A is correct but **R** is not correct.

(*b*) A is not correct but **R** is correct.

(*c*) Both **A** and **R** are correct and **R** is NOT the correct explanation of **A**.

(*d*) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

65. The smallest division on the main scale of a Vernier callipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this calliper with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is **[JEE Adv, 2021]**



(*a*) 3.07 cm (*b*) 3.11 cm (*c*) 3.15 cm (*d*) 3.17 cm

66. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as **[3 Sep, 2020 (Shift-I)]**

(*a*) 2.121 cm (*b*) 2.123 cm
(*c*) 2.124 cm (*d*) 2.125 cm

67. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is **[12 Jan, 2019 (Shift-I)]**

(*a*) 50 (*b*) 200
(c) 100 (*d*) 500

> *"Gratitude can transform common days into thanksgivings, turn routine jobs into joy, and change ordinary opportunities into blessings."*
>
> **– William Arthur Ward**

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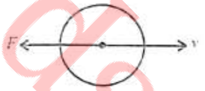
SINGLE CORRECT TYPE QUESTIONS

1. In a certain fluid, measurements indicate that drag force F is proportional to velocity of the object v in power of $\frac{3}{2}$.

$F \propto v^{3/2}$



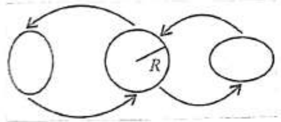
If two spheres of radii r_1 and r_2 are moving with equal velocity through same fluid they will experience drag force F_1 and F_2 respectively. If $r_2 = 4r_1$ find $\frac{F_2}{F_1}$.

(The density of fluid is ρ and coefficient of viscosity is η)

(a) 2 (b) 4

(c) 6 (d) 8

2. A small droplet of water floating at a space station usually has a spherical shape due to surface tension which changes to ellipsoidal shape with some regular time interval T

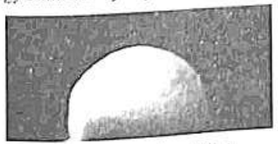


Find ratio $\frac{T_2}{T_1}$ of two period of oscillation T_1 and T_2 for water droplets with radii r_1 and r_2 respectively where $r_2 = 2r_1$. Assume no artificial gravity is present at the space station.

(a) 2 (b) 4

(c) $\sqrt{2}$ (d) $2\sqrt{2}$

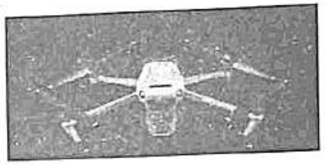
3. British physicist G.I Taylor was able to estimate equivalent energy of E of first atomic bomb by looking at pictures that were published in a popular magazine. From the pictures it was clearly established that after time $t = 0.006$s, radius of the cloud was $r \simeq 80$ m. If the density of air $\rho = 1.2$ kg/m^2, the energy estimated by Taylor was



(a) 10^{14} J (b) 10^{10} J

(c) 10^{16} J (d) 10^{8} J

4. A certain drone is able to hover if the mechanical power input is P_1. If an exact replica is made where all linear dimensions have been doubled, then new power P_2 of the mechanical input is



(a) $2^{5/2}P_1$ (b) $2^{3/2}P_1$

(c) $2^{7/2}P_1$ (d) $2^{1/2}P_1$

5. In cosmology, geometrized system of units (GU) is used where we assume

$G = c = 1$

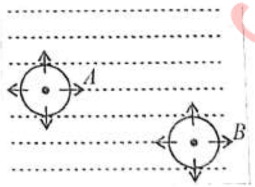
Angular momentum L_{GU} in geometrized units is measured in m^2. For conversion between geometrized units and SI units for angular momentum we need a conversion factor such that following relation is true $L_{SI} = c^x G^y L_{GU}$

Find the value of x and y

(a) 3, −1 (b) −1, 3

(c) 2, −3 (d) −2, 3

6. Two identical amounts of explosive are burst under water at two different location A and B where hydrostalic pressure are P_A and P_B respectively ($P_B = 1.3\ P_A$). A hollow space occurring due to under water explosion expands and shrinks with some regular period of time T which does not depend on size of the hollow bubble. Assuming that sizes of hollow spaces are much smaller than depths of explosion A and B, determine ratio x which is defined as $x = \frac{T_B}{T_A}$ where T_B and T_A are periods of oscillations of two bubbles occurring at location A and B respectively.



(a) 0.2 (b) 0.5

(c) 0.8 (d) 0.4

7. The pitch of a screw gauge is 1 mm and its cap is divided into 100 divisions. When nothing is placed between its studs, the zero marking of the circular scale is getting 4 divisions above the reference line. When a wire is placed between its studs, the main scale reading is 2 mm and 50th division of circular scale coincides with the reference line. If the length of the wire is 10.5 cm, then find the curved surface area (in cm^2) of the wire in three significant figures.
(*a*) 8.50 cm^2
(*b*) 8.54 cm^2
(*c*) 8.58 cm^2
(*d*) 9.00 cm^2

8. Suppose that you measure three independent variables as $x = 10.0 \pm 0.2$, $y = 7.0 \pm 0.1$, $\theta = 40.0 \pm 0.3°$
and use these values to compute $q = \dfrac{x+2}{x + y\cos 40}$.
What should be your answer for q and its uncertainty?
(*a*) 3.5 ± 0.1
(*b*) 2.5 ± 0.1
(*c*) 3.5 ± 0.3
(*d*) 2.5 ± 0.3

9. The Atwood machine consists of two masses M and m (with $M > m$) attached to the ends of a light string that passes over a light, frictionless pulley. When the masses are released, the mass M is easily shown to accelerate down with an acceleration

$$a = g\frac{M-m}{M+m}$$

Suppose that M and m are measured as $M = 100 \pm 1$ and $m = 50 \pm 1$, both in grams. Find a and the uncertainty δa.
(*a*) 3.3 ± 0.1
(*b*) 3.3 ± 0.2
(*c*) 3.3 ± 0.3
(*d*) 3.3 ± 0.01

10. If a narrow collimated beam of monoenergetic γ-ray of intensity I_0 is incident on a thin sheet of material of thickness 'x', the intensity of the emerging beam is given by
$I = I_0 \exp(-\mu x)$
where μ is a quantity known as the linear attenuation coefficient. The following values are obtained for γ-rays of energy 1 MeV incident on lead:
$I = (0.926 \pm 0.010) \times 10^{10}$ γ-rays $m^{-2}s^{-1}$
$I_0 = (2.026 \pm 0.012) \times 10^{10}$ γ-rays $m^{-2}s^{-1}$
$x = (10.00 \pm 0.02)$ mm
Find out μ and corresponding absolute error in μ.

Why was the physicist grateful for vectors?
They always pointed him in the right direction!

ANSWER KEY

CONCEPT APPLICATION

1. $[A] = [M^0L^0T^0]$, $[B] = [M^0L^0T^0]$ 2. $[A] = [L^{1/2}T^{-1}]$, $[B] = [LT^{-1}]$ 3. $[A] = [L]$, $[B] = M^0L^0T^2$, $[C] = [M^0L^0T^0]$

4. $[L^2T^{-1}]$ 5. (c) 6. (d) 7. $m = Kc^{\frac{1}{2}}h^{\frac{1}{2}}G^{-\frac{1}{2}}$ 8. (a) 9. (c) 10. (a) 11. (d) 12. (d)

13. (a) 14. (d) 15. (d) 16. (c) 17. (b) 18. (d) 19. (c) 20. (d) 21. [0.002]

22. [1.004 ± 0.001] 23. (c) 24. (d) 25. (a) 26. (c) 27. [1] 28. (b) 29. [0.005] 30. [0.215]

31. (a)

SCHOOL LEVEL PROBLEMS

1. (c) 2. (a) 3. (b) 4. (c) 5. (a) 6. (b) 7. (a) 8. (b) 9. (d) 10. (a)

11. (c) 12. (c) 24. (i) (c), (ii) (c), (iii) (d), (iv) (a), (v) (c) 25. (i) (a), (ii) (c), (iii) (d), (iv) (a), (v) (c)

PRARAMBH (TOPICWISE)

1. (b) 2. (a) 3. (a) 4. (c) 5. (b) 6. (c) 7. (b) 8. (b) 9. (d) 10. (a)

11. (d) 12. (d) 13. (c) 14. (c) 15. (b) 16. (b) 17. (b) 18. (d) 19. (b) 20. (a)

21. (c) 22. (c) 23. (c) 24. (a) 25. (b) 26. (a) 27. (c) 28. (a) 29. (d) 30. (b)

31. (b) 32. (b) 33. (c) 34. (b) 35. (c) 36. (a) 37. (b) 38. (a) 39. (b) 40. (a)

41. (c) 42. (c) 43. (a) 44. (a) 45. (a) 46. (a) 47. (c) 48. (b) 49. (b) 50. (b)

51. (c) 52. (c) 53. (a) 54. (a) 55. (b) 56. (b) 57. (c) 58. (c) 59. (c) 60. (a)

61. (b)

PRABAL (JEE MAIN LEVEL)

1. (b) 2. (d) 3. (c) 4. (b) 5. (c) 6. (d) 7. (a) 8. (b) 9. (c) 10. (d)

11. (d) 12. (c) 13. (d) 14. (d) 15. (d) 16. (a) 17. (c) 18. (c) 19. (d) 20. (c)

21. (b) 22. (b) 23. (a) 24. (d) 25. (c) 26. (b) 27. (b) 28. (b) 29. (d) 30. (a)

31. (c) 32. (a) 33. (a) 34. (d) 35. (b) 36. (c) 37. (b) 38. (a) 39. (a) 40. (b)

41. (c) 42. (a) 43. [1] 44. [3] 45. [4] 46. [0] 47. [9] 48. [3] 49. [6] 50. [6]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (a,b,c) 2. (c,d) 3. (a,b,c) 4. (a,b,c,d) 5. (a,c) 6. (b,d) 7. (a,b,c,d) 8. (a,b,c,d) 9. (b) 10. (a)

11. (c) 12. (c) 13. (d) 14. (b) 15. (d) 16. (d) 17. (a) 18. (c) 19. (a) 20. (a)

21. (a) 22. (c) 23. (c) 24. (c) 25. (a) 26. (b) 27. (d) 28. (c) 29. (a) 30. (c)

31. [2.59] 32. [92.14] 33. [0.41] 34. [1.67] 35. [0] 36. [1] 37. [4] 38. (1) 39. (10) 40. [928]

41. (d) 42. (c) 43. (b) 44. (b) 45. (c) 46. (a)

PYQ's (PAST YEAR QUESTIONS)

1. (b) 2. (c) 3. (c) 4. (b,d) 5. (a) 6. (c) 7. (b) 8. (c) 9. (b) 10. (b)

11. (b) 12. (d) 13. (a) 14. (c) 15. (a) 16. (b) 17. (a) 18. (a) 19. (b) 20. (c)

21. (b) 22. (a) 23. (a) 24. (b) 25. (a) 26. (a) 27. [4] 28. (c) 29. (b) 30. (a)

31. (c) 32. (a,b,d) 33. (c) 34. (b) 35. (d) 36. (c) 37. (b) 38. (c) 39. (b) 40. (c)

41. (b) 42. (a) 43. (c) 44. (c) 45. (c) 46. (b) 47. (a) 48. (c) 49. (b) 50. (b)

51. (b) 52. (d) 53. (c) 54. (a) 55. (b) 56. (c) 57. (d) 58. (d) 59. (b) 60. (b)

61. (b) 62. (a) 63. (b) 64. (b) 65. (c) 66. (c) 67. (b)

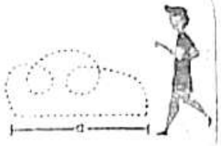
PW CHALLENGERS

1. (d) 2. (d) 3. (a) 4. (c) 5. (a) 6. (c) 7. (c) 8. (c) 9. (a) 10. [78.3 ± 1.9]

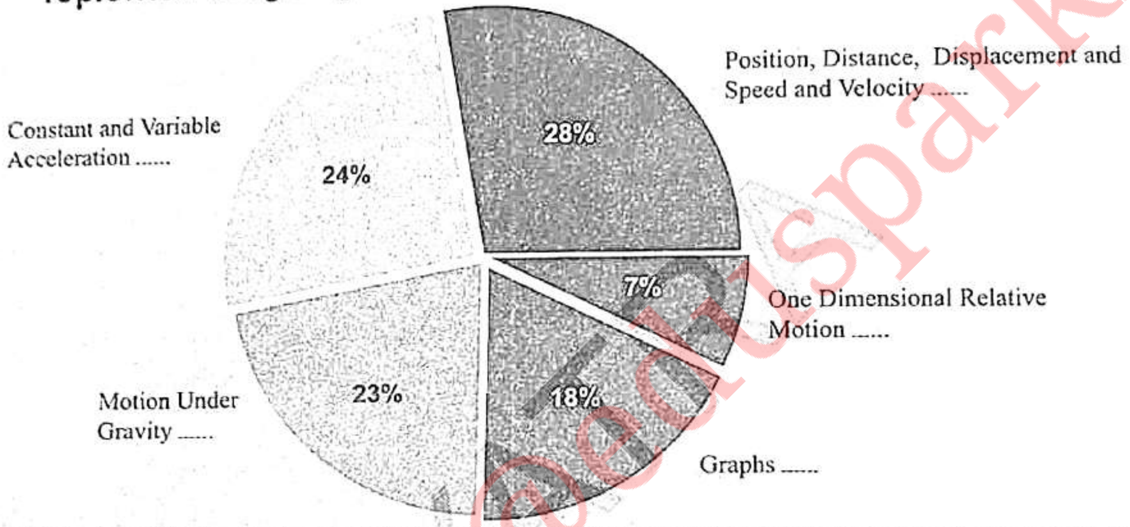
CHAPTER

3

Motion in a Straight Line



Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

INTRODUCTION

A body is at rest when it does not change its position with time and is in motion if it changes its position with time in the frame of reference of the observer.

All motion is relative. There is no meaning of rest or motion without reference to the observer.

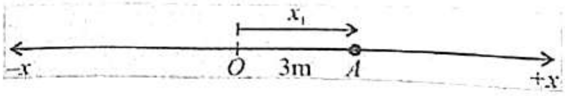
A passenger in a moving train is at rest with respect to another passenger in the same train while both are in motion with respect to observer on the ground. Therefore nothing is at absolute rest or in absolute motion.

To describe the motion of a particle, we introduce four important quantities namely position, displacement, velocity and acceleration. In general motion of a particle in three dimensions these quantities are vectors which have direction as well as magnitude. But for a particle moving in a straight line, there are only two directions, distinguished by designating one as positive and other as negative.

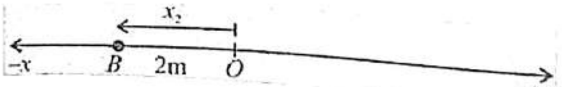
DISTANCE AND DISPLACEMENT

Position

The position of a particle is the location of particle measured with respect to some reference point. It is a vector quantity.



Position of A wrt O, $x_1 = 3$ m



Position of B wrt O, $x_2 = -2$ m

If particle lies towards +ve side of the chosen reference, then its position is also +ve and vice-versa.

Distance

The length of the actual path between initial and final positions of a particle is called distance covered by the particle. It is the actual length of the path covered by the body.

Characteristics of Distance

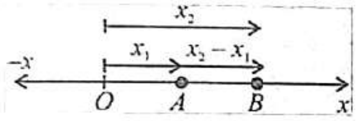
- It is a scalar quantity
- It depends on the path
- It never reduces with time
- Distance covered by a particle is always positive or zero and can never be negative
- Dimension: $[M^0 L^1 T^0]$
- Unit: In CGS: centimeter (cm). In SI system: meter (m).

Displacement

The shortest distance from the initial position to the final position of the particle is called displacement. The displacement of a particle is measured as the change in the position of the particle in a particular direction over a given time interval. It depends only on final and initial positions.

Position of A w.r.t. $O = OA = x_1$

Position of B w.r.t. $O = OB = x_2$



Displacement $= AB = x_2 - x_1$

The direction of displacement here is from A to B. If direction of displacement is along +ve direction of the chosen reference then displacement is +ve and vice versa.

Characteristics of Displacement

- It is a vector quantity.
- The displacement of a particle between any two points is equal to the shortest distance between them.
- The displacement of an object in a given time interval may be +ve, –ve or zero.
- The actual distance travelled by a particle in the given interval of time is always equal to or greater than the magnitude of the displacement and in no case, it is less than the magnitude of the displacement, i.e. Distance ≥ |Displacement|
- Dimension: $[M^0 L^1 T^0]$
- Unit: In CGS: centimeter (cm). In SI: system: meter (m).

Note: Distance is always positive but displacement may be +ve, – ve or zero.

AVERAGE SPEED AND INSTANTANEOUS SPEED

Average speed is the ratio of total distance covered by a particle in a given time interval divided by the time interval.

$$v_{avg} = \frac{\Delta s}{\Delta t} \qquad (\text{where } \Delta t = t_2 - t_1)$$

Instantaneous speed at any instant t is

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

- The slope of the distance-time graph provides the value of instantaneous speed.
- The average speed is defined for a time interval while the instantaneous speed is defined at an instant. The word speed normally implies instantaneous speed.
- Average speed and instantaneous speed both are scalar quantities.
- For any moving object, the average speed can never be zero or negative, i.e., $v_{av} > 0$, as total distance covered is always +ve only.
- If a particle travels distances s_1, s_2, s_3 , ..., etc., at different speeds v_1, v_2, v_3, ... etc., respectively, then

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{\Sigma s_i}{\Sigma(s_i / v_i)}$$

If $s_1 = s_2 = ... = s_n = s$,

Then $\frac{1}{v_{av.}} = \frac{1}{n}\left[\frac{1}{v_1} + \frac{1}{v_2} + ...\right] = \frac{1}{n}\Sigma\frac{1}{v_i}$

Special case: If a particle moves a distance at speed v_1 and comes back to initial position with speed v_2, then

$$v_{av} = \frac{2v_1v_2}{v_1 + v_2}$$

- If a particle travels at speeds v_1, v_2, ..., etc., for time intervals t_1, t_2, ..., then

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{v_1t_1 + v_2t_2 + ...+ v_nt_n}{t_1 + t_2 + ...t_n} = \frac{\Sigma v_i t_i}{\Sigma t_i}$$

Special case: If a particle moves for two equal intervals of time at different speeds, then $v_{av} = \frac{v_1 + v_2}{2}$.

AVERAGE VELOCITY

The average velocity is the ratio of displacement from time t_1 to t_2 and the time interval $t_2 - t_1$.

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

INSTANTANEOUS VELOCITY

The instantaneous velocity at any instant t is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The magnitude of instantaneous velocity is always equal to the instantaneous speed.

$$\text{Instantaneous speed} = |\vec{v}| = \lim_{\Delta t \to 0} \frac{|\Delta x|}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Instantaneous velocity is called simply velocity.

If an object moves along a straight line without changing its direction, then the magnitude of the average velocity is equal to the average speed, otherwise magnitude of average velocity < average speed.

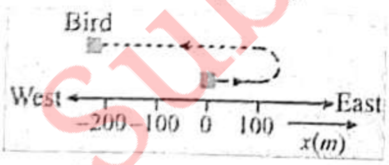
- Velocity can be +ve or –ve as it is a vector but speed can never be negative as it is the magnitude of velocity.
- If a body is moving with a constant velocity, then the average velocity and instantaneous velocity are equal.
- The velocity of a body is uniform, if both magnitude and direction do not change.
- If a body moves with non-uniform velocity, then magnitude of velocity or direction of velocity may change or both.
- A body can have non-zero speed and zero average velocity when a body completes one revolution around a circle. The average velocity is zero since the displacement is zero. But the average speed is not zero since the distance travelled ≠ 0.
- If a body is moving with constant speed then its velocity may or may not be constant. In case of uniform circular motion though speed remains constant but velocity changes instant to instant because of change in direction.

Train Your Brain

Example 1: A bird flies towards east at 10 m/s for 100 m. It then turns around and flies at 20 m/s for 15s. Find

(*a*) Its average speed (*b*) Its average velocity

Sol. Let us take the x axis to point eastwards A sketch of the path is shown in the figure. To find the required quantities, we need the total time interval. The first part of the journey took



$\Delta t_1 = (100\ \text{m})/(10\ \text{m/s}) = 10\text{s}$, and we are given $t_2 = 15\text{s}$ for the second part. Hence the total time interval is $\Delta t = \Delta t_1 + \Delta t_2 = 25\ \text{s}$

The bird flies 100 m east and then $(20\ \text{m/s}) \times (15\text{s}) = 300$ m west.

(*a*) $\text{Average speed} = \dfrac{\text{Distance}}{\Delta t}$

$$= \frac{100\,\text{m} + 300\,\text{m}}{25\text{s}} = 16\ \text{m/s}$$

(*b*) The net displacement is

$\Delta x = -200$ m

$$\text{So, } v_{av} = \frac{\Delta x}{\Delta t} = \frac{-200\text{m}}{25\text{s}} = -8\ \text{m/s}.$$

The negative sign means that v_{av} is directed toward the west.

Example 2: A particle moves with speed v_1 along a particular direction. After some time it turns back and reaches the starting point again travelling with speed v_2. Find (for the whole journey).

(*a*) Average velocity (*b*) Average speed

Sol. (*a*) Since the particle reaches the starting point again, its displacement is zero.

$$\therefore \text{Average velocity} = \frac{\text{Net displacement}}{\text{total time}} = 0$$

(*b*) Let it travel distance x while moving away and distance x while moving towards the starting point.

Time taken to go away is $t_1 = \dfrac{x}{v_1}$

Time taken while return journey $t_2 = \dfrac{x}{v_2}$

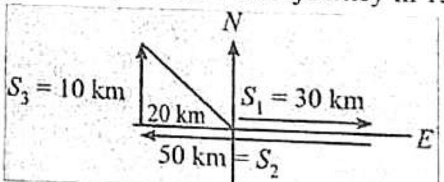
$$\therefore \text{Average speed} = \frac{x + x}{t_1 + t_2} = \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}}$$

$$v_{av} = \frac{2v_1v_2}{v_1 + v_2}$$

i.e., harmonic mean of individual speeds.

Example 3: A person goes 30 km east, then he walks 50 km west and then he goes 10 km north. Find average speed and average velocity for the whole journey in 15 hrs.

Sol.



$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{30 + 50 + 10}{15}$$

$$= 6\ \text{km/hr}$$

$$\text{Average velocity} = \frac{\text{Total Displacement}}{\text{Total Time}}$$

$$= \frac{\sqrt{(20^2 + 10^2)}}{15} = \frac{10\sqrt{5}}{15}$$

$$\text{Average velocity} = \frac{2\sqrt{5}}{3}\ \text{km/hr north of west}$$

Concept Application

1. A car moves 30 km with 20 km/hr and then 30 km with 30 km/hr. Find average speed and average velocity for the whole journey in the same straight line.
2. A person goes 20 km N, then 20 km E and then 20 km N-E, find average speed and average velocity if total time taken is 6 hr.

ACCELERATION

The rate of change of velocity of an object with time is called acceleration of the object.

Let v and v' be the velocity of the object at time t and t' respectively, then acceleration of the body is given by

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- Acceleration is a vector quantity.
- **Unit:** In SI system: m/s^2

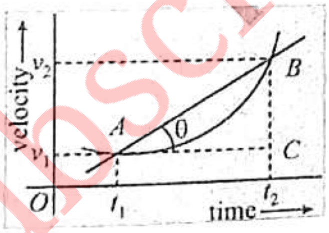
  In CGS system: cm/s^2
- **Dimension :** $[M^0L^1T^{-2}]$

Average Acceleration

When an object is moving with a variable acceleration in a straight line, then the average acceleration of the object for the given motion is defined as the ratio of the change in velocity of the object during motion to the time taken i.e.,

$$\text{Average Acceleration} = \frac{\text{change in velocity}}{\text{total time taken}}$$

Suppose the velocity of a particle is v_1 at time t_1 and v_2 at time t_2.



Then, Change in velocity $= v_2 - v_1 = \Delta v$

Elapsed time in changing the velocity $= t_2 - t_1 = \Delta t$

Thus, $a_{av} = \dfrac{v_2 - v_1}{t_2 - t_1} = \dfrac{\Delta v}{\Delta t}$

$\Rightarrow a_{av} = \dfrac{BC}{AC} = \tan\theta =$ the slope of chord of $v - t$ graph is average acceleration.

Instantaneous Acceleration

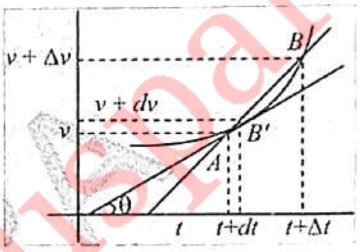
The acceleration of the object at a given instant of time or at a given point of motion, is called its instantaneous acceleration. Suppose the velocity of a particle at time $t_1 = t$ is $v_1 = v$ and becomes $v_2 = v + \Delta v$ at time $t_2 = t + \Delta t$.

Then, $a_{av} = \dfrac{\Delta v}{\Delta t}$

If Δt approaches to zero, then the rate of change of velocity will be instantaneous acceleration.

Instantaneous acceleration $a_{\text{inst}} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$

Instantaneous acceleration at a point is equal to slope of tangent at that point on velocity time graph in the graph shown.



Slope $= \dfrac{dv}{dt} =$ acceleration

As $v = \dfrac{dx}{dt}$ therefore $a = \dfrac{d}{dt}\left(\dfrac{dx}{dt}\right) = \dfrac{d^2x}{dt^2}$

Thus, instantaneous acceleration of an object is equal to the second derivative of the position w.r.t. time of the object at the given instant.

Note:

(*i*) It is not essential that when velocity is zero acceleration must be zero.

e.g. In vertical motion under gravity at the top point $v = 0$ but $a \neq 0$

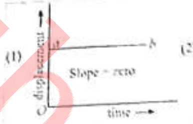
(*ii*) If a and v are both positive or both negative, speed of a body increases. If a and v have opposite signs then speed decreases, this is called retardation.

DISPLACEMENT-TIME GRAPHS AND THEIR CHARACTERISTICS

If the displacement-time graph is:

- A straight line parallel to time-axis, shown by line ab, it means that the body is at rest, i.e., $v = 0$.
- A straight line inclined to time-axis (such as Oc and fg) shows that body is moving with a constant velocity.
- A straight line inclined to time-axis by an angle > 90° (line fg) represent negative velocity.
- Of the type of curve Od (graph-3) whose slope decreases with time, the velocity goes on decreasing, i.e., motion is retarded.

- Of the type of curve Oe (graph-4) whose slope increases with time, the velocity goes on increasing, i.e., motion is accelerated.

(1)


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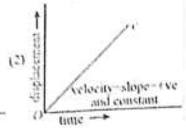

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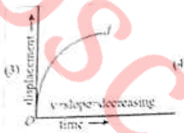
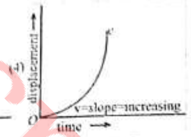


(4)


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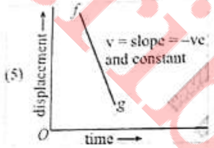


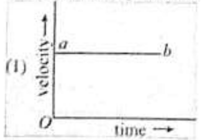
No line can ever be perpendicular to the time axis because it implies infinite velocity.

Slope of displacement-time graph at a point gives velocity of the body at that instant.

VELOCITY-TIME GRAPHS AND THEIR CHARACTERISTICS

If the velocity-time graph is:

- A straight line parallel to time axis shown by line ab, it means that the body is moving with a constant velocity or acceleration (a) is zero.
- A straight line inclined to the time-axis with $+ve$ slope (line Oc) it means that the body is moving with constant positive acceleration.
- A straight line inclined to time-axis with negative slope (line fg) it means that the body is moving with constant negative acceleration.
- A curve like Od (graph 3) whose slope decreases with time, the acceleration goes on decreasing.
- A curve like Oe (graph 4) whose slope increases with time, the acceleration goes on increasing.

(1)


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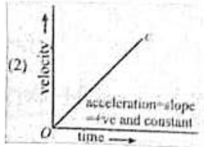

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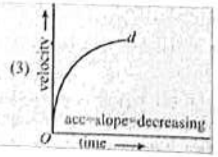

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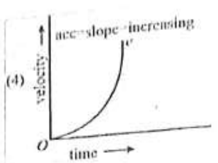

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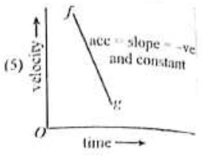


Note:

1. No velocity-time graph can ever be perpendicular to the time-axis because it implies infinite acceleration.
2. The area of velocity-time graph with time axis represents the displacement of that body.
3. Slope of velocity time graph at an instant gives the acceleration of the body at that instant.

MOTION WITH CONSTANT ACCELERATION

In different types of motions, the acceleration is either constant or approximately so. For example, near the surface of earth all objects fall vertically with constant acceleration if air resistance is neglected. Even when acceleration is not constant, we can learn something about the motion of the body by using constant acceleration results to be developed later in this section.

Let the velocity of body at $t = 0$ is u and it moves with constant acceleration a and acquires velocity v at time t.

$$\frac{dv}{dt} = a \quad \text{or} \quad dv = adt$$

$$\Rightarrow \int_u^v dv = \int_0^t adt = a\int_0^t dt$$

$$\Rightarrow v\Big|_u^v = at\Big|_0^t$$

$$\Rightarrow v - u = at \quad \text{or} \quad v = u + at \qquad \text{...(i)}$$

To find the displacement, we again integrate.

Let body be at x_0 at $t = 0$ and reaches x at time t

$$v = u + at$$

$$\frac{dx}{dt} = u + at$$

$$\text{or } dx = (u + at)dt$$

$$\int_{x_0}^{x} dx = \int_0^t (u + at)dt = u\int_0^t dt + a\int_0^t tdt$$

$$x\Big|_{x_0}^{x} = ut\Big|_0^t + a\frac{t^2}{2}\Big|_0^t$$

$$x - x_0 = ut + \frac{1}{2}at^2$$

$$S = x - x_0 = ut + \frac{1}{2}at^2 \qquad \text{...(ii)}$$

We can also find, relation between velocity and displacement

By using chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{vdv}{dx}$$

$$\Rightarrow vdv = adx$$

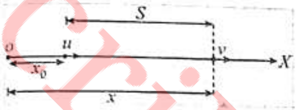
$$\int_u^v v\,dv = a\int_{x_0}^{x} dx$$

$$\left.\frac{v^2}{2}\right|_u^v = a\,x\big|_{x_0}^{x}$$

$$\frac{v^2}{2}-\frac{u^2}{2} = a(x-x_0)$$

$$\Rightarrow v^2-u^2 = 2aS \qquad \text{...(iii)}$$

These relation are very helpful in solving the problems of motion in one dimension. All these relations are given in table below for easy reference.



Equation	Contains		
	S	v	t
$v = u + at$	No	Yes	Yes
$s = ut + \frac{1}{2}at^2$	Yes	No	Yes
$v^2 - u^2 = 2as$	Yes	Yes	No

In simple problems of uniformly accelerated motion, two parameters are given and third is to be found. Depending on convenience one can choose any one of the three relations. The following two relations are also helpful in solving problems.

Displacement of the Body in the n^{th} Second:

$$S_n = S(\text{at } t = n) - S(\text{at } t = n-1)$$

$$= \left(un + \frac{1}{2}an^2\right) - \left(u(n-1) + \frac{1}{2}a(n-1)^2\right) = u + \frac{a}{2}(2n-1)$$

Average velocity:

$$V_{avg} = \frac{S}{t} = \frac{ut + \frac{1}{2}at^2}{t} = u + \frac{at}{2}$$

$$= u + \frac{v-u}{2} = \frac{u+v}{2}$$

$$\text{or} \quad S = \left(\frac{u+v}{2}\right)t$$

This relation is only valid for uniform acceleration.

Note:

- These equations can be applied only when acceleration is constant.
- If a body moves with uniform acceleration and velocity changes from u to v in a time interval, then average velocity $= \frac{v+u}{2}$.
- If a body moving with uniform acceleration has velocities u and v at two points in its path, then the velocity at the midpoint of given two points $= \sqrt{\frac{u^2+v^2}{2}}$.

- In position time graph, slope is equal to velocity.
- In velocity time graph area under the curve is displacement and slope is equal to acceleration.
- In acceleration time graph area under the curve is equal to change in velocity.
- **For a body starting from rest and moving with uniform acceleration,**
  - (*a*) The ratio of distances covered in first one sec, two sec, three sec, ... is :
    $1^2 : 2^2 : 3^2 : \ldots$, i.e., $1 : 4 : 9 : \ldots$
    Ratio of distances covered in
    1st, 2nd, 3rd sec, ... is $1 : 3 : 5 : \ldots$
  - (*b*) The ratio of velocities after
    1 sec, 2 sec, 3 sec, ... is $1 : 2 : 3 : \ldots$

Train Your Brain

Example 4: The displacement of a particle, moving in a straight line, is given by $S = 2t^2 + 2t + 4$ where S is in metres and t in seconds. The acceleration of the particle is

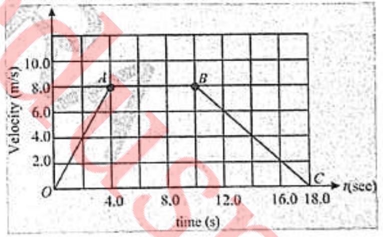
(*a*) $2\ m/s^2$ (*b*) $4\ m/s^2$
(*c*) $6\ m/s^2$ (*d*) $8\ m/s^2$

Sol. Given $S = 2t^2 + 2t + 4$

$$\therefore \text{Velocity } (v) = \frac{dS}{dt} = 4t + 2$$

$$\text{Acceleration } (a) = \frac{dv}{dt} = 4(1) + 0 = 4\text{m/s}^2$$

Example 5: What is the acceleration for each graph segment in figure? Describe the motion of the object over the total time interval. Also calculate displacement.



Sol. Segment OA; $a = \frac{8-0}{4-0} = 2\ m/s^2$

Segment AB; graph horizontal i.e., slope zero i.e., $a = 0$

Segment BC; $a = \frac{0-8}{18-10} = -1\ m/s^2$

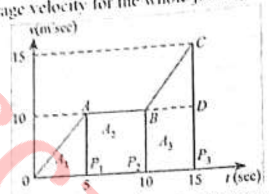
The graph is trapezium. Its area between $t = 0$ to $t = 18$s is displacement.

Area of v-t graph = displacement $= \frac{1}{2}(18+6) \times 8 = 96$ m

Particle accelerates uniformly for first 4 sec., then moves with uniform velocity for next 6 sec. and then retards uniformly to come to rest in next 8 sec.

Example 6: The motion of a body is described in ($v-t$) graph as given under.
Find the following:

(*a*) Max and Min acceleration
(*b*) Displacement from $t = 10$ to $t = 15$
(*c*) Average velocity for the whole journey.



Sol. (*a*) We know slope of ($v-t$) graph gives acceleration

$$\text{Slope}_{OA} = \frac{AP_1}{OP_1} = \frac{10}{5} = 2\,\text{m/sec}^2 \text{ (Max-acceleration)}$$

$$\text{Slope}_{AB} = 0\,\text{m/sec}^2 \quad \text{(min}-\text{acceleration)}$$

$$\text{Slope}_{BC} = \frac{CD}{BD} = \frac{5}{5}$$
$$= 1\,\text{m/sec}^2$$

(*b*) Displacement = Area ($v-t$)graph
from $t = 10$ to $t = 15$ sec

$$= \frac{1}{2}(10+15)\times 5 = 62.5\text{ m}$$

(*c*) $\text{Average Velocity} = \dfrac{\text{Total Displacement}}{\text{Total Time}} = \dfrac{\text{Area }(v-t)\text{graph}}{t_{total}}$

$$= \frac{A_1 + A_2 + A_3}{t_{total}} = \frac{25+50+62.5}{15}$$

$$= \frac{137.5}{15} = 9.17\text{ m/sec}$$

Example 7: How long does it take for a particle to travel 100 m if it begins from rest and accelerates at 10 m/s^2? What is its velocity when it has travelled 100 m? What is the average velocity during this time?

Sol. $u = 0$, $a = 10\text{ m/s}^2$, $S = 100$ m

Applying $S = ut + \frac{1}{2}at^2$

we get $100 = \frac{1}{2}\times 10\times t^2$

$\Rightarrow t = \sqrt{20} = 2\sqrt{5}$ s

$v = u + at = 0 + 10\times 2\sqrt{5} = 20\sqrt{5}$ m/s

$v_{avg} = \frac{u+v}{2} = \frac{0+20\sqrt{5}}{2} = 10\sqrt{5}$ m/s

Example 8: A car travelling with 72 km/hr is 30 m from a barrier when the driver slams the breaks. The car hits barrier 2.0 seconds later.

(*a*) What is the car's constant deceleration before impact?
(*b*) How fast is car travelling at impact?

Sol. (*a*) $u = 72\text{ km/hr} = 72\times\frac{5}{18}\text{ m/s} = 20\text{ m/s}$

$S = 30$ m
$t = 2$s
$a = ?$

$S = ut + \frac{1}{2}at^2$

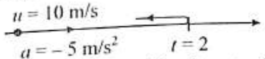
$30 = 20\times 2 + \frac{1}{2}\times a\times 2^2$

$\Rightarrow a = -\frac{10}{2} = -5\ \text{m/s}^2$

(*b*) $v = u + at = 20 + (-5)\times 2 = 10$ m/s

Example 9: A particle moving with initial velocity of 10 towards East has an acceleration of 5 m/s^2 towards west, the displacement and distance travelled by the particle in 4 seconds?

Sol.



$v = u + at \Rightarrow 0 = 10 - 5t \Rightarrow t = 2s$

The direction of velocity changes after two seconds

$S = 10\times 4 + \frac{1}{2}(-5)\times 4^2 = 0 = \text{displacement}$

Distance travelled is not equal to displacement because during course of journey, velocity changes direction

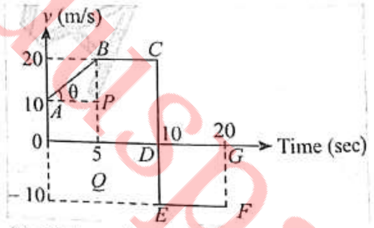
$D = S(\text{at }2s) + |S(\text{at }4s) - S(\text{at }2s)|$

$= \left(10\times 2 - \frac{1}{2}\times 5\times 2^2\right) + \left|0 - (10\times 2) - \frac{1}{2}\times 5\times 2^2\right|$

$= 10 + 10 = 20$ m

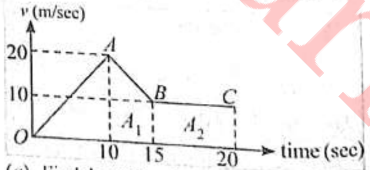
Concept Application

3.



(*a*) Find acceleration between $t = 0$ to $t = 5$
(*b*) Find total displacement between $t = 0$ to $t = 20$ sec

4.



(*a*) Find the ratio of acceleration for OA and AB in the graph shown.
(*b*) Total distance covered between 10 to 20 sec.

VERTICAL MOTION UNDER GRAVITY (FREE FALL)

Motion that occurs solely under the influence of gravity is called free fall. Thus a body projected upward or downward or released from rest are all under free fall.

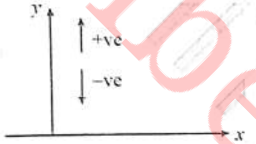
In the absence of air resistance all falling bodies have the same acceleration due to gravity, regardless of their sizes or shapes.

The value of the acceleration due to gravity depends on both latitude and altitude. It is approximately 9.8 m/s² near the surface of the earth. For simplicity a value of 10 m/s² is used. To do calculations regarding motion under gravity, we follow a proper sign convention. If we take upward direction as positive then $a = -g$

Thus the equation of kinematics may be modified as

$v = u - gt$ ...(i)

$\Delta y = y - y_0 = ut - \frac{1}{2}gt^2$ ...(ii)



$v^2 = u^2 - 2g(y - y_0)$ ...(iii)

y_0 = position of particle at time $t = 0$

y = position of particle at time t.

u = velocity of particle at time $t = 0$

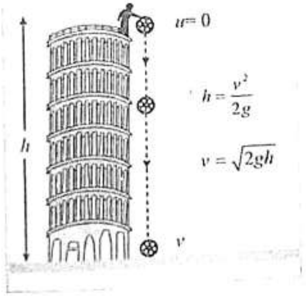
v = velocity of particle at time t.

(i) **A body dropped from some height (initial velocity zero)**

- Equation of motion: Taking initial position as origin and downward direction as negative. Here we have,

  $u = 0$ [As body starts from rest]

  $a = -g$ as acceleration is in the downward direction



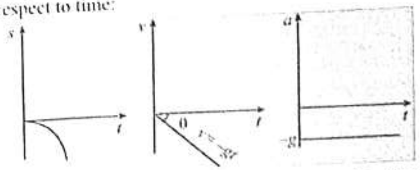
$v = -gt$ ...(i)

$\Delta y = -h = -\frac{1}{2}gt^2$ ...(ii)

$v^2 = 2(-g)(-h) = 2gh$ ...(iii)

$h_n = \frac{g}{2}(2n-1)$ = Height covered in nth second. ...(iv)

- Graph of displacement, velocity and acceleration with respect to time:



- As $h = (1/2)gt^2$, i.e., $h \propto t^2$, distance covered in time t, $2t$, $3t$, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of integers.

- The distance covered in the nth sec, $h_n = \frac{1}{2}g(2n-1)$

  So distance covered in 1st sec, 2nd sec, 3rd sec .... etc., will be in the ratio of 1 : 3 : 5. This is called 'Galileo's Law of odd numbers'.

(ii) **A body projected vertically downward with some initial velocity:** The initial velocity is downward and will be negative.

Equation of motion: $v = -u - gt$

$\Delta y = -h = -ut - \frac{1}{2}gt^2$

$v^2 = u^2 + 2gh$

$h_n = u + \frac{g}{2}(2n-1)$

(iii) **A body is projected vertically upwards:**

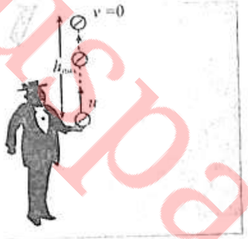
Equation of motion: Taking initial position as origin and vertically up as positive.

$a = -g$ [As acceleration is downwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

$v = u - gt$; $h = ut - \frac{1}{2}gt^2$; $v^2 = u^2 - 2gh$;

$h_n = u - \frac{g}{2}(2n-1)$



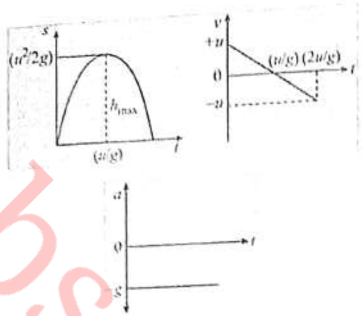
For maximum height $v = 0$

So from above equation

$u = gt \Rightarrow t = \frac{u}{g}$

$h_{max} = \frac{u^2}{g} - \frac{u^2}{2g} = \frac{u^2}{2g}$ Now, $h_{max} = \frac{u^2}{2g} \Rightarrow u = \sqrt{2gh_{max}}$

- Graph of displacement, velocity and acceleration with respect to time (for maximum height):



Important Points

(i) In case of motion under gravity for a given body, mass, acceleration, and mechanical energy remain constant while speed, velocity, momentum, kinetic energy and potential energy changes.

(ii) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e., $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$

(iii) In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance. Time of ascent (t_1) = time of descent $(t_2) = u/g$

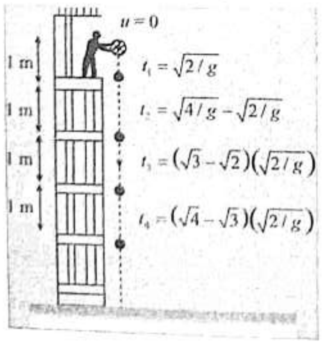
Total time of flight $T = t_1 + t_2 = \dfrac{2u}{g}$

(iv) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

Acceleration at any point on the path is same whether the body is moving in upward or downward direction.

(v) A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of 1 m each will then be in the ratio of the difference in the square roots of the integers i.e.,

$\sqrt{1}, (\sqrt{2}-\sqrt{1}), (\sqrt{3}-\sqrt{2}), (\sqrt{4}-\sqrt{3}), \ldots\ldots$


u = 0
1 m
1 m
1 m
1 m
$t_1 = \sqrt{2/g}$
$t_2 = \sqrt{4/g} - \sqrt{2/g}$
$t_3 = (\sqrt{3}-\sqrt{2})(\sqrt{2/g})$
$t_4 = (\sqrt{4}-\sqrt{3})(\sqrt{2/g})$

Note:

(i) During ascent, $a = -g$, velocity becomes less positive i.e. speed decreases since velocity and acceleration are in opposite direction.

(ii) During descent, $a = -g$, but now it is in the direction of velocity so it is not retardation. It makes velocity more negative i.e. speed increases in negative direction.

Train Your Brain

Example 10: A man standing on the top of a building throws a ball with speed 5 m/s in upward direction from 30 m height above the ground level. How much time does it takes to reach the ground?

Sol. $u = 5$ m/s

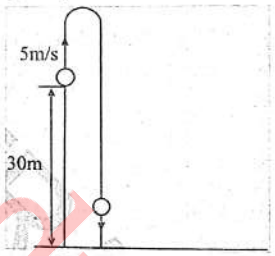
When it reaches the ground, $\Delta y = -30$ m

$S = ut - \frac{1}{2}gt^2$

$-30 = 5t - \frac{1}{2}(10)t^2$

$\Rightarrow t^2 - t - 6 = 0$

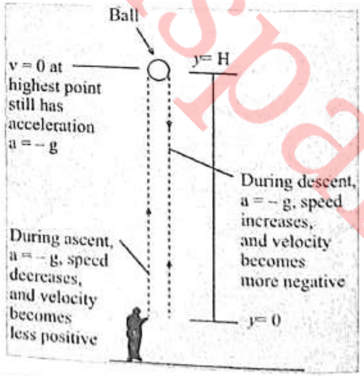
On solving, we get $t = 3$ and -2



Rejecting $t = -2$ sec, we get $t = 3$ sec

Example 11: A kid throws a ball up, with some initial speed. Comment on magnitudes and signs of acceleration and velocity of the ball.

Sol.


Ball
v = 0 at highest point still has acceleration a = – g
y = H
During descent, a = – g, speed increases, and velocity becomes more negative
During ascent, a = – g, speed decreases, and velocity becomes less positive
y = 0

Example 12: If a body is thrown up with the velocity of 15 m/s then maximum height attained by the body is ($g = 10$ m/s^2)

(*a*) 11.25 m (*b*) 16.2 m
(*c*) 24.5 m (*d*) 7.62 m

Sol. $H_{max} = \frac{u^2}{2g} = \frac{(15)^2}{2\times10} = 11.25$ m

Example 13: A body falls from rest in the gravitational field of the earth. The distance travelled in the fifth second of its motion is ($g = 10$ m/s^2)

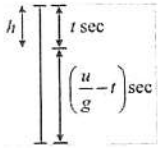
(*a*) 25 m (*b*) 45 m
(*c*) 90 m (*d*) 125 m

Sol. $h_n = \frac{g}{2}(2n-1) \Rightarrow h_{5th} = \frac{10}{2}(2\times5-1) = 45$ m

Example 14: If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

(*a*) $\frac{1}{2}gt^2$ (*b*) $ut - \frac{1}{2}gt^2$
(*c*) $(u-gt)t$ (*d*) ut

Sol. If ball is thrown with velocity u, then time of ascent $= \frac{u}{g}$



Velocity after $\left(\frac{u}{g}-t\right)$sec, $v = u - g\left(\frac{u}{g}-t\right) = gt$.

So, distance in last 't' sec, $0^2 = (gt)^2 - 2(g)h$.

$h = \frac{1}{2}gt^2$.

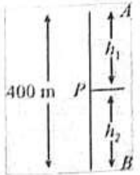
Example 15: A man drops a ball downside from the roof of a tower of height 400 m. At the same time another ball is thrown upside with a velocity 50 m/s from the foot of tower. What is the height from the foot of the tower where the two balls would meet?

(*a*) 100 meters (*b*) 320 meters
(*c*) 80 meters (*d*) 240 meters

Sol. Let both balls meet at point P after time t.

The distance travelled by ball A

$h_1 = \frac{1}{2}gt^2$ ...(i)



The distance travelled by ball B

$h_2 = ut - \frac{1}{2}gt^2$ ...(ii)

By adding (i) and (ii) $h_1 + h_2 = ut = 400$

(Given $h = h_1 + h_2 = 400$)

$\therefore t = 400/50 = 8$ s and $h_1 = 320$ m, $h_2 = 80$ m

Example 16: Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant?

(*a*) 2.50 m (*b*) 3.75 m
(*c*) 4.00 m (*d*) 1.25 m

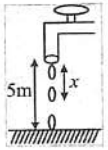
Sol. Let the interval between each drop be t then from question

For first drop $\frac{1}{2}g(2t)^2 = 5$ ...(i)

For second drop $x = \frac{1}{2}gt^2$ ...(ii)

By solving (i) and (ii) $x = \frac{5}{4}$ and

Hence required height $h = 5 - \frac{5}{4} = 3.75m$.



Example 17: A balloon is at a height of 81 m and is ascending vertically upward with a velocity of 12 m/sec. A body of 2 kg weight is dropped from it. If $g = 10$ m/s^2 the body will reach the surface of the earth in

(*a*) 1.5 s (*b*) 4.025 s
(*c*) 5.4 s (*d*) 6.75 s

Sol. As the balloon is going up so initial velocity of balloon $= +12$ m/s,

$\Delta y = -81$ m; $a = -g = -10$ m/s^2

By applying $h = ut + \frac{1}{2}gt^2$; $-81 = 12t - \frac{1}{2}(10)t^2$

$\Rightarrow 5t^2 - 12t - 81 = 0$

$\Rightarrow t = \frac{12 \pm \sqrt{144+1620}}{10} = \frac{12 \pm \sqrt{1764}}{10}$

$= 5.4$ s

Example 18: A particle is dropped under gravity from rest from a height h ($g = 9.8$ m/s^2) and it travels a distance $\frac{9h}{25}$ in the last second, the height h is

(*a*) 100 m (*b*) 122.5 m
(*c*) 145 m (*d*) 167.5 m

Sol. Distance travelled in n sec $= \frac{1}{2}gn^2 = h$ ...(i)

Distance travelled in n^{th} sec.

$= \frac{g}{2}(2n-1) = \frac{9h}{25}$ ...(ii)

Solving (i) and (ii)

We get, $h = 122.5$ m

Example 19: A stone is thrown vertically upward with a speed u from the top of the tower, reaches the ground with a velocity $3u$. The height of the tower is

(*a*) $3u^2/g$ (*b*) $4u^2/g$ (*c*) $6u^2/g$ (*d*) $9u^2/g$

Sol. Initial velocity $= -u$.

By applying $v^2 = u^2 + 2gh$,

$(3u)^2 = (-u)^2 + 2gh$,

$\Rightarrow h = \frac{4u^2}{g}$

Example 20: A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is

(*a*) 4.9 m (*b*) 9.8 m

(*c*) 19.6 m (*d*) 24.5 m

Sol. The separation between two bodies, two seconds after the release of second body is given by

$$s = \frac{1}{2}g(t_1^2 - t_2^2) = \frac{1}{2} \times 9.8 \times (3^2 - 2^2) = 24.5\ m$$

Concept Application

5. A body is projected from top of a tower vertically upward with 5 m/sec, the body lands the ground in 4 sec. Total height of tower would be

(*a*) 45 m (*b*) 60 m (*c*) 30 m (*d*) 50 m

6. A body is thrown vertically upwards and if returns back to hand in 3 sec, its velocity of throw and max-height attained are,

(*a*) 15 m/sec, 11.25 m (*b*) 10 m/sec, 12 m

(*c*) 30 m/sec, 22.5 m (*d*) 20 m/sec, 30 m

7. Water drops are released from the bottom of an overhead tank at regular interval. When 1st drop touches the ground, fifth drop is just about to release. The height of overhead tank is 8m from the ground. Find the separation between 2nd and 3rd drop just when 1st drop touches the ground.

(*a*) 1.5 m (*b*) 2 m

(*c*) 2.5 m (*d*) 4.5 m

8. Two balls A & B of masses m & 5 m are dropped from the towers of height 36 m and 64 m respectively. The ratio of time taken by them to reach the ground

(*a*) 2/3 (*b*) 7/4 (*c*) 5/2 (*d*) 3/4

9. A splash is heard 3 sec after the stone is released into a well of depth 20 m. The velocity of sound is

(*a*) 20 m/sec (*b*) 40 m/sec

(*c*) 10 m/sec (*d*) 35 m/sec

MOTION WITH VARIABLE ACCELERATION

In previous section, we studied rectilinear motion when acceleration is constant. In general acceleration can vary as depend on time, position and velocity of the particle.

Let us consider some simple cases.

(*i*) **Acceleration only depends on time t.**

$$\frac{dv}{dt} = a(t)$$

$$\int_u^v dv = \int_0^t a(t)dt \Rightarrow v - u = \int_0^t a(t)dt$$

$$\text{or } v = u + \int_0^t a(t)dt$$

(*ii*) **Acceleration only depends on position x.**

$$\frac{dv}{dt} = a(x)$$

We can use chain rule to eliminate time.

$$\frac{dv}{dx}\frac{dx}{dt} = a(x) \Rightarrow v\frac{dv}{dx} = a(x)$$

$$\int_u^v vdv = \int_{x_0}^x a(x)dx$$

$$\text{or } \frac{v^2}{2} - \frac{u^2}{2} = \int_{x_0}^x a(x)dx$$

(*iii*) **Acceleration only depends on velocity.**

$$\frac{dv}{dt} = a(v) \Rightarrow \int_u^v \frac{dv}{a(v)} = \int_0^t dt$$

$$\text{or } \int_u^v \frac{dv}{a(v)} = t$$

This gives us velocity as a function of time.

In case we want velocity as a function of position, we can use chain rule.

$$\frac{dv}{dx}\frac{dx}{dt} = a(v) \Rightarrow \frac{v\,dv}{a(v)} = dx$$

$$\int_u^v \frac{v\,dv}{a(v)} = \int_{x_0}^x dx = x - x_0$$

Train Your Brain

Example 21: The acceleration a of a particle moving in one dimension is given by $a = 6 - 2t$. If the particle is initially at $x = 0$ and its velocity is 2 m/s, find its position and velocity at time t.

Sol. $\dfrac{dv}{dt} = 6 - 2t$

$$\int_2^v dv = \int_0^t (6-2t)dt$$

$$\Rightarrow v - 2 = (6t - t^2)\Big|_0^t = 6t - t^2 \Rightarrow v(t) = 2 + 6t - t^2$$

To find position, we integrate velocity.

$$v = \frac{dx}{dt} = 2 + 6t - t^2 \Rightarrow dx = (2 + 6t - t^2)dt$$

$$\int_0^x dx = \int_0^t (2 + 6t - t^2)dt = 2t + 3t^2 - \frac{t^3}{3}$$

$$\text{or } x(t) = 2t + 3t^2 - \frac{t^3}{3}$$

Example 22: The retardation of a car when its engine is shut off depends on its velocity as $a = -\alpha v$ where α is positive constant. Find the total distance travelled by the car if its initial velocity is 20 m/s and $\alpha = 0.5$/s.

Sol. $\dfrac{dv}{dt} = -\alpha v$

$$\frac{dv}{dx}\left(\frac{dx}{dt}\right) = -\alpha v \Rightarrow \frac{vdv}{dx} = -\alpha v$$

$$\text{or } dv = -\alpha dx$$

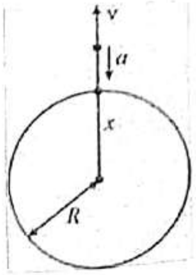
$$\int_{20}^{0} dv = -\alpha\int_0^d dx \Rightarrow v\Big|_{20}^{0} = -\alpha\, x\Big|_0^d$$

$$-20 = -\alpha d$$

$$d = \frac{20}{\alpha} = \frac{20}{0.5} = 40 \text{ m}$$

Example 23: With what velocity in vertical upward direction should a body be projected from the surface of earth so that it reaches a height equal to radius of earth? The acceleration of body is given by $a = -\dfrac{GM}{x^2}$ where x is the distance from centre of earth and M is the mass of earth.

Sol. Note that acceleration due to gravity is nearly constant near the surface of earth. But if the height become too large its dependence on distance can not be ignored.



$$a = \frac{dv}{dt} = -\frac{GM}{x^2}$$

$$\text{or } \frac{dv}{dx}\cdot\frac{dx}{dt} = -\frac{GM}{x^2} \Rightarrow vdv = = -\frac{GM}{x^2}dx$$

At the highest point, velocity is zero. Also note $x_i = R$ and $x_f = 2R$.

$$\int_u^0 vdv = -GM\int_R^{2R}\frac{dx}{x^2}$$

$$\frac{v^2}{2}\Bigg|_u^0 = -GM\int_R^{2R} x^{-2}dx = \frac{GM}{x}\Bigg|_R^{2R}$$

$$\Rightarrow -\frac{u^2}{2} = GM\left[\frac{1}{2R} - \frac{1}{R}\right]$$

$$\Rightarrow u^2 = \frac{GM}{R} \Rightarrow u = \sqrt{\frac{GM}{R}} = \sqrt{\frac{GM}{R^2}R}$$

$$\because g = \left(\frac{GM}{R^2}\right)$$

$$= \sqrt{gR} = 8 \text{ km/s} \quad [\because R = 6400 \text{ km, g} = 10 \text{ m/s}^2]$$

Concept Application

10. Starting from rest at $t = 0$, a particle moves in a straight line with an acceleration a given by $a = t^3$ m/s^2 where t is in seconds. Then the velocity of particle after 4 seconds is

(*a*) 32 m/s (*b*) 64 m/s
(*c*) 128 m/s (*d*) 16 m/s

11. A particle moves in a straight line with acceleration $a = -\dfrac{1}{3v^2}$ where v is its velocity at time t. If initial velocity is 5 m/s then time t at which its velocity becomes zero is

(*a*) 5 sec (*b*) 25 sec (*c*) 125 sec (*d*) 50 sec

12. The acceleration of a particle as a function of its position x is given $a = -2x$. If velocity at $x = 0$ is 20 m/s, find the position x where its velocity becomes zero.

(*a*) $10\sqrt{2}$ m (*b*) $5\sqrt{2}$ m
(*c*) $20\sqrt{2}$ m (*d*) 20 m

RELATIVE MOTION

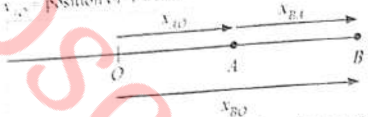
Every motion is actually only relative motion. There is nothing like absolute motion. Whether a body is moving or is at rest is not a quality of body itself rather it is always with respect to an observer. If the observer finds that the position of an object is not changing when observed by him then the object is 'actually' at rest (and vice-versa). Therefore, when a passenger (A) in a train observes another passenger (B) then he finds that passenger (and the train) to be not moving. Therefore, it means that the passenger and train both are in reality at rest for passenger (A). However, another observer on platform finds the train and passenger (B) to be moving when they are in reality moving for this observer only. Due to this duality we say that motion is not absolute rather it is dependent on observer.

Relative position of B with respect to A,

$$x_{BA} = x_{BO} - x_{AO} \qquad \text{...(i)}$$

where, x_{BO} = position of B w.r.t. O

x_{AO} = position of A w.r.t. O



Differentiating eq. (i) gives us the relation for relative velocity

$$\frac{d}{dt}x_{BA} = \frac{d}{dt}x_{BO} - \frac{d}{dt}x_{AO}$$

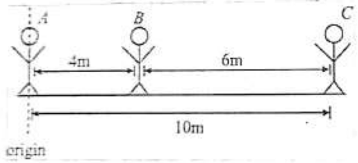
$$V_{BA} = V_{BO} - V_{AO}$$

Further differentiating,

$$a_{BA} = a_{BO} - a_{AO}$$

Train Your Brain

Example 24: The position of three men A, B and C is shown in figure. Then find position of one man with respect to other (take +ve direction towards right and –ve towards left)



Sol. Here,

Position of B w.r.t. A is 4 m towards right. ($x_{BA} = +4$ m)

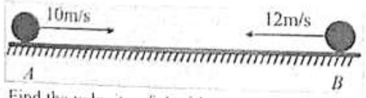
Position of C w.r.t. A is 10 m towards right. ($x_{CA} = -10$m)

Position of C w.r.t. B is 6 m towards right ($x_{CB} = +6$m)

Position of A w.r.t. B is 4 m towards left. ($x_{AB} = -4$m)

Position of A w.r.t. C is 10 m towards left. ($x_{AC} = -10$m)

Example 25: Two objects A and B are moving towards each other with velocities 10 m/s and 12 m/s respectively as shown.



(i) Find the velocity of A with respect to B.

(ii) Find the velocity of B with respect to A

Sol. $v_A = +10$ m/s, $v_B = -12$ m/s

(i) $v_{AB} = v_A - v_B = (10) - (-12) = 22$ m/s.

(ii) $v_{BA} = v_B - v_A = (-12) - (10) = -22$ m/s.

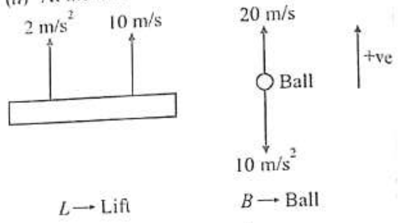
Example 26: An open lift is moving upwards with velocity 10m/s. It has an upward acceleration of 2 m/s². A ball is projected upwards with velocity 20 m/s relative to ground. Find :

(a) Time when ball again meets the lift.

(b) Displacement of lift and ball at that instant.

(c) Distance travelled by the ball upto that instant. Take $g = 10$ m/s²

Sol. (a) At the time when ball again meets the lift,



$$\therefore 10t + \frac{1}{2} \times 2 \times t^2 = 20\,t - \frac{1}{2} \times 10t^2$$

Solving this equation, we get

$$t = 0 \text{ and } t = \frac{5}{3}\text{ s}$$

$\therefore$ Ball will again meet the lift after $\frac{5}{3}$ s.

(b) At this instant

$$S_L = S_B = 10 \times \frac{5}{3} + \frac{1}{2} \times 2 \times \left(\frac{5}{3}\right)^2 = \frac{175}{9} = 19.4\text{m}$$

(c) For the ball $u \uparrow \downarrow a$. Therefore, we will first find t_0, the time when its velocity becomes zero.

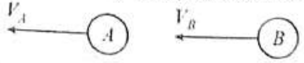
As $t\left(= \frac{5}{3}s\right) < t_0$ distance and displacement are equal

or $d = 19.4$ m

Concept of relative motion is more useful in two body problem in two (or three) dimensional motion. This can be understood by the following example.

Concept Application

13. Two particles are moving along a straight line as shown. The velocity of approach between A and B is

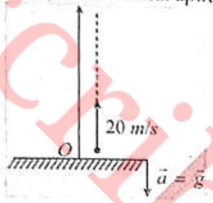


(a) $V_A + V_B$
(b) $|V_A - V_B|$
(c) $V_A - V_B$
(d) $V_B - V_A$

Aarambh (Solved Examples)

1. A stone is thrown vertical upwards from ground level with $u = 20$ m/s.
   (*a*) Find the maximum height attained by the stone.
   (*b*) time interval t after which it returns to the point of projection.
   (*c*) The velocity with which it strikes the ground.

Sol. Let us choose our origin O at the point of projection with +ve X-axis pointing in the vertical upwards direction.



Note that in this coordinate system, acceleration due to gravity is negative because it points in the downward direction.

Thus $a = -9.8$ m/s^2, $u = 20$ m/s

(*a*) At the highest point, velocity of the particle will become zero. Let h be the maximum height.

Thus $S = h$.

Using the relation,

$v^2 - u^2 = 2a\,S$

we get $0 - 20^2 = 2 \times (-9.8) \times h$

$$\Rightarrow h = \frac{400}{19.6} = 20.4\,\text{m}$$

Therefore, 20.4 m is the correct answer.

(*b*) $S = 0 = 20t - \frac{1}{2}(9.8)t^2$

$$\Rightarrow t = \frac{40}{9.8} = 4.08\ \text{sec}$$

Therefore, 4.08 s is the correct answer.

(*c*) Since, the particle returns to the initial position, $S = 0$.

$$\Rightarrow v = 20 - 9.8 \times \frac{40}{9.8} \quad \text{(we know } t \text{ from part } (b))$$

$= -20$ m/s

Here, minus sign indicates that particle moves in the downward direction.

Therefore, −20 m/s is the correct answer.

Note: It returns with same speed with which it was thrown.

2. A body is thrown down from the top of a tower of height h with velocity 10 m/s. Simultaneously, another body is projected upward from bottom. They meet at a height $2h/3$ from the ground level. If $h = 60$ m, find the initial velocity of the lower body.
   (*a*) 19.23 m/s (*b*) 38.27 m/s
   (*c*) 55.16 m/s (*d*) None of these

Sol. Let us choose the origin at the ground level with +ve y-axis pointing in the upward direction.

Let us refer lower and upper body as 1 and 2 respectively.

Then,

$a = -g$

$x_{1_i} = 0,\ x_{1_f} = 2h/3,\ u_1 = ?$

$x_{2_i} = h,\ x_{2_f} = 2h/3,\ u_2 = -10$ m/s

From eqn of motion , we have

$$x_{1_f} = 0 + u_1 t - \frac{1}{2} \times 9.8 \times t^2 \quad \text{...(i)}$$

$$x_{2_f} = h - 10t - \frac{1}{2} \times 9.8 \times t^2 \quad \text{...(ii)}$$

But , $x_{1_f} = x_{2_f} = 2h/3$

Hence, equating eqn (i) and (ii) we have

$$u_1 t = h - 10\,t \quad \Rightarrow t = \frac{h}{u_1 + 10}$$

Putting this value in eqn (ii), we get

$$\frac{h}{3} = \frac{10h}{u_1 + 10} + 4.9\left(\frac{h}{u_1 + 10}\right)^2$$

But, $h = 60$ m.

$$\Rightarrow 20 = \frac{600}{u_1 + 10} + 4.9\frac{60^2}{(u_1 + 10)^2}$$

$$\Rightarrow (u_1 + 10)^2 - 30(u_1 + 10) - 882 = 0$$

Solving this quadratic eqn, we find

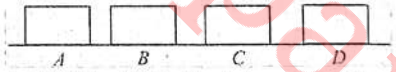
$$u_1 + 10 = \frac{30 \pm \sqrt{100 + 3528}}{2} \qquad \Rightarrow u_1 = 38.27\ \text{m/s}$$

The other value is not possible because body is thrown upwards and is positive in the chosen coordinate system.

Therefore, option (*b*) is the correct answer.

3. A car starts moving on a straight road, first with acceleration $a = 5$ m/s^2, then moves uniformly, and finally decelerating at the same rate, comes to rest. The total time of motion equals 25 sec. The average velocity during that time is 72 km/hr. How long did the car move uniformly?
   (*a*) 30 s (*b*) 50 s (*c*) 15 s (*d*) 20 s

Sol.



Let AB, BC and CD be the displacements of the car when it accelerates, moves with constant velocity and decelerates respectively.

$$< v > = 72\ \text{km/hr} = 72 \times \frac{5}{18}\ \text{m/s} = 20\ \text{m/s}$$

Total distance travelled $= < v > \times$ time $= 20 \times 25 = 500$ m.

From $A \to B$

$$AB = ut_{AB} + \frac{1}{2}a(t_{AB})^2$$

$= 2.5(t_{AB})^2$

$v_B = 0 + at_{AB} = 5t_{AB}$

From $B \rightarrow C$

Since velocity is constant

$BC = v_B \times t_{BC} = 5t_{AB} \times t_{BC}$

From $C \rightarrow D$

In this interval

$u = v_B = 5t_{AB},\ v = 0,\ a = -5\ \text{m/s}^2$

$v = u + at \Rightarrow 0 - 5t_{AB} - 5t_{CD} = 0$

$\Rightarrow t_{AB} = t_{CD}$ and

$S = ut - \frac{1}{2}at^2$

$\Rightarrow CD = 5t_{AB}^2 - 2.5(t_{AB})^2 \quad = 2.5t^2_{AB} = AB$

Total time $= t_{AB} + t_{BC} + t_{CD} = 25$

$\Rightarrow 2t_{AB} + t_{BC} = 25$ ...(i)

Also,

Total distance $= AB + BC + CD = 500$

$\Rightarrow 2.5t_{AB}^2 + 5t_{AB}\, t_{BC} + 2.5t_{AB}^2 = 500$ ...(ii)

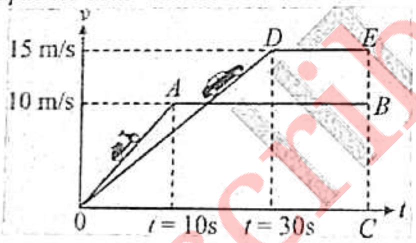
Solving eqn (i) and eqn (ii) we find $t_{BC} = 15\,\text{sec}$

Therefore, option (*c*) is the correct answer.

4. A motorcycle and a car start their rectilinear motion from rest from the same place at the same time and travel in the same direction. The motorcycle accelerates at $1.0\ \text{m/s}^2$ up to a speed of 36 km/hr and the car at $0.5\ \text{m/s}^2$ up to a speed of 54 km/hr. Their velocities remain constant after that. Draw *v-t* graph of both. Calculate the distance at which the car would overtake the motorcycle.

(*a*) 150 m (*b*) 900 m (*c*) 300 m (*d*) 100 m

Sol. *v-t* graphs for both vehicles is as below



For motorcycle

$10\ \text{m/s} = 0 + (1\text{m/s}^2)\, t$

$t = 10\text{s}$

For car

$15\ \text{m/s} = 0 + (0.5\ \text{m/s}^2)\, t$

$t = 30\ \text{s}$

Suppose after *t* time car over takes motorcycle.

Area under *v-t* graph till that time for both will be same

Area of $OABC$ = Area of $ODEC$

$\frac{1}{2}(t + t - 10)\, 10 = \frac{1}{2}(t + t - 30)\, 15$

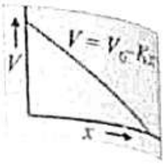
$4t - 20 = 6t - 90$

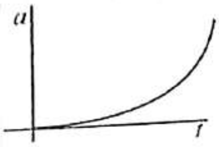
$2t = 70 \Rightarrow t = 35$ sec.

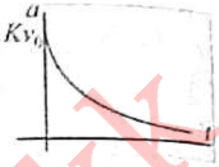
Area under the graph = 300 m = distance at which car overtakes motorcycle.

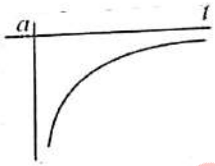
Therefore, option (*c*) is the correct answer.

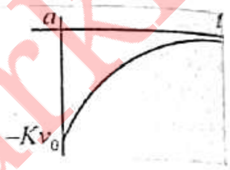
5. A particle is moving along *x*-axis with velocity V which varies according to the law $V = V_0 - Kx$ here V_0 and K are constants. Choose the correct acceleration *vs* time plot for the time interval when particle moves from $x = 0$ to $x = \frac{V_0}{K}$.



(*a*)


(*b*)


(*c*)


(*d*)


Sol. $V = V_0 - Kx$

$$\frac{dx}{dt} = (V_0 - Kx) \Rightarrow \int_0^x \frac{dx}{(V_0 - Kx)} = \int_0^t dt$$

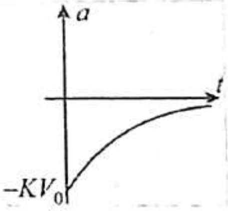
$$x = \frac{V_0}{K}(1 - e^{-Kt})$$

$$\therefore \frac{dx}{dt} = +V_0\, e^{-Kt} \Rightarrow a = \frac{d^2x}{dt^2} = -KV_0\, e^{-Kt}$$

At $t = 0,\ a = -KV_0$

At $t = \infty,\ a = 0$

Therefore, graph is as shown



Therefore, option (*d*) is the correct answer.

6. A steel ball bearing is released from the roof of a building. An observer standing in front of a window 120 cm high observes that the ball takes 0.125 sec to fall from top to the bottom of the window. The ball continues to fall and makes a completely elastic collision with side walk and reappears at the bottom of the window 2 s after passing it on the way down. How tall is the building? (In elastic collision the kinetic energy remains conserved before and after collision)

(*a*) 50 m (*b*) 41.5 m

(*c*) 75 m (*d*) 20.5 m

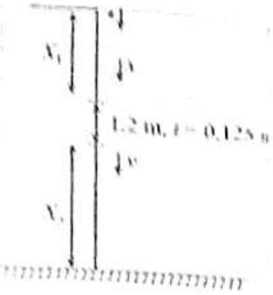
Sol. $1.2 = ut + \frac{1}{2}gt^2$

$1.2 = u \times 0.125 + \frac{1}{2} \times 10 \times (0.125)^2$

$1.2 = u \times 0.125 + 5 \times (0.125)^2$

$u = \frac{1.2 - 0.078125}{0.125} = 8.975\ \text{m/s}$

$v = 8.975 + 10 \times 0.125 = 10.225\ \text{m/s}$



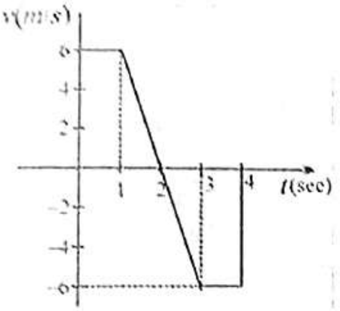
$x_2 = 10.225 \times 1 + \frac{1}{2} \times 10 \times 1^2 = 15.225$ m

$0 - u^2 = -2 \times 10 \times x_1 \Rightarrow x_1 = \frac{u^2}{20} = \frac{(8.975)^2}{20} = 4.028$ m

$H = x_1 + x_2 + 1.2 = 4.028 + 15.225 + 1.2$
$= 20.4525 \approx 20.5$ m

Therefore, option (a) is the correct answer.

A particle moves along a straight line along x-axis. At time $t = 0$, its position is at $x = 0$. The velocity v m/s of the object changes as a function of time t seconds as shown in the figure.



(i) What is x at t = 1 sec?
(a) 2 m (b) 4 m (c) 6 m (d) 8 m

(ii) What is the acceleration at t = 2 sec?
(a) 10 m/s^2 (b) 20 m/s^2 (c) −12 m/s^2 (d) −6 m/s^2

(iii) What is x at t = 4 sec?
(a) 0 m (b) 1 m (c) 5 m (d) 10 m

(iv) What is the average speed between t = 0 and t = 3 sec?
(a) 8 m/s (b) 4 m/s (c) 2 m/s (d) 1 m/s

Sol. (i) x is displacement at t = 1 sec.

Area under the v – t curve gives displacement

From t = 0 to t = 1 sec.

$x = 6 \times 1 = 6$ m

Therefore, option (c) is the correct answer.

(ii) Slope of the v–t curve gives acceleration from the given v–t curve

Slope at t = 2 sec. gives acceleration at t = 2 sec.

$\tan\theta = a = -\frac{6}{1} = -6$ m/s^2

Therefore, option (d) is the correct answer.

(iii) x (at t = 4 sec):

Area under the curve from t = 0 to t = 4 sec

$= 6 \times 1 + \frac{1}{2} \times 6 \times 1 - \frac{1}{2} \times 6 \times 1 - 6 \times 1 = 0$

$\Rightarrow x\ (t = 4) = 0$ m

Therefore, option (a) is the correct answer.

(iv) Average speed from t = 0 to t = 3 sec.

Displacement from t = 0 to t = 2 sec = Area under the curve $= 6 + \frac{1}{2} \times 6 \times 1 = 9$ m

Displacement form t = 2 to t = 3 sec $= -\frac{1}{2} \times 6 \times 1 = -3$ m

Distance from t = 0 to t = 3 sec = |9| + |−3| = 12 m

Average speed $= \frac{\text{Distance}}{\text{Time}} = \frac{12}{3} = 4$ m/s

Therefore, option (b) is the correct answer.

8. When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. For example, if a person is driving and suddenly a boy appears on the road, then the time elasped before he slams the breaks of the car is the reaction time. Reaction time depends on complexity of the situation and on individual. You can measure your reaction time by a simple experiment. Take a ruler and ask your friend to drop it vertically through the gap between your thumb and forefinger. After you catch it, find the distance d travelled by the ruler. In a particular case, d was found to be 20 cm. [$g = 10$ m/s^2]

(i) Estimate reaction time.
(a) 0.1 s (b) 0.2 s (c) 0.4 s (d) 1 s

(ii) Now if you are driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0 m/s^2, find the distance travelled by the car after you see the need to put the brakes on.
(a) 21.75 m (b) 42.5 m (c) 10.875 m (d) 85 m

Sol.

(i) $0.2 = \frac{1}{2} \times 10t^2 \Rightarrow t^2 = \frac{0.4}{10} \Rightarrow t = 0.2\ s$

Reaction time = 0.2 sec

Therefore, option (b) is the correct answer.

(ii) $54 \text{ km/hr} = \frac{54 \times 5}{18} = 15$ m/s

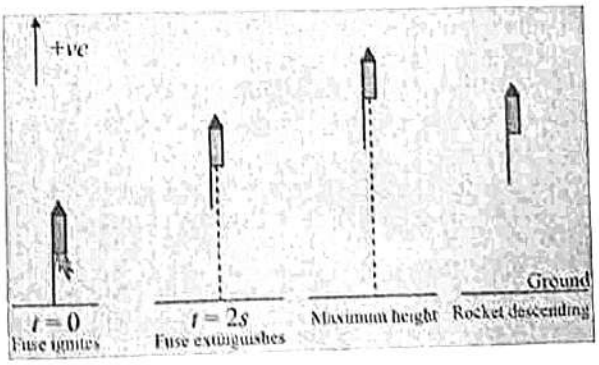
Total distance = 15 × reaction time + ($v^2/2a$)

Total distance $= 0.2 \times 15 + \frac{15^2}{2 \times 6} = 3 + \frac{225}{12}$

$= 21.75$ m

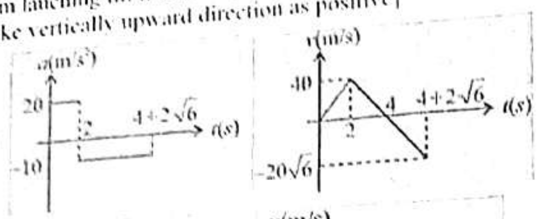
Therefore, option (a) is the correct answer.

9. A Diwali rocket is launched vertically with its fuse ignited at time $t = 0$, as shown. The charge provides constant acceleration for 2 sec. till rocket attains $V = 40$ ms^{-1}. Afterwards rocket continues to move freely under gravity.

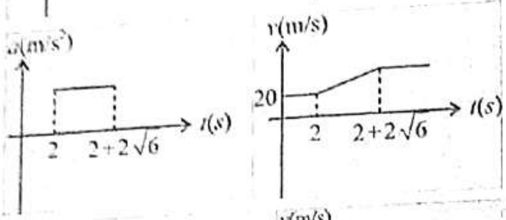


The acceleration-time and velocity-time graphs for the rocket from lauching till it reaches ground are
[Take vertically upward direction as positive]

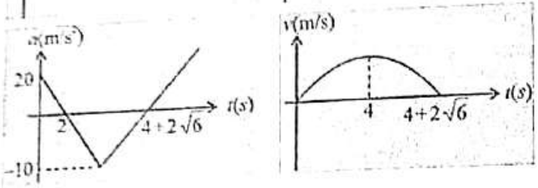
(a)



(b)



(c)



(d) None of these

Sol. $v = u + at$

$\Rightarrow 40 = 0 + a \times 2 \Rightarrow a = 20 \text{ m/s}^2$

$v^2 = u^2 + 2ah \Rightarrow (40)^2 = 0 + 2 \times 20h$

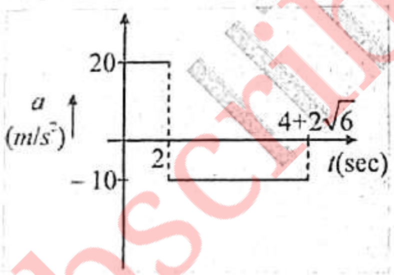
$\Rightarrow h = \dfrac{40 \times 40}{40} = 40 \text{ m}$

Time taken in reaching from $h = 40$ m to ground

$h = ut - \dfrac{1}{2}gt^2 \Rightarrow -40 = 40t - \dfrac{1}{2} \times 10 \times t^2$

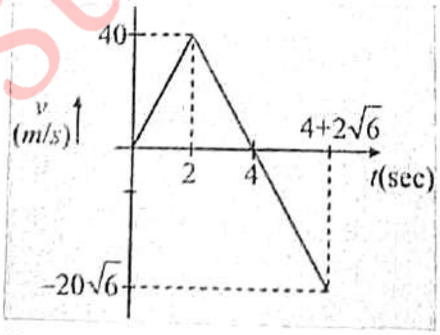
On solving

$t = 4 + 2\sqrt{6}$ sec



$v^2 = 40^2 + 2(-10)(-40)$

$\Rightarrow v = -20\sqrt{6}$ m/s



Therefore, option (a) is the correct answer.

10. A block of mass m is fired horizontally along a level surface that is lubricated with oil. The oil provides a viscous resistance that varies as the 3/2 power of the speed. If the initial speed of the block is v_0 at $x = 0$, find the maximum distance reached by the block. Assume no resistance to motion other than that provided by the oil.

(a) $mv_0^{1/3}$ (b) $2mv_0^{1/3}$

(c) $3mv_0^{1/3}$ (d) $4mv_0^{1/3}$

Sol. $F = -v^{3/2}$

$a = -\dfrac{1}{m}v^{3/2}$

$v\dfrac{dv}{dx} = -\dfrac{1}{m}v^{3/2}$

$\displaystyle\int_{v_0}^{0} v^{-\frac{1}{2}}dv = -\frac{1}{m}\int_{0}^{d} dx$

$2mv_0^{1/2} = d$ or $d = 2mv_0^{1/2}$

Therefore, option (b) is the correct answer.

11. Acceleration of particle moving rectilinearly is $a = 4 - 2x$ (where x is position in metre and a in m/s²). It is at rest at $x = 0$. At what position x (in metre) will the particle again come to instantaneous rest?

(a) 2 m (b) 3 m (c) 4 m (d) 5 m

Sol. $\dfrac{vdv}{dx} = 4 - 2x$

$\displaystyle\int_0^v vdv = \int_0^x (4-2x)dx \Rightarrow \frac{v^2}{2} = 4x - x^2$

when $v = 0$, $4x - x^2 = 0$

$x = 0, 4$

∴ At $x = 4$ m, the particle will again come to rest.

Therefore, option (c) is the correct answer.

12. The driver of a train A running at 25 ms^{-1} sights a train B moving in the same direction on the same track with 15 ms^{-1}. The driver of train A applies brakes to produce a deceleration of 1.0 ms^{-2}. To avoid accident the minimum distance between two trains should be

(a) 20 m (b) 30 m (c) 50 m (d) 70 m

Sol. $v_r = 25 - 15 = 10$ m/s and $a_r = -1$ m/s² so by $v_r^2 = u_r^2 + 2a_rS$

$S = \dfrac{100}{2 \times 1} = 50 \text{ m}.$

Therefore, option (c) is the correct answer.

13. The driver of a car travelling at a speed of 20 m/s, wishes to overtake a truck that is moving with a constant speed of 20 m s^{-1} in the same lane. The car's maximum acceleration is 0.5 m s^{-2}. Initially the vehicles are separated by 40 m, and the car returns back into its lane after it is 40 m ahead of the truck. The car is 3 m long and the truck 17 m. Based on the above comprehension, answer the following questions

(i) Find the minimum time required for the car to pass the truck and return back to its lane?

(a) 30 s (b) 20 s (c) 40 s (d) 70 s

(*ii*) What distance does the car travel during this time?

(*a*) 250 m (*b*) 350 m (*c*) 600 m (*d*) 500 m

(*iii*) What is the final speed of the car ?

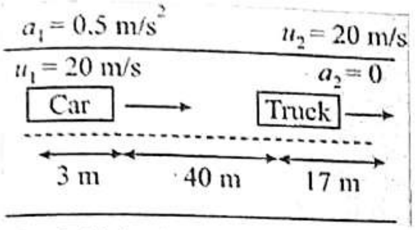
(*a*) 30 ms^{-1} (*b*) 25 ms^{-1} (*c*) 35 ms^{-1} (*d*) 20 ms^{-1}

Sol. (*i*) Displacement of car relative to truck

$x_r = 40 + 17 + 3 + 40$

$= 100$ m.

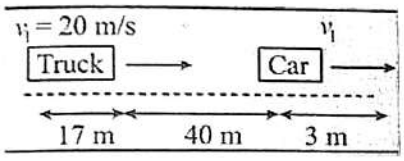
$t = 0$ (Initilly)



Relative initial velocity between car and truck

$u_r = 20 - 20 = 0$

$t = t$ (Finally)



Relative acceleration between car and truck

$a_r = 0.5 - 0 = 0.5$ m/s^2

Let required time = t.

∴ II equation of motion

$$x_r = u_r t + \frac{1}{2} a_r t^2 \Rightarrow 100 = 0 + \frac{1}{2} \times 0.5 \times t^2$$

$\Rightarrow t = 20$ sec.

Therefore, option (*b*) is the correct answer.

(*ii*) Distance travelled by car

$$x_c = ut + \frac{1}{2} at^2$$

$$= 20 \times 20 + \frac{1}{2} \times 0.5 \times 20^2 = 500 \text{ m}$$

Therefore, option (*d*) is the correct answer.

(*iii*) Final speed of the car

$= u + at$

$= 20 + 0.5 \times 20 = 30$ m/s.

Therefore, option (*a*) is the correct answer.

14. Men are running along a road at 15 km/h behind one another at equal intervals of 20 m. Cyclists are riding in the same direction at 25 km/h at equal intervals of 30 m. What should be the speed of an observer travelling along the road in opposite direction so that whenever he meets a runner he also meets a cyclist? (neglect the size of cycle)

(*a*) 10 kmh^{-1} (*b*) 7 kmh^{-1} (*c*) 15 kmh^{-1} (*d*) 5 kmh^{-1}

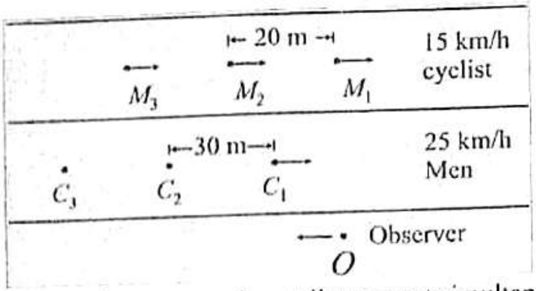
Sol. Let u = speed of observer.

Relative velocity between observer and a man

$= u + 15$ km/h.

Relative velocity between observer and a cyclist.

$= u + 25$ km/h.



Hence, to for a man and a cyclist to meet simultaneously

$$\frac{20 \text{ m}}{(u+15)\text{ km/h}} = \frac{30 \text{ m}}{(u+25)\text{ km/h}}$$

$\Rightarrow u = 5$ km/h

Therefore, option (*d*) is the correct answer.

15. Car *A* and car *B* start moving simultaneously in the same direction along the line joining them. Car *A* moves with a constant acceleration $a = 4$ m/s^2, while car *B* moves with a constant velocity $v = 1$ m/s. At time $t = 0$, car *A* is 10 m behind car *B*. Find the time when car *A* overtakes car *B*. How much time *A* take to overtake.

(*a*) 2.5 s (*b*) 7.5 s (*c*) 5 s (*d*) 1.25 s

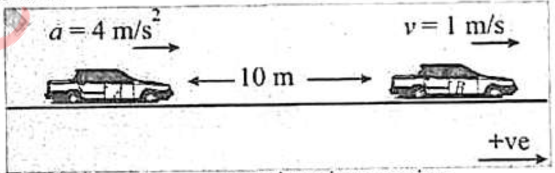
Sol. Given: $u_A = 0$, $u_B = 1$ m/s, $a_A = 4$ m/s^2 and $a_B = 0$

Assuming car *B* to be at rest, we have

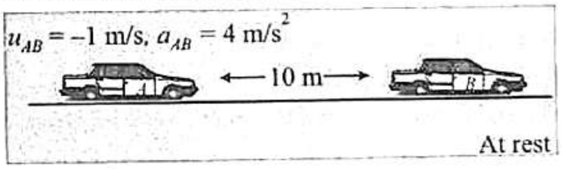
$u_{AB} = u_A - u_B = 0 - 1 = -1$ m/s

$a_{AB} = a_A - a_B = 4 - 0 = 4$ m/s^2

Now, the problem can be assumed in simplified form as follow:



Substituting the proper values in equation



$$S = ut + \frac{1}{2} at^2$$

we get $10 = -t + \frac{1}{2}(4)(t^2)$ or $2t^2 - t - 10 = 0$

Ignoring the negative value, the desired time is 2.5s.

Therefore, option (*a*) is the correct answer.

Note: The above problem can also be solved without using the concept of relative motion as under. At the time when *A* overtakes *B*,

$$S_A = S_B + 10$$

$$\therefore \frac{1}{2} \times 4 \times t^2 = 1 \times t + 10$$

or $2t^2 - t - 10 = 0$

which on solving gives $t = 2.5$ s and -2 s, the same as we found above.

School Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. A body in one dimensional motion has zero speed at an instant. At that instant, it must have
   (*a*) Zero velocity
   (*b*) Zero acceleration
   (*c*) non-zero velocity
   (*d*) non-zero acceleration

2. A particle is moving along a circle such that it completes one revolution in 40 seconds. In 2 minutes, 20 seconds, the ratio $\frac{\text{displacement}}{\text{distance}}$ is
   (*a*) 0 (*b*) $\frac{1}{7}\pi$
   (*c*) $\frac{2}{7}\pi$ (*d*) $\frac{1}{11}\pi$

3. Consider the motion of the tip of the second hand of a clock. In one minute (R be the length of second hand), its
   (*a*) Displacement is $2\pi R$
   (*b*) Distance covered is $2R$
   (*c*) Displacement is zero
   (*d*) Distance covered is zero

4. A train starts from rest from a station with acceleration $0.2\ \text{m/s}^2$ on a straight track and then comes to rest after attaining maximum speed on another station due to retardation $0.4\ \text{m/s}^2$. If total time spent is half an hour, then distance between two stations is [Neglect length of train]
   (*a*) 216 km (*b*) 512 km (*c*) 728 km (*d*) 1296 km

5. The position x of particle moving along x-axis varies with time t as $x = A\sin(\omega t)$ where A and ω are positive constants. The acceleration a of particle varies with its position (x) as
   (*a*) $a = Ax$ (*b*) $a = -\omega^2 x$
   (*c*) $a = A\omega x$ (*d*) $a = -\omega^2 \times A$

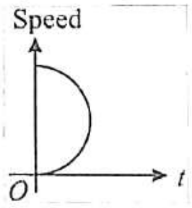
6. A particle moves in a straight line and its position x at time t is given by $x^2 = 2 + t$. Its acceleration is given by
   (*a*) $\frac{-2}{x^3}$ (*b*) $-\frac{1}{4x^3}$
   (*c*) $-\frac{1}{4x^2}$ (*d*) $\frac{1}{x^2}$

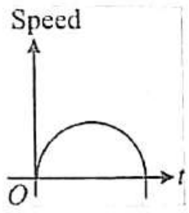
7. A body is moving with variable acceleration (a) along a straight line. The average acceleration of body in time interval t_1 to t_2 is
   (*a*) $\frac{a[t_2+t_1]}{2}$ (*b*) $\frac{a[t_2-t_1]}{2}$
   (*c*) $\frac{\int_{t_1}^{t_2} a\,dt}{t_2+t_1}$ (*d*) $\frac{\int_{t_1}^{t_2} a\,dt}{t_2-t_1}$

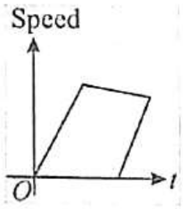
8. A body is projected vertically upward with speed 10 m/s and other at same time with same speed in downward direction from the top of a tower. The magnitude of acceleration of first body w.r.t. second is {take $g = 10\ \text{m/s}^2$}
   (*a*) Zero (*b*) $10\ \text{m/s}^2$
   (*c*) $5\ \text{m/s}^2$ (*d*) $20\ \text{m/s}^2$

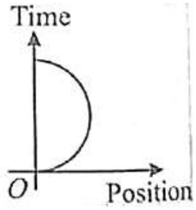
9. The position of a particle moving along x-axis given by $x = (-2t^3 + 3t^2 + 5)$m. The acceleration of particle at the instant its velocity becomes zero is
   (*a*) $12\ \text{m/s}^2$ (*b*) $-12\ \text{m/s}^2$
   (*c*) $-6\ \text{m/s}^2$ (*d*) Zero

10. Which one of the following graphs for a body moving along a straight line is possible?

(*a*)


(*b*)


(*c*)


(*d*)


Directions: These questions consist of two statements each, printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.

(*a*) Both Assertion and Reason are True and the Reason is a correct explanation of the Assertion.
(*b*) Both Assertion and Reason are True but Reason is not a correct explanation of the Assertion.
(*c*) Assertion is True but the Reason is False.
(*d*) Assertion is False but Reason is True.

11. **Assertion:** The motion of body projected under the effect of gravity without air resistance is uniformly accelerated motion.
    Reason: If a body is projected upwards or downwards, then the direction of acceleration is downward.

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22. (*i*)
(*ii*)

12. **Assertion:** The relative acceleration of two objects moving under the effect of gravity, only is always zero, irrespective of direction of motion.

Reason: The acceleration of object moving under the effect of gravity have acceleration always in downward direction and is independent from size and mass of object

VERY SHORT ANSWER TYPE QUESTIONS

13. Two persons A and B are walking with speed 5 km/h and 7 km/h respectively in the same direction. Find the relative displacement of B w.r.t. A after 2 hours.

14. Explain that a body can have zero average velocity but not zero average speed

15. The displacement x (in metre) of the body is given by $x = 5t - 3t^2$. Calculate the velocity and acceleration of the body at time $t = 2$s.

16. The acceleration a of an object at time t is given by $a = 2t$. If initial velocity of the object is 2 m/s, then find its velocity after 5 s.

17. Which of the two, velocity or acceleration decides the direction of motion of a body? Explain, with the help of an example.

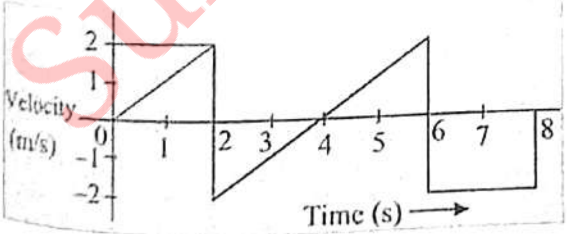
18. Differentiate between average acceleration and instantaneous acceleration.

19. The distance travelled by a particle moving in a straight line is found to be proportional to the square of the time elapsed. Is it moving with constant speed or constant acceleration? Explain.

SHORT ANSWER TYPE QUESTIONS

20. A car travelling at 50 km/h overtakes another car travelling at 32 km/h. Assuming each car to be 5 m long. find the time taken during the overtake.

21. The velocity-time graph of a particle moving along a straight line is as shown below. Calculate the distance covered between $t = 0$ to $t = 8$ seconds. Also calculate the displacement between the same interval.



LONG ANSWER TYPE QUESTIONS

22. (*i*) What do you mean by free fall?

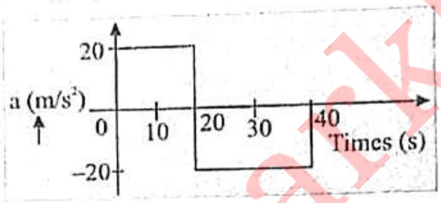
(*ii*) Taking upwards direction positive, point of projection as origin and neglecting air resistance, draw the position-time, velocity-time and acceleration-time graphs of an object under free fall.

23. The acceleration-time graph of a body starting from rest is shown below. Calculate the

(*i*) Average acceleration

(*ii*) Average velocity

(*iii*) Average speed in the time interval $t = 0$ to $t = 40$s



CASE STUDY BASED QUESTIONS

24. A body is allowed to fall from a height of 100 m. If the time taken for the first 50 m is t_1 and for the remaining 50 m is t_2.

(*i*) Which is correct?

(*a*) $t_1 = t_2$

(*b*) $t_1 > t_2$

(*c*) $t_1 < t_2$

(*d*) Depends upon the mass

(*ii*) The ratio of t_1 and t_2 is nearly

(*a*) 5 : 2 (*b*) 3 : 1

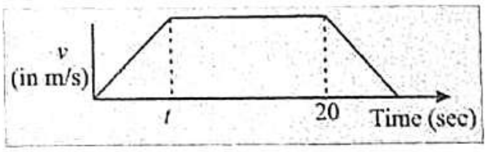
(*c*) 3 : 2 (*d*) 5 : 3

(*iii*) The ratio of times to reach the ground and to reach first half of the distance is

(*a*) $\sqrt{3} : 1$ (*b*) $\sqrt{2} : 1$

(*c*) 5 : 2 (*d*) $1 : \sqrt{3}$

25. The velocity-time graph of a particle moving along a straight line is shown in figure The rate of acceleration and deceleration is constant and it is equal to 5 ms^{-2}. If the average velocity during the motion is 20 ms^{-1}, then



(*i*) The value of t is

(*a*) 5 s (*b*) 10 s

(*c*) 20 s (*d*) $5\sqrt{2}$ s

(*ii*) The maximum velocity of the particle is

(*a*) 20 ms^{-1} (*b*) 25 ms^{-1}

(*c*) 30 ms^{-1} (*d*) 40 ms^{-1}

(*iii*) The distance travelled with uniform velocity is

(*a*) 375 m (*b*) 125 m

(*c*) 300 m (*d*) 450 m

Prarambh (Topicwise)

POSITION, DISTANCE AND DISPLACEMENT

1. A body moves 6 m north, 8 m east and 10 m vertically upwards, what is its resultant displacement from initial position?

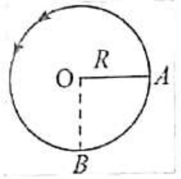
(a) $10\sqrt{2}$ m (b) 10 m

(c) $\frac{10}{\sqrt{2}}$ m (d) 20 m

2. An aeroplane flies from $P(-4\text{m}, -5\text{m}, +8\text{m})$ to $Q(7\text{m}, -2\text{m}, -3\text{m})$ in xyz coordinate system. The displacement vector of aeroplane is

(a) 3 m (b) $\sqrt{43}$ m (c) $\sqrt{251}$ m (d) $\sqrt{83}$ m

3. A body moves in circular path of radius R from A to B as shown. Its displacement and distance covered are



(a) $R, \frac{3\pi R}{2}$ (b) $\sqrt{2}R, \frac{\pi R}{2}$

(c) $\sqrt{2}R, \frac{3\pi R}{2}$ (d) None of these

4. A particle covers half of the circle of radius r. Then the displacement and distance of the particle are respectively

(a) $2\pi r, 0$ (b) $2r, \pi r$ (c) $\frac{\pi r}{2}, 2r$ (d) $\pi r, r$

SPEED AND VELOCITY

5. A person travels along a straight road for half the distance with velocity v_1 and the remaining half distance with velocity v_2. The average velocity is given by

(a) v_1v_2 (b) $\frac{v_2^2}{v_1^2}$

(c) $\frac{v_1+v_2}{2}$ (d) $\frac{2v_1v_2}{v_1+v_2}$

6. A car travels the first half of a distance between two places at a speed of 30 km/hr and the second half of the distance at 50 km/hr. The average speed of the car for the whole journey is

(a) 42.5 km/hr (b) 40.0 km/hr

(c) 37.5 km/hr (d) 35.0 km/hr

7. A person travels along a straight road for the first half time with a velocity v_1 and the next half time with a velocity v_2. The mean velocity V of the man is

(a) $\frac{2}{V} = \frac{1}{v_1} + \frac{1}{v_2}$ (b) $V = \frac{v_1+v_2}{2}$

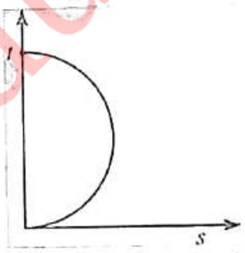
(c) $V = \sqrt{v_1v_2}$ (d) $V = \sqrt{\frac{v_1}{v_2}}$

8. If a car covers $2/5^{\text{th}}$ of the total distance with v_1 speed and $3/5^{\text{th}}$ distance with v_2 then average speed is

(a) $\frac{1}{2}\sqrt{v_1v_2}$ (b) $\frac{v_1+v_2}{2}$

(c) $\frac{2v_1v_2}{v_1+v_2}$ (d) $\frac{5v_1v_2}{3v_1+2v_2}$

9. Which of the following options is correct for the object having a straight line motion represented by the following graph?



(a) The object moves with constantly increasing velocity from O to A and then it moves with constant velocity

(b) Velocity of the object increases uniformly

(c) Average velocity is zero

(d) The graph shown is impossible

CONSTANT ACCELERATION

10. A particle experiences a constant acceleration for 20 sec after starting from rest. If it travels a distance S_1 in the first 10 sec and a distance S_2 in the next 10 sec, then

(a) $S_1 = S_2$ (b) $S_1 = S_2/3$

(c) $S_1 = S_2/2$ (d) $S_1 = S_2/4$

11. A body is moving from rest under constant acceleration and let S_1 be the displacement in the first $(p-1)$ sec and S_2 be the displacement in the first p sec. The displacement in $(p^2-p+1)^{\text{th}}$ sec. will be

(a) $S_1 + S_2$ (b) $S_1 S_2$

(c) $S_1 - S_2$ (d) S_1 / S_2

12. The displacement of body moving with constant acceleration in 3rd seconds is 2m and in 5th second is 9m. Find the acceleration of body.

(a) $\frac{5}{2}$ ms^{-2} (b) $\frac{7}{2}$ ms^{-2}

(c) $\frac{9}{2}$ ms^{-2} (d) $\frac{11}{2}$ ms^{-2}

---

13. A point m
denote the
of time t_1.
correct?
(a) $(v_1 -$
(b) $(v_1 -$
(c) $(v_1 -$
(d) $(v_1 -$

14. A motor c
to stop on
of 10 m.
(a) 20 m
(c) –40

15. Which of
(a) A bo
(b) A b
vary
(c) A b
vary
(d) The
its a

16. A car m
the appl
If the ve
force in
(a) $\frac{20}{3}$

17. A car s
4ms^{-2},
before i
starting
(a) 23.
(c) 24.

18. A car s
on a str
deceler
speed
(a) $\frac{a}{4}$

19. A car, s
a dista
thaen
distanc
(a) S
(c) S

13. A point moves with uniform acceleration and v_1, v_2 and v_3 denote the average velocities in the three successive intervals of time t_1, t_2 and t_3. Which of the following relations is correct?

(*a*) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$

(*b*) $(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$

(*c*) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 - t_3)$

(*d*) $(v_1 - v_2) : (v_2 - v_3) = (t_1 - t_2) : (t_2 + t_3)$

14. A motor car moving with a uniform speed of 20 m/sec comes to stop on the application of brakes after travelling a distance of 10 m. Its acceleration is

(*a*) 20 m/sec^2 (*b*) –20 m/sec^2
(*c*) –40 m/sec^2 (*d*) +2 m/sec^2

15. Which of the following four statements is false?

(*a*) A body can have zero velocity and still be accelerated.

(*b*) A body can have a constant velocity and still have a varying speed.

(*c*) A body can have a constant speed and still have a varying velocity.

(*d*) The direction of the velocity of a body can change when its acceleration is constant.

16. A car moving with a velocity of 10 m/s can be stopped by the application of a constant force F in a distance of 20 m. If the velocity of the car is 30 m/s, it can be stopped by this force in

(*a*) $\frac{20}{3}$ m (*b*) 20 m (*c*) 60 m (*d*) 180 m

17. A car starting from rest and moving with acceleration of 4ms^{-2}, covers half the distance during last second of motion before it strikes a vertical wall. Find the distance of wall from starting point.

(*a*) 23.3 m (*b*) 24 m
(*c*) 24.3 m (*d*) 26.3 m

18. A car starts from rest and moves with uniform acceleration a on a straight road from time $t = 0$ to $t = T$. After that, constant deceleration brings it to rest. In this process the average speed of the car is

(*a*) $\frac{aT}{4}$ (*b*) $\frac{3aT}{2}$ (*c*) $\frac{aT}{2}$ (*d*) aT

19. A car, starting from rest, accelerates at constant rate f through a distance S, then continues at constant speed for time t and thaen decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S, then

(*a*) $S = \frac{1}{2}ft^2$ (*b*) $S = \frac{1}{4}ft^2$

(*c*) $S = \frac{1}{72}ft^2$ (*d*) $S = \frac{1}{6}ft^2$

MOTION UNDER GRAVITY

20. A stone falls from a balloon that is descending at a uniform rate of 12 m/s. The displacement of the stone from the point of release after 10 sec is

(*a*) 490 m (*b*) 510 m (*c*) 610 m (*d*) 725 m

21. Two bodies of different masses m_a and m_b are dropped from two different heights a and b. The ratio of the time taken by the two to cover these distances are

(*a*) $a : b$ (*b*) $b : a$ (*c*) $\sqrt{a} : \sqrt{b}$ (*d*) $a^2 : b^2$

22. Two balls are projected simultaneously with the same speed from the top of a tower, one vertically upwards and the other vertically downwards. If the first ball strikes the ground with speed 20 m/s then speed of second ball when it strikes the ground is

(*a*) 10 m/s (*b*) 20 m/s
(*c*) 40 m/s (*d*) Data insufficient

23. A stone dropped from the top of the tower touches the ground in 4 sec. The height of the tower is about

(*a*) 80 m (*b*) 40 m (*c*) 20 m (*d*) 160 m

24. A ball is released from the top of a tower of height h. It takes t sec to reach the ground. Where will be the ball after time $t/2$ sec?

(*a*) At $h/2$ from the ground

(*b*) At $h/4$ from the ground

(*c*) Depends upon mass and volume of the ball

(*d*) At $3h/4$ from the ground

25. A body is slipping from an inclined plane of height h and length l. If the angle of inclination is θ, the time taken by the body to come from the top to the bottom of this inclined plane is

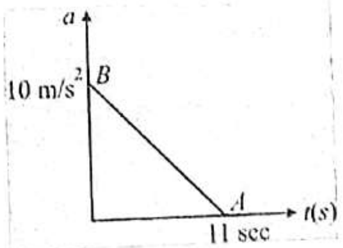
(*a*) $\sqrt{\frac{2h}{g}}$ (*b*) $\sqrt{\frac{2h}{g}}$ (*c*) $\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$ (*d*) $\sin\theta\sqrt{\frac{2h}{g}}$

26. A man in a balloon rising vertically with an acceleration of 4.9 m/sec^2 releases a ball 2 sec after the balloon is let go from the ground. The greatest height above the ground reached by the ball is (g = 9.8 m/sec^2)

(*a*) 14.7 m (*b*) 19.6 m (*c*) 9.8 m (*d*) 24.5 m

VARIABLE ACCELERATION

27. A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be



(*a*) 110 m/s (*b*) 55 m/s (*c*) 550 m/s (*d*) 660 m/s

28. If the velocity of a particle is given by $v = (180 - 16x)^{1/2}$ m/s, then its acceleration will be

(a) Zero (b) 8 m/s² (c) −8 m/s² (d) 4 m/s²

29. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

(a) Go on decreasing with time

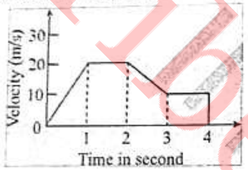
(b) Will be independent of α and β

(c) Drop to zero when $\alpha = \beta$

(d) Go on increasing with time

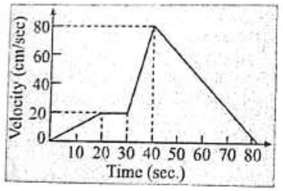
GRAPHS

30. The variation of velocity of a particle with time moving along a straight line is illustrated in the following figure. The distance travelled by the particle in four seconds is



(a) 60 m (b) 55 m (c) 25 m (d) 30 m

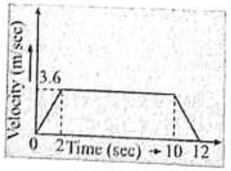
31. The $v - t$ graph of a moving object is given in figure. The maximum acceleration is



(a) 1 cm/sec² (b) 2 cm/sec²

(c) 3 cm/sec² (d) 6 cm/sec²

32. A lift is going up. The variation in the speed of the lift is as given in the graph. What is the height to which the lift takes the passengers?



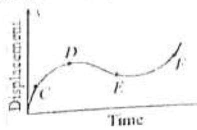
(a) 3.6 m

(b) 28.8 m

(c) 36.0 m

(d) Cannot be calculated from the above graph

33. The displacement time graph of moving particle is shown below

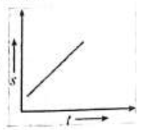


The instantaneous velocity of the particle is negative at the point.

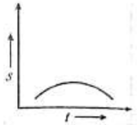
(a) D (b) F (c) C (d) E

34. Which of the following graphs represents uniform
velocity?

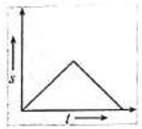
(a)



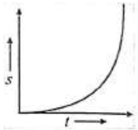
(b)



(c)

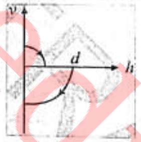


(d)

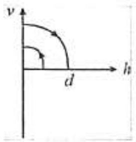


35. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

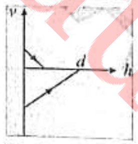
(a)



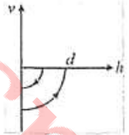
(b)



(c)

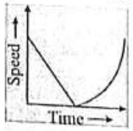


(d)

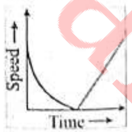


36. A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its motion if the air resistance (constant) is not ignored?

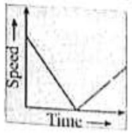
(a)



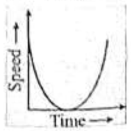
(b)



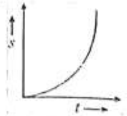
(c)

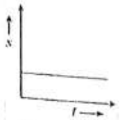


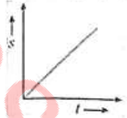
(d)

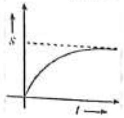


37. Which graph must represent non-uniform acceleration (s is displacement)?

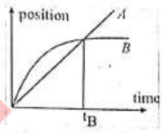
(a)


(b)


(c)


(d)


38. The graph shows position as a function of time for two trains running on parallel tracks. Which one of the following statements is true?



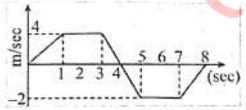
(a) At time t_B, both trains have the same velocity

(b) Both trains have the same velocity at some time after t_B

(c) Both trains have the same velocity at some time before t_B

(d) Somewhere on the graph, both trains have the same acceleration

39. The v-t graph of a linear motion is shown in adjoining figure. The distance from origin after 8 seconds is



(a) 18 meters (b) 16 meters
(c) 8 meters (d) 6 meters

ONE DIMENSIONAL RELATIVE MOTION

40. Two trains, each 50 m long are travelling in opposite direction with velocity 10 m/s and 15 m/s. The time of crossing is

(a) 2 s (b) 4 s (c) $2\sqrt{3}$ s (d) $4\sqrt{3}$ s

41. A 210 meter long train is moving due North at a of 25m/s. A small bird is flying due South a little above the train with speed 5m/s. The time taken by the bird to cross the train is

(a) 6 s (b) 7 s
(c) 9 s (d) 10 s

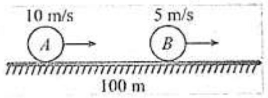
42. A stone is dropped from a building, and 2 seconds later another stone is dropped. (Both a are dropped from rest.) How far apart are the two stones by the time the first one has reached a speed of 30 m/s ?

(a) 80 m (b) 100 m
(c) 60 m (d) 40 m

43. Two trains each of length 50 m are approaching each other on parallel rails. Their velocities are 10 m/sec and 15 m/sec. They will cross each other in

(a) 2 sec (b) 4 sec (c) 10 sec (d) 6 sec

44. An object A is moving with 10 m/s and B is moving with 5 m/s in the same direction of positive x-axis. A is 100 m behind B as shown. Find time taken by A to Meet B



(a) 18 sec (b) 16 sec (c) 20 sec (d) 17 sec

45. A thief is running away on a straight road with a speed of 9 ms^{-1}. A police man chases him on a jeep moving at a speed of 10 ms^{-1}. If the instantaneous separation of the jeep from the motorcycle is 100m, how long will it take for the police man to catch the thief?

(a) 1s (b) 19s (c) 90s (d) 100s

DISTANCE OF NEAREST APPROACH

46. A body is projected vertically up at $t = 0$ with a velocity of 98 m/s. Another body is projected from the same point with same velocity after 4 seconds. Both bodies will meet after:

(a) 6 s (b) 8 s
(c) 10 s (d) 12 s

Prabal (JEE Main Level)

1. A car runs at constant speed on a circular track of radius 100 m taking 62.8 s on each lap. What is the average speed and average velocity on each complete lap?

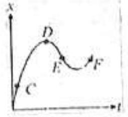
(a) Average velocity 10 m/s, average speed 10 m/s

(b) Average velocity zero, average speed 10 m/s

(c) Average velocity zero, average speed zero

(d) Average velocity 10 m/s, average speed zero

2. The displacement-time graph of a moving particle is shown below. The instantaneous velocity of the particle is zero at the point



(a) C (b) D (c) E (d) F

3. A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10s is x_1, in next 10 s is x_2 and in last 10 s is x_3. Then $x_1 : x_2 : x_3$ is

(a) 1 : 2 : 4 (b) 1 : 2 : 5

(c) 1 : 3 : 5 (d) 1 : 3 : 9

4. A body is dropped from a height h under acceleration due to gravity g. If t_1 and t_2 are time intervals for its fall for first half and the second half distance, the relation between them is

(a) $t_1 = t_2$ (b) $t_1 = 2t_2$

(c) $t_1 = 2.414\,t_2$ (d) $t_1 = 4t_2$

5. A body is thrown upward and reaches its maximum height. At that position

(a) Its velocity is zero and its acceleration is also zero

(b) Its velocity is zero but its acceleration is maximum

(c) Its acceleration is minimum

(d) Its velocity is zero and its acceleration is the acceleration due to gravity

6. The motion of a body is given by the equation $\frac{dv(t)}{dt} = 6.0 - 3v(t)$, where $v(t)$ is speed in m/s and t in sec. If body was at rest at $t = 0$ choose the wrong option.

(a) The terminal speed is 2.0 m/s

(b) The speed varies with the time as $v(t) = 2(1 - e^{-3t})$ m/s

(c) The speed is 0.1 m/s when the acceleration is half the initial value

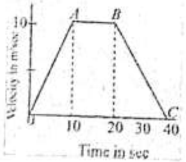
(d) The magnitude of the initial acceleration is 6.0 m/s^2

7. The displacement time graphs of motion of two particles A and B are straight lines making angles of 30° and 60° respectively with the time axis. If the velocity of A is v_A and that of B is v_B then the value of $\frac{v_A}{v_B}$ is

(a) 1/2 (b) $1/\sqrt{3}$

(c) $\sqrt{3}$ (d) 1/3

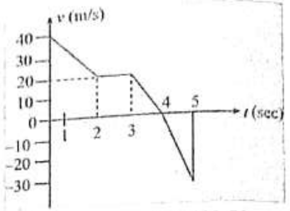
8. The curve shown represents the velocity-time graph of a particle, its acceleration values along OA, AB and BC in metre/sec^2 are respectively



(a) 1, 0, −0.5 (b) 1, 0, 0.5

(c) 1, 1, 0.5 (d) 1, 0.5, 0

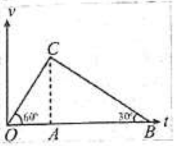
9. In the following velocity-time graph of a body, the distance and displacement travelled by the body in 5 second in meters will be



(a) 75, 115 (b) 105, 75

(c) 45, 75 (d) 95, 55

10. The velocity-time graph of body is shown in figure. Which of the following statements is wrong?



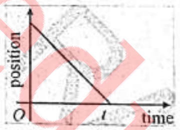
(a) Ratio of average velocities for parts OA and OB = 1

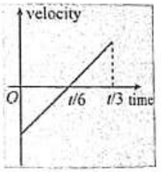
(b) $\frac{OA}{OB} = \frac{1}{4}$

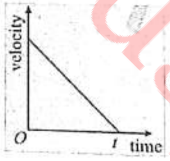
(c) Magnitude of average accelerations for parts OA and OB is in inverse of ratio of distances covered

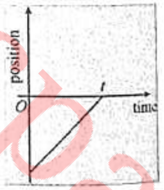
(d) Ratio of distance covered for parts OA and OB is 1 : 3

11. For which of the following graphs the average velocity of a particle moving along a straight line for time interval $(0, t)$ must be negative?

(a)


(b)


(c)


(d)


12. Four particles move along x-axis. Their coordinates (in meters) as functions of time (in seconds) are given by

Particle 1 : $x(t) = 3.5 - 2.7t^3$

Particle 2 : $x(t) = 3.5 + 2.7t^3$

Particle 3 : $x(t) = 3.5 + 2.7t^2$

Particle 4 : $x(t) = 3.5 - 3.4t - 2.7t^2$

Which of these particles have constant acceleration?

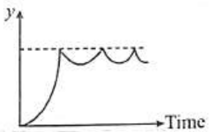
(a) All four (b) Only 1 and 2

(c) Only 2 and 3 (d) Only 3 and 4

13. A particle is projected up from ground with initial speed v_0. Starting from time $t = 0$ to $t = t_1$.
(*a*) Distance travelled and magnitude of displacement are not equal if $t_1 < \frac{v_0}{g}$
(*b*) Distance travelled and magnitude of displacement are equal if $\frac{v_0}{g} < t_1 < \frac{2v_0}{g}$
(*c*) Distance travelled and magnitude of displacement may not be equal if $0 < t_1 < \frac{2v_0}{g}$
(*d*) The magnitude of displacement is greater than the distance travelled if $\frac{v_0}{g} < t_1 < \frac{2v_0}{g}$

14. Two bodies P and Q have to move equal distances starting from rest. P is accelerated with $2a$ for first half distance, then its acceleration becomes a for last half, whereas Q has acceleration a for first half and acceleration $2a$ for last half, then for whole journey.
(*a*) Average speed of P is more than that of Q
(*b*) Average speed of both will be same
(*c*) Maximum speed during the journey is more for P
(*d*) Maximum speed during the journey is more for Q

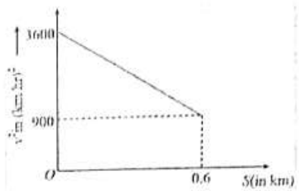
15. The graph below describes the motion of a ball rebounding from a horizontal surface being released from a point above the surface. The quantity represented on the y-axis is



(*a*) Displacement of ball
(*b*) Velocity of ball
(*c*) Acceleration of ball
(*d*) Momentum of ball

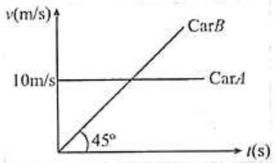
16. A car is moving along a straight line. It is taken from rest to a velocity of 20 ms^{-1} by a constant acceleration of 5ms^{-2}. It maintains a constant velocity of 20 ms^{-1} for 5 seconds and then is brought to rest again by a constant acceleration of -2 ms^{-2}. Find the distance covered by the car.
(*a*) 120 m (*b*) 200 m
(*c*) 240 m (*d*) 400 m

17. A graph between the square of the velocity of a particle and the distance 'S' moved by the particle is shown in the figure. The acceleration of the particle in kilometer per hour square is



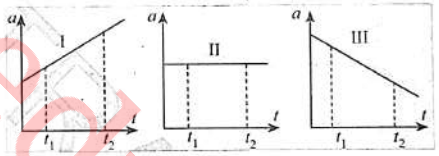
(*a*) 2250 (*b*) 225
(*c*) -2250 (*d*) -225

18. Initially car A is 10.5 m ahead of car B. Both start moving at time $t = 0$ in the same direction along a straight line. The velocity time graph of two cars is shown in figure. The time when the car B will catch the car A, will be



(*a*) $t = 21$ s (*b*) $t = 2\sqrt{5}$ s
(*c*) $t = 20$ s (*d*) None of these

19. Each of the three graphs represents acceleration versus time for an object that already has a positive velocity at time t_1. Which graphs show an object whose speed is increasing for the entire time interval between t_1 and t_2?

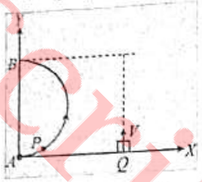


(*a*) graph I, only (*b*) graphs I and II, only
(*c*) graphs I and III, only (*d*) graphs I, II, and III

20. From the top of a tower, a stone is thrown up. It reaches the ground in time t_1. A second stone thrown down with the same speed reaches the ground in time t_2. A third stone released from rest reaches the ground in time t_3. Then:
(*a*) $t_3 = \frac{t_1 + t_2}{2}$ (*b*) $t_3 = \sqrt{t_1 t_2}$
(*c*) $\frac{1}{t_3} = \frac{1}{t_1} - \frac{1}{t_2}$ (*d*) $t_3^2 = t_1^2 - t_2^2$

21. A bus is moving with a velocity 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100s. If the bus is at a distance of 1 km from the scooterist, with what velocity should the scooterist chase the bus?
(*a*) 50 ms^{-1} (*b*) 40 ms^{-1}
(*c*) 30 ms^{-1} (*d*) 20 ms^{-1}

22. A particle P starts from origin as shown and moves along a circular path. Another particle Q crosses x-axis at the instant particle P leaves origin. Q moves with constant speed V parallel to y-axis and is all the time having y-coordinate same as that of P. When P reaches diametrically opposite at point B, its average speed is



(a) πV (b) $\frac{\pi V}{2}$

(c) $\frac{V}{2}$ (d) None of these

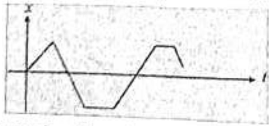
INTEGER TYPE QUESTIONS

23. A thief in a stolen car passes through a police check post at his top speed of 90 kmh^{-1}. A motorcycle cop, reacting after 2 s, accelerates from rest at 5 ms^{-2}. His top speed being 108 kmh^{-1}. Find the maximum separation between policemen and thief is $K \times 10^{-1}$m. Find K.

24. Two trains are moving with velocities $v_1 = 10$ ms^{-1} and $v_2 = 20$ ms^{-1} on the same track in opposite directions. After the application of brakes if their retarding rates are $a_1 = 2$ ms^{-2} and $a_2 = 1$ ms^{-2} respectively, then find the minimum distance of separation (in m) between the trains to avoid collision.

25. Two trains, each travelling with a speed of 37.5 kmh^{-1}, are approaching each other on the same straight track. A bird that can fly at 60 kmh^{-1} flies off from one train when they are 90 km apart and heads directly for the other train. On reaching the other train, it flies back to the first and so on. Total distance covered by the bird is ______ km.

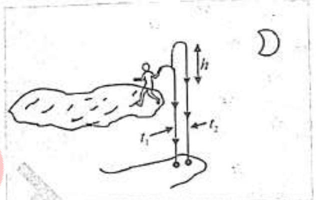
26. Consider a particle moving on a straight line with varying velocity. Its position time graph is as shown.



Find the number of times its velocity changes during motion.

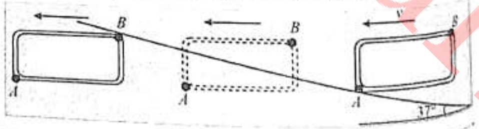
27. A large procession of people is moving along a road of width 10 m. There is a railway track across the road. The number of people present per square meter of the road is 6 (on an average). The average speed at which the procession is moving is 0.4 m/s. Find the number of people crossing the railway track per second.

28. An astronaut on the starship Enterprise is roaming around on a distant planet. He drops a rock from the top of a cliff and observes that it takes time $t_1 = \sqrt{2}$ sec to reach the bottom. He now throws another rock vertically upwards so that it reaches a height $h = 10$ m above the cliff before dropping down the cliff. The second rock takes a total time $t_2 = 2$ sec to reach the bottom of the cliff, starting from the time it leaves the astronaut's hand. The planet has a very thin atmosphere which offers negligible air resistance. What is the value of acceleration (in m/s^2) due to gravity on this planet?



29. A stone is dropped from a certain height and can reach the ground in 5s. But in its fall, the stone is stopped after 3s of the fall for a moment and is dropped again at once. Now the stone reaches the ground in total time of 't' seconds. Find the value of t.

30. A train is travelling at v m/s along a level straight track. Very near and parallel to the track is a wall. On the wall a naughty boy has drawn a straight line that slopes upward at 37° angle with the horizontal. A passenger in the train is observing the line out of window (0.90 m high, 1.8 m wide as shown in figure). The line first appears at window corner A and finally disappears at window corner B. If it takes 0.4 sec between appearance at A and disappearance of the line at B, what is the value of v (in cm/s)?



MULTIPLE

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(c) V
(d) A

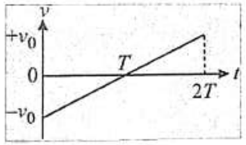
Parikshit (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

1. Mark the correct statements for a particle going on a straight line
   (*a*) If the velocity is zero at any instant, the acceleration should also be zero at that instant.
   (*b*) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.
   (*c*) If the velocity and acceleration have opposite sign, the object is slowing down.
   (*d*) If the position and velocity have opposite sign, the particle is moving towards the origin.

2. Let $\vec{v}$ and $\vec{a}$ denote the velocity and acceleration respectively of a body in one-dimensional motion then which among the following is/are true?
   (*a*) $|\vec{v}|$ must decrease when $\vec{a} < 0$.
   (*b*) Speed must increase when $\vec{a} > 0$.
   (*c*) Speed will increase when both $\vec{v}$ and $\vec{a}$ are < 0.
   (*d*) Speed will decrease when $\vec{v} < 0$ and $\vec{a} > 0$.

3. The figure shows the velocity (v) of a particle plotted against time (t)



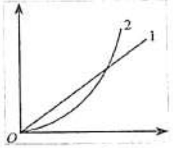
   (*a*) The particle changes its direction of motion at some point.
   (*b*) The acceleration of the particle remains constant.
   (*c*) The displacement of the particle is zero.
   (*d*) The initial and final speeds of the particle are the same.

4. A particle moves with constant speed v along a regular hexagon $ABCDEF$ in the same order. Then the magnitude of the average velocity for its motion from A to
   (*a*) F is $v/5$ (*b*) D is $v/3$
   (*c*) C is $\frac{v\sqrt{3}}{2}$ (*d*) B is v

5. Path of a particle moving in x-y plane is $y = 3x + 4$. At some instant suppose x-component of velocity is 1 m/s and it is increasing at a rate of 1 m/s^2. Then
   (*a*) At this instant the speed of particle is $\sqrt{10}$ m/s.
   (*b*) At this instant the acceleration of particle is $\sqrt{10}$ m/s^2.
   (*c*) Velocity time graph is a straight line.
   (*d*) Acceleration-time graph is a straight line.

6. A particle having a velocity $v = v_0$ at $t = 0$ is brought to rest by decelerating at the rate $|a| = \alpha\sqrt{v}$, where α is a positive constant.
   (*a*) The particle comes to rest at $t = \frac{2\sqrt{v_0}}{\alpha}$
   (*b*) The particle will come to rest at infinity.
   (*c*) The distance travelled by the particle is $\frac{2v_0^{3/2}}{\alpha}$.
   (*d*) The distance travelled by the particle is $\frac{2}{3}\frac{v_0^{3/2}}{\alpha}$.

7. A particle is resting over a smooth horizontal floor. At $t = 0$, a horizontal force starts acting on it. Magnitude of the force increases with time as $F = kt$, where k is a constant. Two curves are drawn for this particle as shown.



   (*a*) Curve-1 shows acceleration versus time.
   (*b*) Curve-2 shows velocity versus time.
   (*c*) Curve-2 shows velocity versus acceleration.
   (*d*) Curve-1 shows velocity versus acceleration.

8. The minimum speed with respect to air that a particular jet aircraft must have in order to keep aloft is 300 km/hr. Suppose that as its pilot prepares to take off, the wind blows eastward at a ground speed that can vary between 0 and 30 km/hr. Ignoring any other fact, a safe procedure to follow, consistent with using up as little fuel as possible, is to:
   (*a*) Take off eastward at a ground speed of 320 km/hr
   (*b*) Take off westward at a ground speed of 320 km/hr
   (*c*) Take off westward at a ground speed of 300 km/hr
   (*d*) Take off westward at a ground speed of 280 km/hr

9. A block is thrown with a velocity of 2 ms^{-1} (relative to ground) on a belt, which is moving with velocity 4 ms^{-1} in opposite direction of the initial velocity of block. If the block stops slipping on the belt after 4 sec of the throwing then choose the correct statements
   (*a*) Displacement with respect to ground is zero after 2.66s and magnitude of displacement with respect to ground is 12 m after 4 sec.
   (*b*) Magnitude of displacement with respect to ground in 4 sec is 4 m.
   (*c*) Magnitude of displacement with respect to belt in 4 sec is 12 m.
   (*d*) Displacement with respect to ground is zero in 8/3 sec

10. A particle has initial velocity 10 m/s. It moves under constant retarding force along the line of velocity which produces a retardation of 5 m/s². Then

(*a*) The maximum displacement in the direction of initial velocity is 10 m

(*b*) The distance travelled in first 3 seconds is 7.5 m.

(*c*) The distance travelled in first 3 seconds is 12.5 m.

(*d*) The distance travelled in first 3 seconds is 17.5 m.

11. The displacement x of a particle depends on time t as $x = \alpha t^2 - \beta t^3$

(*a*) Particle will return to its starting point after time α/β.

(*b*) The particle will come to rest after time $\frac{2\alpha}{3\beta}$.

(*c*) The initial velocity of the particle was zero but its initial acceleration was not zero.

(*d*) No net force acts on the particle at time $\frac{\alpha}{3\beta}$.

12. A body starts from the origin and moves along the x-axis such that position at any instant is given by $X = \frac{t^3}{3} - \frac{3t^2}{2} + 2t + 18$, where *t* is in second and position is in m. Choose the correct options"

(*a*) At $t = 2$ sec, the particle is at its local maximum position

(*b*) At $t = 1$ sec, the particle is at its local minimum position

(*c*) At $t = 2$ sec, the particle is at its local minimum position

(*d*) At $t = 1$ sec, the particle is at its local maximum position

13. The position of a particle moving in a string line is given by

$x = 3t^3 - 18t^2 + 36t$

Here, *x* is in m and t in second. Then

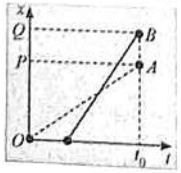
(*a*) Direction of velocity and acceleration both change at $t = 2$s

(*b*) The distance travelled by particle is equal to magnitude of displacement, for $t = 0$ to $t = 5$s

(*c*) The speed of particle is decreasing in $t = 0$ to $t = 2$s then it is increasing for $t > 2$

(*d*) The magnitudes of velocity and acceleration are equal at $t = 0$

14. The position-time $(x - t)$ graphs for two rabbits *A* and *B* moving from their carrot field *O* to their homes *P* and *Q* respectively along straight line path (taken as *x* axis) are shown in figure below. Choose the correct statement (s):



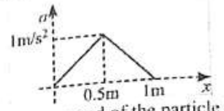
(*a*) *A* lives closer to the carrot field than *B*

(*b*) *A* starts from the carrot field earlier than *B*

(*c*) *A* and *B* have equal average velocities from 0 to t_0

(*d*) *B* overtakes *A* on the way

15. A body initially at rest, starts moving along x-axis in such a way so that its acceleration vs displacement plot is as shown in figure.



(*a*) The maximum speed of the particle is 1 m/s

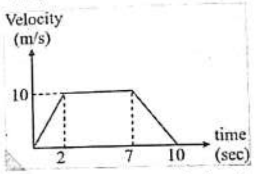
(*b*) The maximum speed of the particle is 6 m/s

(*c*) The maximum acceleration of the particle is 1 m/s²

(*d*) The rate of change of acceleration of particle at $x = 0.2$ m is 2m/s³

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 16 to 18): The velocity-time graph of a car moving on a straight track is given below. The car weighs 1000 kg. (Use $F = ma$)



16. The distance travelled by the car during the whole motion is

(*a*) 50 m (*b*) 75 m (*c*) 100 m (*d*) 150 m

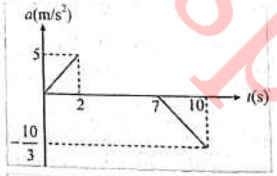
17. The braking force required to bring the car to a stop within one second from the maximum speed is

(*a*) $\frac{10000}{3}$ *N* (*b*) 5000 *N*

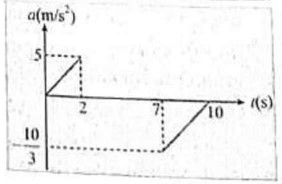
(*c*) 10000 *N* (*d*) $\frac{5000}{3}$ *N*

18. Correct acceleration-time graph representing the motion of car is

(*a*)



(*b*)



(*c*)

(*d*)

Comprehen

Usain Bolt 9.683 s). Th he was wel who finishe favourable celebrate b coach repo he could h analysis of at the Uni predicted a acceleratio Thompson ± 0.04 s ha

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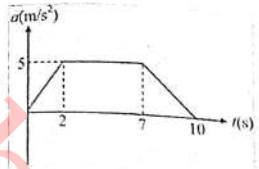
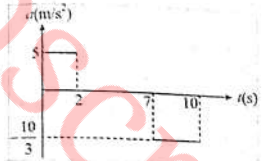
Motion

(c)

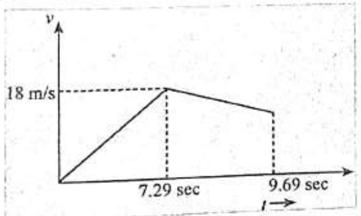


(d)



Comprehension (Q. 19 to 20): In the 2008 Olympic 100 m final, Usain Bolt broke new ground, winning in 9.69 s (unofficially 9.683 s). This was an improvement upon his own world record, and he was well ahead of second-place finisher Richard Thompson, who finished in 9.89 s. Not only was the record set without a favourable wind (+0.0 m/s), but he also visibly slowed down to celebrate before he finished and his shoelace was untied. Bolt's coach reported that, based upon the speed of Bolt's opening, he could have finished with a time of 9.52 s. After scientific analysis of Bolt's run by the Institute of Theoretical Astrophysics at the University of Oslo, Hans Eriksen and his colleagues also predicted a 9.60 s time. Considering factors such as Bolt's position, acceleration and velocity in comparison with second-place-finisher Thompson, the team estimated that Bolt could have finished in 9.55 ± 0.04 s had he not slowed to celebrate before the finishing line.

Let us also analyse the motion of Bolt. Assume that the velocity time graph of Usain Bolt is as shown below.



19. What was the initial acceleration of Bolt?

(*a*) 4.5 m/s^2 (*b*) 3.1 m/s^2

(*c*) 2.5 m/s^2 (*d*) 1.2 m/s^2

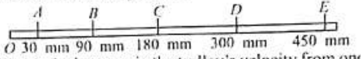
20. What was the final velocity of Bolt?

(*a*) 10.1 m/s (*b*) 10.6 m/s

(*c*) 13.4 m/s (*d*) 14.6 m/s

Comprehension (Q. 21 to 22): A time ticker apparatus oscillates at a fixed frequency. A paper ribbon is attached to it so that in every period it strikes & makes a mark on the ribbon. For example, if ribbon is pulled with constant velocity v the marks on the ribbon are at a distance of vT from each other. T is time period of time ticker.

The frequency of a time-ticker apparatus is 20 Hz. The ticker ribbon is attached to a trolley which has pulled the ribbon through the ticker apparatus. The markings on the ribbon are made at 10-tick intervals. The ribbon is shown below, and all the distances are measured from the starting point 0.



21. What is the increase in the trolley's velocity from one ticker mark to the next?

(*a*) 0.06 m.s^{-1} (*b*) 0.03 m.s^{-1}

(*c*) 0.6 m.s^{-1} (*d*) 0.3 m.s^{-1}

22. What is the acceleration of the trolley that pulled the ticker ribbon through the time ticker?

(*a*) 0.06 m.s^{-1} (*b*) 0.6 m.s^{-1}

(*c*) 0.12 m.s^{-1} (*d*) 1.2 m.s^{-1}

Comprehension (Q. 23 to 24): When a particle is subjected to a simple harmonic motion, its x-coordinate is given by x = A sin (ωt) where A and ω are constants. When it is subjected to two simultaneous SHM, its x-coordinate is given by x = A sin ωt + B cos ωt where B is a constant.

23. Which of the following quantities does not have dimensions of velocity?

(*a*) Aω (*b*) Bω

(*c*) $\frac{B^2\omega^2}{A}$ (*d*) Bωsin(ωt)

24. The maximum *x*-coordinate when particle is subjected to two simultaneous SHM is

(*a*) $A + B$ (*b*) $\sqrt{A^2 + B^2}$

(*c*) $A - B$ (*d*) A or B whichever is greater

Comprehension (Q. 25 to 26): Two long horizontal test tracks at Edward Air force base, running parallel and next to each other were used to compare the performance of rocket motor and a jet motor.

The rocket motor start from rest and accelerate constantly along the first track until it reaches exactly half the test distance $L/2$ at time t_1 after this point rocket run out of fuel, and then continued at constant speed to cover the remaining distance $L/2$.

A jet motor started at the same instant as the rocket at the same starting co-ordinate $s = 0$ along the second track and ran along the track with constant acceleration for the whole length L

It was observed that both the rocket and jet motor covered the test distance in exactly same time T.

25. The ratio of $\frac{t_1}{T}$ is

(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$

(*c*) $\frac{3}{2}$ (*d*) 3

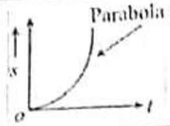
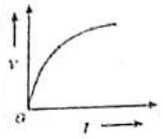
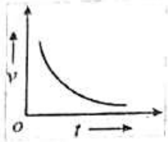
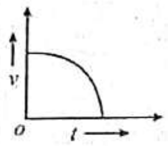
26. The ratio of acceleration of jet motor (a_2) to the rocket motor (a_1)

(*a*) $\frac{8}{9}$ (*b*) $\frac{9}{8}$

(*c*) $\frac{4}{9}$ (*d*) $\frac{9}{4}$

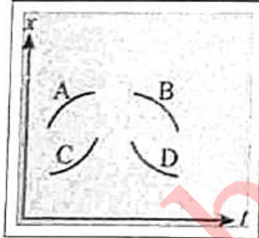
MATCH THE COLUMN TYPE QUESTIONS

27. Match Column-I with Column-II and select the correct answer using the codes given below the lists.

Column-I		Column-II	
A.	Acceleration decreasing with time	p.	
B.	Velocity increasing with time	q.	
C.	Magnitude of acceleration increasing with time	r.	
D.	Body going farther away from the starting point with time	s.	

(*a*) A-(q,s); B-(p,q); C-(s); D-(p,q,r,s)
(*b*) A-(r,s); B-(r,q); C-(p,q,r,s); D-(s)
(*c*) A-(r,q); B-(q); C-(s,r); D-(p,q,r,s)
(*d*) A-(r,s); B-(p,r); C-(p,q,r,s); D-(p,q)

28. Column-I shows the position-time graph of particles moving along a straight line and column-II lists the conclusion that follow from graphs. Match column-I with column-II and choose the correct option given below the columns.

Column-I		Column-II
	p.	Acceleration $a > 0$
	q.	Acceleration $a < 0$
	r.	Speeding up
	s.	Slowing down

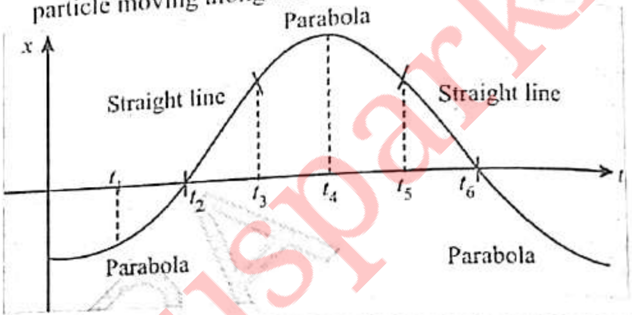
(*a*) A-(q,p); B-(r,p); C-(s,q); D-(r,s)
(*b*) A-(q,s); B-(q,r); C-(p,r); D-(p,s)
(*c*) A-(r,q); B-(s,r); C-(q,r); D-(p,r)
(*d*) A-(p,s); B-(r,s); C-(p,r); D-(p,q)

29. The position of a particle along x-axis is given by $x = (2t^3 - 21t^2 + 60t)m$. Then match the Column-I with Column-II.

	Column-I		Column-II
A.	Velocity of particle is zero	p.	2 sec
B.	Acceleration of particle is zero	q.	3 sec
C.	Acceleration of particle is negative	r.	3.5 sec
D.	Velocity of particle is towards the origin	s.	4 sec
		t.	5 sec

(*a*) A-(p,r,t); B-(r); C-(p,q); D-(q,r,s)
(*b*) A-(p,t,r); B-(r,t); C-(q); D-(s)
(*c*) A-(p,r); B-(s); C-(p,r,q); D-(r,s)
(*d*) A-(p,t); B-(r); C-(p,q); D-(q,r,s)

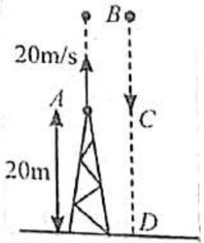
30. Figure shows a graph of position versus time graph for a particle moving along x-axis.



	Column-I		Column-II
A.	Slowing down	p.	$t_1 \to t_2$
B.	Returning towards origin	q.	$t_2 \to t_3$
C.	Moving away from origin	r.	$t_3 \to t_4$
D.	Speeding up	s.	$t_4 \to t_5$
		t.	$t_5 \to t_6$

(*a*) A-(r); B-(p,s,t); C-(q,r); D-(p,s)
(*b*) A-(r); B-(r,t); C-(q); D-(s)
(*c*) A-(p,r); B-(s); C-(p,r,q); D-(r,s)
(*d*) A-(r); B-(p,s); C-(p,q); D-(q,r,s)

31. A particle is projected vertically upwards with speed 20 m/s from top of a tower of height 20 m as shown in figure. Given B is top most point of trajectory and C is at same height as A :



	Column-I		Column-II
A.	ratio of maximum height from ground (BD) to the initial height from ground (AD) is	p.	$\frac{1}{\sqrt{2}}$
B.	Ratio of distance travelled in 1st second to the distance travelled in 2nd second is	q.	1

C.	Ratio of initial speed at A to the final just before reaching to ground (D) is	r.	2
D.	Ratio of time taken from A to C and time taken from A to B is	s.	3

(*a*) A-(r); B-(s); C-(q); D-(p)

(*b*) A-(r); B-(s); C-(p); D-(r)

(*c*) A-(p); B-(s); C-(p); D-(r)

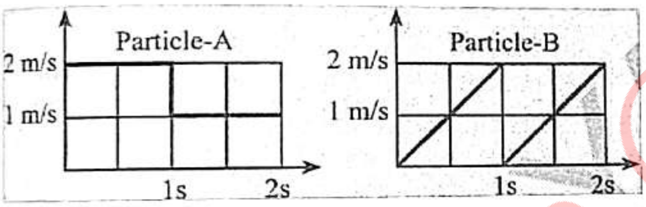
(*d*) A-(p); B-(r); C-(p); D-(q)

NUMERICAL TYPE QUESTIONS

Answer should be rounded off upto two decimal places

32. A particle is thrown upwards from ground. It experiences a constant resistance force which can produce retardation 2 m/s^2. The ratio of time of ascent to the time of descent is____. [$g = 10\ m/s^2$]

33. Two particles A and B start from the same point and move in the positive x-direction. Their velocity-time relationships are shown in the following figures. What is the maximum separation (in m) between them during the time interval shown?



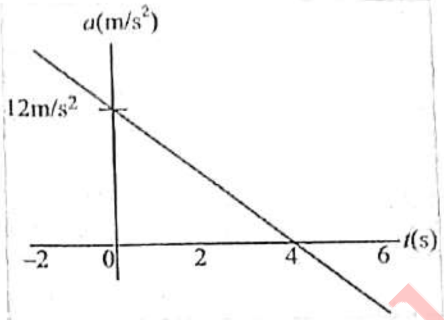
34. A train takes 2 minutes to acquire its full speed 60kmph from rest and 1 minute to come to rest from the full speed. If somewhere in between two stations 1 km of the track be under repair and the limited speed on this part be fixed to 20kmph, find the time (in s) of late running of the train on account of this repair work, assuming otherwise normal running of the train between the stations.

35. A man walking from town A to another town B at the rate of 4 km/hour starts one hour before a coach (also travelling from A to B). The coach is travelling at the rate of 12 km/hr and on the way he is picked up by the coach. On arriving at B, he finds that his coach journey lasted 2 hours. Find the distance (in km) between A and B.

36. A falling stone takes 0.2 seconds to fall past a window which is 1 m high. From how far above the top of the window was the stone dropped (in m)?

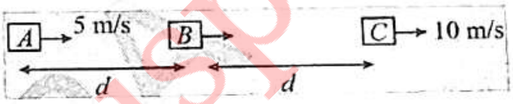
INTEGER TYPE QUESTIONS

37. Figure gives the acceleration a versus time t for a particle moving along an x-axis. At $t = -2.0$ s, the particle's velocity is 7.0 m/s. What is its velocity (in m /s) at $t = 6.0$ s?



38. A particle moves in xy-plane according to the equation $x = 3t$, $y = 25 - 4t$. What is the minimum distance of the particle (in m) from the origin? Both x and y are in m.

39. Three persons A, B, C are moving along a straight line, as shown, with constant but different speeds. When B catches C, the separation between A and C becomes $4d$, then the speed of B is u. Find $2u$.

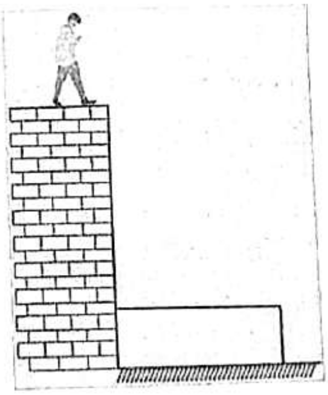


40. Two cars are moving in same direction with speed of 30 kmh^{-1}. They are separated by a distance of 5 km. What is the speed of a car (in kmh^{-1}) moving in opposite direction if it meets the two cars at an interval of 4 min?

41. The acceleration of a marble in a certain fluid is proportional to the speed of the marble squared and is given (in SI units) by $a = -\frac{v^2}{3}$ for $v > 0$. If the marble enters this fluid with a speed of 1.50 m/s, how long (in sec.) will it take before the marble's speed is reduced to half of its initial value?

SINGLE CORRECT TYPE QUESTIONS

42. Suppose that a man jumps off a building 202 m high onto cushions having a total thickness of 2 m. If the cushions are crushed to a thickness of 0.5 m, what is the man's acceleration (assumed constant) as he slows down?



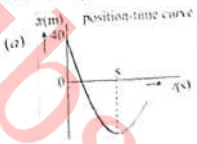
(*a*) 10 m/s^2

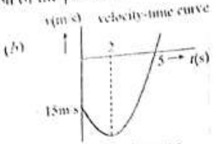
(*b*) $\frac{4000}{3}$ m/s^2

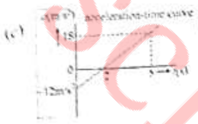
(*c*) 50 m/s^2

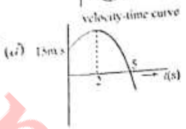
(*d*) 20 m/s^2

43. The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$, where x is expressed in meters and t in seconds. Which of the graph does not represent the motion of the particle?

(a)


(b)


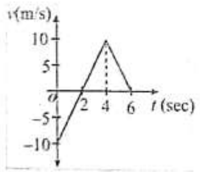
(c)


(d)


44. A trolley is moving away from a stop with an acceleration $a = 0.2\ \text{m s}^{-2}$. After reaching the velocity $u = 36$ km/h, it moves with a constant velocity for the time of 2 min. Then, it uniformly slows down, and stops after further travelling a distance of 100 m. Find the average speed all the way between stops.

(a) $\frac{76}{17}$ m/s
(b) $\frac{208}{21}$ m/s
(c) $\frac{85}{12}$ m/s
(d) $\frac{155}{19}$ m/s

45. The figure shows the graph of velocity-time for a particle moving in a straight line. If the average speed for 6 sec is 'b' and the average acceleration from 0 sec to 4 sec is 'c' find magnitude of bc (in m²/s³).



(a) 5
(b) 20
(c) 25
(d) 40

46. James bond is standing on a bridge above the road below and his pursuers are getting too close for comfort. He spots a flat bed truck loaded with mattresses approaching at 30 m/s which he measures by knowing that the telephones poles the truck is passing are 20m apart in this country. The bed of truck is 20m below the bridge and bond quickly calculates how many poles away the truck should be when he jumps down the bridge onto the truck making his get away. How many poles is it ?

(a) 3
(b) 4
(c) 5
(d) 6

47. A police party is chasing a dacoit in a jeep which is moving at a constant speed v while the dacoit is on a motorcycle. When he is at a distance x from the jeep, he accelerates from rest at a constant rate α. Which of the following relations is true if the police is able to catch the dacoit?

(a) $v^2 \le \alpha x$
(b) $v^2 \le 2\alpha x$
(c) $v^2 \ge 2\alpha x$
(d) $v^2 \ge \alpha x$

48. Acceleration of particle moving rectilinearly is $a = 4 - 2x$ (where x is position in metre and a in m/s²). It is at instantaneous rest at $x = 0$. At what position x will the particle again come to instantaneous rest?

(a) 8 m
(b) 4 m
(c) 2 m
(d) 1 m

49. A car leaves regularly from point A for point B every 10 minutes. The distance between A and B is 60 km. The cars travel at a speed of 60 km/hr.

Find the number of cars that a man driving from B to A will meet en route if he starts from B simultaneously with one of the cars leaving A. The car from B travels at a speed of 60 km/hr.

(a) 10
(b) 11
(c) 20
(d) 21

50. A car is moving along a straight line. It is taken from rest to a velocity of 20 ms^{-1} by a constant acceleration of 5ms^{-2}. It maintains a constant velocity of 20 ms^{-1} for 5 seconds and then is brought to rest again by a constant acceleration of −2 ms^{-2}. Find the distance covered by the car.

(a) 120 m
(b) 200 m
(c) 240 m
(d) 400 m

51. A rock is shot vertically upward from the edge of the top of a tall building. The rock reaches its maximum height above the top of the building 1.75s after being shot. Then, after barely missing the edge of the building as it falls downward, the rock strikes the ground 6.0s after it is launched. How tall is the building?

(a) 30 m
(b) 75 m
(c) 105 m
(d) 150 m

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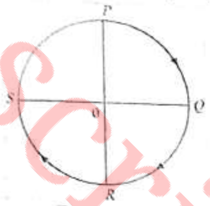
PYQ's (Past Year Questions)

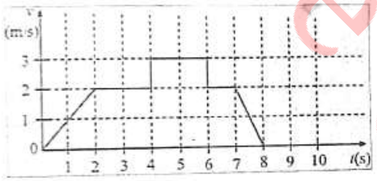
POSITION, DISTANCE AND DISPLACEMENT

1. A cyclist starts from the point P of a circular ground of radius 2 km and travels along its circumference to the point S. The displacement of cyclist is: [04 April, 2024 (Shift-II)]



(a) 6 km (b) $\sqrt{8}$ km (c) 4 km (d) 8 km

2. A particle starts from the origin at time $t = 0$ and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5s$? [10 Jan, 2019 (Shift-II)]



(a) 10 m (b) 6 m (c) 3 m (d) 9 m

SPEED AND VELOCITY

3. A particle is moving in a straight line. The variation of position 'x' as a function of time 't' is given as $x = (t^3 - 6t^2 + 20t + 15)$ m. The velocity of the body when its acceleration becomes zero is:
(a) 4 m/s (b) 8 m/s (c) 10 m/s (d) 6 m/s
[29 Jan, 2024 (Shift-II)]

4. A particle is moving in one dimension (along x axis) under the action of a variable force. It's initial position was 16 m right of origin. The variation of its position (x) with time (t) is given as $x = -3t^3 + 18t^2 + 16t$, where x is in m and t is in s. The velocity of the particle when its acceleration becomes zero is ________ m/s.
[1 Feb, 2024 (Shift-I)]

5. A train starting from rest first accelerates uniformly up to a speed of 80 km/h for time t, then it moves with a constant speed for time $3t$. The average speed of the train for this duration of journey will be (in km/h). [6 April, 2024 (Shift-I)]
(a) 80 (b) 70 (c) 30 (d) 40

6. A particle moves in a straight line so that its displacement x at any time t is given by $x^2 = 1 + t^2$. Its acceleration at any time t is x^{-n} where n = ______. [6 April, 2024 (Shift-II)]

7. A particle moving in a straight line covers half the distance with speed 6 m/s. The other half is covered in two equal time intervals with speeds 9 m/s and 15 m/s respectively. The average speed of the particle during the motion is: [09 April, 2024 (Shift-I)]
(a) 8.8 m/s (b) 10 m/s (c) 9.2 m/s (d) 8 m/s

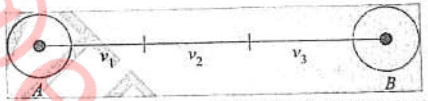
8. A horse rider covers half the distance with 5 *m/s* speed. The remaining part of the distance was travelled with speed 10 *m/s* for half the time and with speed 15 *m/s* for other half of the time. The mean speed of the rider averaged over the whole time of motion is $x/7$ *m/s*. The value of x is ______.
[30 Jan, 2023 (Shift-I)]

9. A car travels a distance of 'x' with speed v_1 and then same distance 'x' with speed v_2 in the same direction. The average speed of the car is: [25 Jan, 2023 (Shift-I)]

(a) $\dfrac{v_1 v_2}{2(v_1 + v_2)}$ (b) $\dfrac{v_1 + v_2}{2}$

(c) $\dfrac{2x}{v_1 + v_2}$ (d) $\dfrac{2v_1 v_2}{v_1 + v_2}$

10. A car covers AB distance with first one-third at velocity v_1 ms^{-1}, second one-third at v_2 ms^{-1} and last one-third at v_3 ms^{-1}. If $v_3 = 3v_1$, $v_2 = 2v_1$ and $v_1 = 11$ ms^{-1} then the average velocity of the car is ________ms^{-1}.
[28 June, 2022 (Shift-II)]



11. A car is moving with speed of 150 km/h and after applying the break it will move 27m before it stops. If the same car is moving with a speed of one third the reported speed then it will stop after travelling ________ m distance.
[25 July, 2022 (Shift-I)]

12. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$; then its displacement after time ($t = 1$) is:
[17 March, 2021 (Shift-II)]

(a) $v_0 + g + f$ (b) $v_0 + \frac{g}{2} + \frac{F}{3}$

(c) $v_0 + 2g + 3F$ (d) $v_0 + \frac{g}{2} + F$

13. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be: [9 April, 2019 (Shift-II)]

(a) $a + \dfrac{b^2}{4c}$ (b) $a + \dfrac{b^2}{c}$

(c) $a + \dfrac{b^2}{2c}$ (d) $a + \dfrac{b^2}{3c}$

CONSTANT ACCELERATION

14. A bullet is fired into a fixed target looses one third of its velocity after travelling 4 cm. It penetrates further $D \times 10^{-3}$ m before coming to rest. The value of D is: **[27 Jan, 2024 (Shift-II)]**

(a) 2 (b) 5 (c) 3 (d) 4

15. A body falling under gravity covers two points A and B separated by 80 m in 2s. The distance of upper point A from the starting point is ______ m (use $g = 10\ ms^{-2}$) **[27 Jan, 2024 (Shift-II)]**

16. A particle initially at rest starts moving from reference point $x = 0$ along x-axis, with velocity v that varies as $v = 4\sqrt{x}$ m/s. The acceleration of the particle is ______ ms^{-2}.

[1 Feb, 2024 (Shift-II)]

17. A body travels 102.5 m in nth second and 115.0 m in $(n + 2)$th second. The acceleration is: **[04 April, 2024 (Shift-I)]**

(a) 9 m/s^2 (b) 6.25 m/s^2 (c) 12.5 m/s^2 (d) 5 m/s^2

18. A bus moving along a straight highway with speed of 72 km/h is brought to halt within 4s after applying the brakes. The distance travelled by the bus during this time (Assume the retardation is uniform) is ______ m. **[04 April, 2024 (Shift-II)]**

19. A body moves on a frictionless plane starting from rest. If S_n is distance moved between $t = n - 1$ and $t = n$ and S_{n-1} is distance moved between $t = n - 2$ and $t = n - 1$, then the ratio $\frac{S_{n-1}}{S_n}$ is $\left(1 - \frac{2}{x}\right)$ for $n = 10$. The value of x is ....

[05 April, 2024 (Shift-I)]

20. A body projected vertically upwards with a certain speed from the top of a tower reaches the ground in t_1. If it is projected vertically downwards from the same point with the same speed, it reaches the ground in t_2. Time required to reach the ground, if it is dropped from the top of the tower, is: **[06 April, 2024 (Shift-II)]**

(a) $\sqrt{t_1 t_2}$ (b) $\sqrt{t_1 - t_2}$ (c) $\sqrt{\frac{t_1}{t_2}}$ (d) $\sqrt{t_1 + t_2}$

21. Two cars are travelling towards each other at speed of 20 ms^{-1} each. When the cars are 300 m apart, both the drivers apply brakes and the cars retard at the rate of 2 ms^{-2}. The distance between them when they come to rest is: **[09 April, 2024 (Shift-II)]**

(a) 200 m (b) 50 m (c) 100 m (d) 25 m

22. For a train engine moving with speed of 20 ms^{-1}, the driver must apply brakes at a distance of 500m before the station for the train to come to rest at the station. If the brakes were applied at half of this distance, the train engine would cross the station with speed $\sqrt{x}\,ms^{-1}$. The value of x is ______.

(Assuming same retardation is produced by brakes)

[1 Feb, 2023 (Shift-II)]

23. A particle starts with an initial velocity of 10.0ms^{-1} along x-direction and accelerates uniformly at the rate of 2.0 ms^{-2}. The time taken by the particle to reach the velocity of 60.0 ms^{-1} is **[6 April, 2023 (Shift-II)]**

(a) 6s (b) 3s

(c) 30s (d) 25s

24. A particle [illegible] moving [illegible] [illegible] [25 July, 2022 (Shift-[illegible] particle [illegible] 20 m [illegible]

25. An engine of a train, moving with uniform acceleration, passes the signal post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is: [25 Feb, 2021 (Shift-[illegible]

(a) $\frac{u+v}{2}$ (b) $\sqrt{\frac{v^2+u^2}{2}}$

(c) $\frac{v-u}{2}$ (d) $\sqrt{\frac{v^2-u^2}{2}}$

26. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'V' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to [9 Jan, 2019 (Shift-I)]

(a) $\frac{2a_1a_2}{a_1+a_2}t$ (b) $\sqrt{2a_1a_2}\,t$

(c) $\sqrt{a_1a_2}\,t$ (d) $\frac{a_1-a_2}{2}t$

MOTION UNDER GRAVITY

27. A ball is thrown vertically upward with an initial velocity of 150 m/s. The ratio of velocity after 3s and 5s is $\frac{x+1}{x}$. The value of x is ______. [12 April, 2023 (Shift-I

Take ($g = 10\ m/s^2$).

(a) 6 (b) 5 (c) −5 (d) 10

28. A tennis ball is dropped on to the floor from a height of 9.8m. It rebounds to a height 5.0 m. Ball comes in contact with the floor for 0.2 s. The average acceleration during contact is ______ ms^{-2} [Given $g = 10\ ms^{-2}$]

[29 Jan, 2023 (Shift-I

29. Two balls A and B placed at the top of 180m tall tower. Ball A is dropped from the top at $t = 0$s. Ball B is thrown vertically down with an initial velocity 'u' at $t = 2$s. After a certain time, both balls meet 100m above the ground. Find the value of 'u' in ms^{-1}. **[29 June, 2022 (Shift-I**

[use $g = 10\ ms^{-2}$]:

(a) 10 (b) 15 (c) 20 (d) 30

30. A ball is thrown vertically upwards with a velocity of 19.6 ms^{-1} from the top of a tower. The ball strikes the ground after 6 s. The height from the ground up to which the ball can rise will be $\left(\frac{k}{5}\right)m$. The value of k is ______

(use $g = 9.8\ m/s^2$)

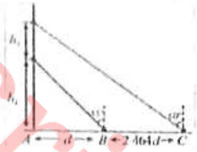
[28 July, 2022 (Shift-II)]

31. Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of ______ m.

(Take $g = 10\ m/s^2$)

[26 Aug, 2021 (Shift-I)]

32. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30° = 0.5774) **[5 Sep, 2020 (Shift-I)]**



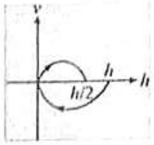
(a) d (b) 0.732 d
(c) 1.464 d (d) 0.464 d

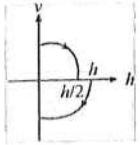
33. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity] **[5 Sep, 2020 (Shift-I)]**

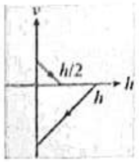
(a) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$ (b) $t = 1.8\sqrt{\frac{h}{g}}$

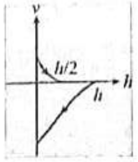
(c) $t = \sqrt{\frac{2h}{3g}}$ (d) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$

34. A tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $h/2$. The velocity versus height of the ball during its motion may be represented graphically by (graph are drawn schematically and on not to scale). **[4 Sep, 2020 (Shift-I)]**

(a)


(b)


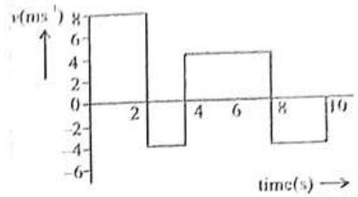
(c)


(d)


35. A ball is dropped from the top of a 100 m high tower on a planet. In the last 1/2 s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is **[8 Jan, 2020 (Shift-II)]**

GRAPHS

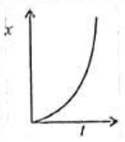
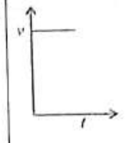
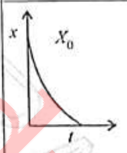
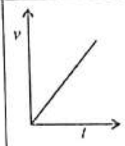
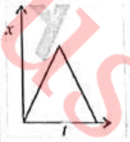
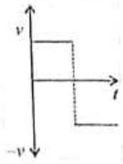
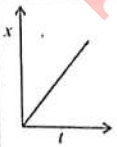
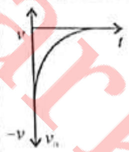
36. The velocity time graph of a body moving in a straight line is shown in figure



The ratio of displacement to distance travelled by the body in time 0 to 10 s is **[24 Jan, 2023 (Shift-II)]**

(a) 1 : 1 (b) 1 : 4
(c) 1 : 2 (d) 1 : 3

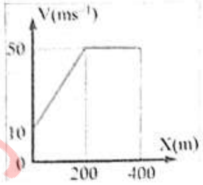
37. Match Column-I with Column-II:

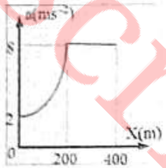
Column-I (x-t graphs)		Column-II (v-t graphs)	
A.		I.	
B.		II.	
C.		III.	
D.		IV.	

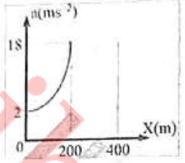
Choose the correct answer from the options given below: **[30 Jan, 2023 (Shift-I)]**

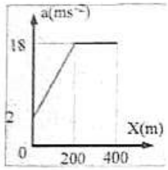
(a) A- II, B-IV, C-III, D-I (b) A- I, B-II, C-III, D-IV
(c) A- II, B-III, C-IV, D-I (d) A- I, B-III, C-IV, D-II

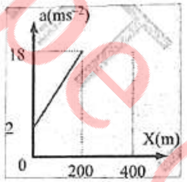
38. The velocity- displacement graph describing the motion of a bicycle is shown in the figure. The acceleration-displacement graph of the bicycle's motion is best described by.
[16 March, 2021 (Shift-I)]



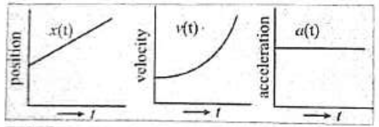
(a)


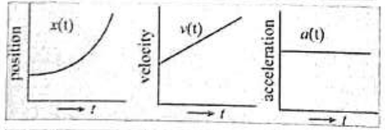
(b)


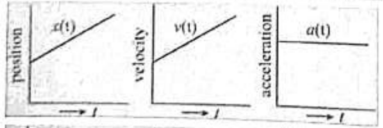
(c)


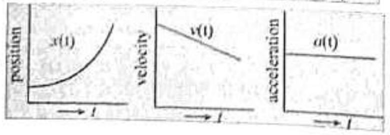
(d)


39. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by
[18 March, 2021 (Shift-I)]

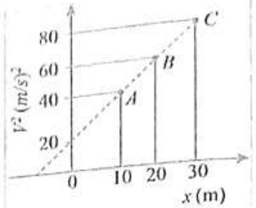
(a)


(b)


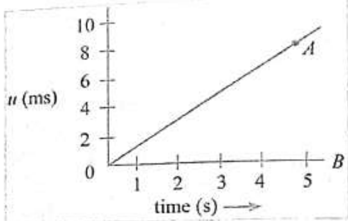
(c)


(d)


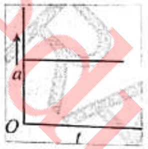
40. A particle is moving with constant acceleration 'a' Following graph shows v^2 versus x (displacement) plot. The acceleration of the particle is ______ m/s^2.
[31 Aug, 2021 (Shift-II)]

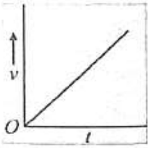


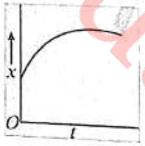
41. The speed versus time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval $t = 0$ to $t = 5$ s will be ______
[3 Sep, 2020 (Shift-II)]

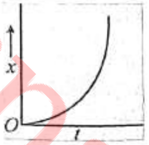


42. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify the figure that is not correctly representing the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)
[8 April, 2019 (Shift-II)]

(A)


(B)


(C)


(D)


(a) (A), (B), (C)
(b) (A)
(c) (C)
(d) (B), (C)

VARIABLE ACCELERATION

43. The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is: (Where v stands for velocity)
[25 July, 2021 (Shift-II)]

(a) $2n^2v^2$
(b) $2mnv^3$
(c) $2mv^3$
(d) $2nv^3$

44. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is ______. [9 Jan, 2020 (Shift-I)]

ONE DIMENSIONAL RELATIVE MOTION

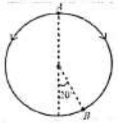
45. Train A is moving along two parallel rail tracks towards north with speed 72 km/h and train B is moving towards south with speed 108 km/h. Velocity of train B with respect to A and velocity of ground with respect to B are (in ms^{-1}): [1 Feb, 2024 (Shift-II)]
(a) −30 and 50 (b) −50 and −30
(c) −50 and 30 (d) 50 and −30

PW Challengers

SINGLE CORRECT TYPE QUESTIONS

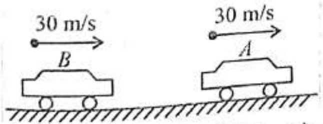
1. A man left his dog sitting on a large slippery ground and walks away with a velocity of 2m/s. When he is 207 m away from the dog, the dog decides to catch him and thereafter they move together. The dog can not develop acceleration more that $2m/s^2$ in any direction due to slippery ground. If the maximum velocity with which dog can move is 18 m/s then the minimum time in which dog will meet the man is
(a) 20 sec (b) 21 sec (c) 22 sec (d) 24 sec

2. Two motorcyclists simultaneously start a race with constant speeds from point A to traverse on a circular track, one clockwise and other in anticlockwise sense. They simultaneously cross point B first time after a time interval of 10 minutes. If they continue to move, how long after they cross at B first time will they again cross at point B.



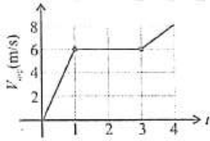
(a) 10 min (b) 95 min (c) 90 min (d) 85 min

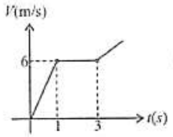
3. Two cars A and B are running on a highway with same velocity of 30 m/s. On application of brakes car A retards at a rate of $3\ m/s^2$ while car B retards at a rate of $4\ m/s^2$. Car A is running ahead of car B. In an emergency when driver of front car A applies brakes, in response the driver of near car B has to apply brakes to avoid accident. The response time of driver of car B is 1 sec. The minimum distance between them to avoid accident is

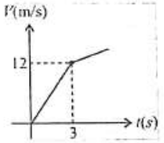


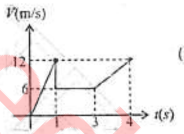
(a) 7.5 m (b) 6 m (c) 8.5 m (d) 10 m

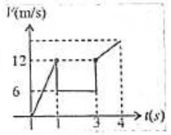
4. The relation between average velocity v_{av} of a particle and time t is shown in the graph. If during the time interval considered, the particle did not change its direction of motion, then plot instantaneous velocity as a function of time?



(a)


(b)


(c)


(d)


5. When a deer was 48 m from a leopard, the leopard starts chasing the deer and the deer immediately starts running away from the leopard with constant velocity. A leopard cannot run at high speeds for a long time and has to slow down due to fatigue. If we assume that the leopard starts with an initial speed of 30 m/s and reduces its speed in equal steps of 5 m/s after every 2 s interval, at what minimum speed must the deer run to escape from the leopard?
(a) $15\ ms^{-1}$ (b) $16\ ms^{-1}$ (c) $17\ ms^{-1}$ (d) $18\ ms^{-1}$

6. At the initial instant, two particles are observed at different locations moving towards each other with velocities u_1 and u_2. If they are subjected to constant accelerations a_1, and a_2 in directions opposite to their initial velocities, they will meet twice. If time interval between these two meetings is Δt, find suitable expression for their initial separation.

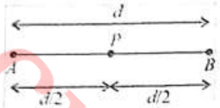
(a) $\dfrac{(u_1+u_2)^2}{a_1+a_2}+\dfrac{a_1+a_2}{8}(\Delta t)^2$

(b) $\dfrac{(u_1+u_2)^2}{2(a_1+a_2)}+\dfrac{a_1+a_2}{8}(\Delta t)^2$

(c) $\frac{(u_1+u_2)^2}{2(a_1+a_2)} - \frac{a_1+a_2}{8}(\Delta t)^2$

(d) $\frac{u_1^2+u_2^2}{2(a_1+a_2)} - \frac{a_1+a_2}{8}(\Delta t)^2$

7. A particle covers distance of d with uniform acceleration between point A & B. If its average velocity is v_{avg}, what could be range of magnitude of its instantaneous velocity v' at $\frac{d}{2}$ from point A.



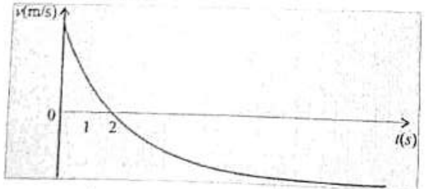
(a) $v_{avg} \le v' \le \sqrt{3}\, v_{avg}$ (b) $\frac{v_{avg}}{\sqrt{3}} \le v' \le \sqrt{3} v_{avg}$

(c) $v_{avg} \le v' \le \sqrt{2}\, v_{avg}$ (d) $\frac{v_{avg}}{\sqrt{2}} \le v' \le \sqrt{2}\, v_{avg}$

8. Mr. Gupta used to walk to his office everyday and it took him 30 minutes. Once on his way he realized that he has forgotten to take his mobile phone. He knew that if he continued walking he will reach office 10 minutes before his office time. He went back home for the mobile phone, increased his speed by 20% and arrived the office 5 minutes late. What fraction of the way to office had he covered till the moment he decided to turn back?

(a) $\frac{7}{11}$ (b) $\frac{6}{11}$ (c) $\frac{5}{11}$ (d) $\frac{4}{11}$

9. To study effect of air resistance, a rubber ball was shot vertically upwards from a spring gun from 20th floor of a tall building. Velocity of the ball was recorded at regular intervals of time and the data obtained were plotted on a graph paper. Some of the marking on the axes are erased as shown in the following figure. With what speed did the ball strike the ground?

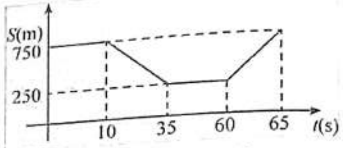


(a) 5 ms^{-1} (b) 10 ms^{-1} (c) 50 ms^{-1} (d) 25 ms^{-1}

10. Two motorboats that can move with velocities 4.0 m/s and 6.0 m/s relative to water are going up-stream in a river. When the faster boat overtakes the slower boat, a buoy is dropped from the slower boat. After lapse of a time interval, both the boats turn back simultaneously and move at the same speeds relative to the water as before. Their engines are switched off when they reach the buoy again. If the maximum separation between the boats is 200 m after the buoy is dropped and water flow velocity in the river is 1.5 m/s, find distance between the places where the faster boat passes by the buoy.

(a) 75 m (b) 300 m (c) 150 m (d) 350 m

11. Two buses travel from one town to the other with constant speed v_1 everywhere on the road except on a narrow bridge where they travel with constant speed v_2. The way separation S between the buses varies with time is shown in the graph below. Find the length of the bridge.



(a) 250 m (b) 300 m

(c) 500 m (d) 600 m

12. At $t = 0$, a balloon start rising from the ground with a constant acceleration of 5 m/s^2. Two stones are dropped from the balloon at $t_1 = 10$ s and $t_2 = 11$ s. The distance between the stones at $t = 13$ s is ($g = 10$ m/s^2)

(a) 18.5 m (b) 27.5 m

(c) 37.5 m (d) 47.5 m

13. A boy is standing on the platform at the beginning of the first coach of a train. The train starts moving with constant acceleration. The first coach passes by the boy in $t_1 = 10$ s and whole train including the first coach in $t_2 = 40$ s. The time interval in which last coach did pass by the boy is

(a) 1.93 s (b) 2.08 s

(c) 1.73 s (d) 1.27 s

14. On a straight section of a highway, sensors are installed to measure traffic density. For each lane of the highway, a sensor runs on an overhead wire. On a particular day, a sensor running with a speed $u = 10$ km/h opposite to the flow of traffic underneath, counts $N = 320$ vehicles in a length $L = 1$ km of the highway. If all the vehicles are moving with the same constant speed 30 km/h and density of the vehicles is uniform, calculate number of vehicles per $l = 100$ m of the lane.

(a) 6 (b) 7

(c) 8 (d) 9

15. An elastic straight strip with initial length L has one end tied to a wall. At $t = 0$, the other end is pulled away from the wall at speed v_0 (assume that the strip stretches uniformly). At the same time, an ant located at the end not attached to the wall begins to crawl toward the wall, with relative velocity $u = \frac{k}{L + v_0 t}$, relative to the point of strip where it is present. Find the time (in sec.) ant will take to reach the wall.

Use : $L = 2$ m, $k = 3$ m^{2}s^{-1} and $v_0 = 0.5$ m/s

ANSWER KEY

CONCEPT APPLICATION

1. (24 km/hr, 24 km/hr) 2. 10 km/h, $\frac{10}{3}\left(\sqrt{2}+1\right)$ km/hr 3. (*a*) 2 m/sec^2, (*b*) 75 m 4. (*a*) 1, (*b*) 125 m
5. (*b*) 6. (*a*) 7. (*c*) 8. (*d*) 9. (*a*) 10. (*b*) 11. (*c*) 12. (*a*) 13. (*b*)

SCHOOL LEVEL PROBLEMS

1. (*a*) 2. (*d*) 3. (*c*) 4. (*a*) 5. (*b*) 6. (*b*) 7. (*d*) 8. (*a*) 9. (*c*) 10. (*d*)
11. (*b*) 12. (*a*) 24. (i)-(*b*), (ii)-(*a*), (iii)-(*b*) 25. (i)-(*a*), (ii)-(*b*), (iii)-(*a*)

PRARAMBH (TOPICWISE)

1. (*a*) 2. (*c*) 3. (*c*) 4. (*b*) 5. (*d*) 6. (*c*) 7. (*b*) 8. (*d*) 9. (*c*) 10. (*b*)
11. (*a*) 12. (*b*) 13. (*b*) 14. (*b*) 15. (*b*) 16. (*d*) 17. (*a*) 18. (*c*) 19. (*c*) 20. (*c*)
21. (*c*) 22. (*b*) 23. (*a*) 24. (*d*) 25. (*c*) 26. (*a*) 27. (*b*) 28. (*c*) 29. (*d*) 30. (*b*)
31. (*d*) 32. (*c*) 33. (*d*) 34. (*a*) 35. (*a*) 36. (*c*) 37. (*d*) 38. (*c*) 39. (*a*) 40. (*b*)
41. (*b*) 42. (*d*) 43. (*d*) 44. (*c*) 45. (*d*) 46. (*d*)

PRABAL (JEE MAIN LEVEL)

1. (*b*) 2. (*b*) 3. (*c*) 4. (*c*) 5. (*d*) 6. (*c*) 7. (*d*) 8. (*a*) 9. (*b*) 10. (*b*)
11. (*a*) 12. (*d*) 13. (*c*) 14. (*a*) 15. (*a*) 16. (*c*) 17. (*c*) 18. (*a*) 19. (*d*) 20. (*b*)
21. (*d*) 22. (*b*) 23. [1125] 24. [225] 25. [72] 26. [5] 27. [24] 28. [80] 29. [7] 30. [750]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*b,c,d*) 2. (*c,d*) 3. (*a,b,c,d*) 4. (*a,c,d*) 5. (*a,b,c,d*) 6. (*a,d*) 7. (*a,b,c*) 8. (*c*) 9. (*b,c,d*) 10. (*a,c*)
11. (*a,b,c,d*) 12. (*c,d*) 13. (*b,c,d*) 14. (*a,b,d*) 15. (*a,c*) 16. (*b*) 17. (*c*) 18. (*d*) 19. (*c*) 20. (*b*)
21. (*a*) 22. (*c*) 23. (*c*) 24. (*b*) 25. (*b*) 26. (*a*) 27. (*a*) 28. (*b*) 29. (*d*) 30. (*a*)
31. (*b*) 32. [0.82] 33. [1.25] 34. [160.00] 35. [30.00] 36. [0.80] 37. [55] 38. [15] 39. [25] 40. [45]
41. [2] 42. (*b*) 43. (*d*) 44. (*d*) 45. (*c*) 46. (*a*) 47. (*c*) 48. (*b*) 49. (*b*) 50. (*c*)
51. (*b*)

PYQ's (PAST YEAR QUESTIONS)

1. (*b*) 2. (*d*) 3. (*b*) 4. [52] 5. (*b*) 6. [3] 7. (*d*) 8. [50] 9. (*d*) 10. [18]
11. [3] 12. (*b*) 13. (*d*) 14. (Bonus) 15. [45] 16. [8] 17. (*b*) 18. [40] 19. [19] 20. (*a*)
21. (*c*) 22. [200] 23. (*d*) 24. [100] 25. (*b*) 26. (*c*) 27. (*b*) 28. [120] 29. (*d*) 30. [392]
31. [50] 32. (*a*) 33. (*a*) 34. (*b*) 35. [8] 36. (*d*) 37. (*a*) 38. (*d*) 39. (*b*) 40. [1]
41. [20] 42. (*c*) 43. (*c*) 44. [3] 45. (*c*)

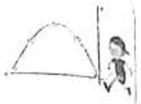
PW CHALLENGERS

1. (*c*) 2. (*c*) 3. (*b*) 4. (*d*) 5. (*c*) 6. (*c*) 7. (*c*) 8. (*d*) 9. (*d*) 10. (*b*)
11. (*c*) 12. (*c*) 13. (*d*) 14. (*c*) 15. [2]

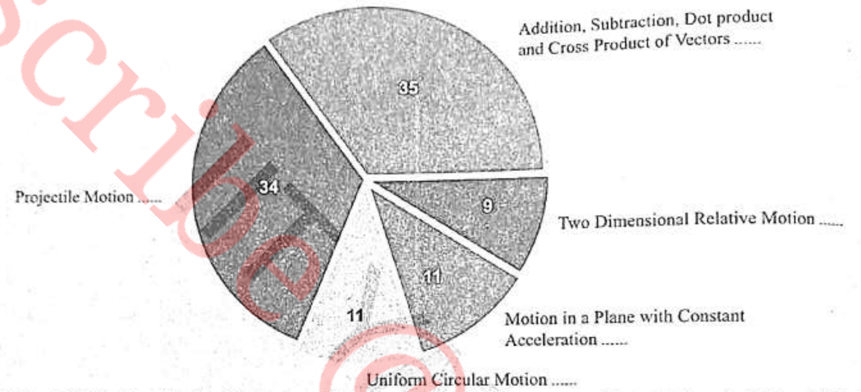
CHAPTER

4

Motion in a Plane



Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

SCALAR AND VECTOR

(*i*) **Scalar:** A scalar quantity requires only a number for its complete description. Mass, volume, density, pressure and temperature are all examples of scalar quantities. The mathematics of scalar quantities is the ordinary algebra of numbers.

(*ii*) **Vector:** A vector quantity requires both magnitude and direction for its complete description. Velocity, acceleration, force and momentum are examples of vector quantities.

A vector quantity is usually represented by an arrow. The arrow points in the direction of vector quantity and length of the arrow is proportional to magnitude of the vector quantity. For example if a displacement of 10 m due east is represented by an arrow of length 1 cm pointing east then displacement of 50 m due north will be represented by an arrow of length 5 cm pointing due north.

A vector quantity is usually denoted by an arrow over the corresponding letter such as $\vec{A}$ or $\vec{a}$ and magnitude of vector by $|\vec{A}|$ or $|\vec{a}|$.

Types of Vector

(*i*) **Equal vectors:** Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitudes and same direction.

(*ii*) **Parallel vector:** Two vectors $\vec{A}$ and $\vec{B}$ are said to be parallel when

(*a*) Both have same direction.

(*b*) One vector is scalar (positive) non-zero multiple of another vector.

(*iii*) **Anti-parallel vectors:** Two vectors $\vec{A}$ and $\vec{B}$ are said to be anti-parallel when

(*a*) Both have opposite direction.

(*b*) One vector is scalar non-zero negative multiple of another vector.

(*iv*) **Zero vector** $(\vec{0})$ **:** A vector having zero magnitude and arbitrary direction (not known to us) is a zero vector.

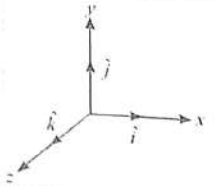
(*v*) **Unit vector:** A vector divided by its magnitude is a unit vector. Unit vector for $\vec{A}$ is $\hat{A}$ (read as A cap/A hat).

Since, $\hat{A} = \dfrac{\vec{A}}{A} \Rightarrow \vec{A} = A\hat{A}$.

Thus, we can say that unit vector gives us the direction.

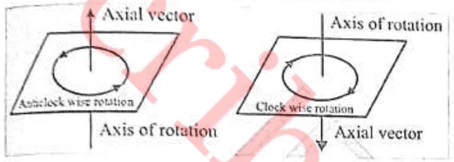
(*vi*) **Orthogonal unit vectors:** If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z axes respectively then $\hat{i}$, $\hat{j}$ and $\hat{k}$ are called orthogonal unit vectors. These vectors must form a

Right Handed Triad (It is a coordinate system such that when we curl the fingers of right hand from x to y then we must get the direction of z along thumb).



y
$\hat{j}$
x
$\hat{i}$
$\hat{k}$
z

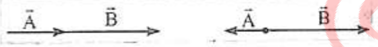
(*vii*) **Axial Vectors:** These represent rotational effects and are always along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque and angular momentum, etc., are examples of axial vectors.



Axial vector
Anticlock wise rotation
Axis of rotation
Axis of rotation
Clock wise rotation
Axial vector

(*viii*) **Coplanar vectors:** Three (or more) vectors are called coplanar vectors if they lie in the same plane. Two (free) vectors are always coplanar.

(*ix*) **Collinear Vectors:** Those vectors which are expressed along same straight line and are parallel or anti-parallel are called collinear vectors.



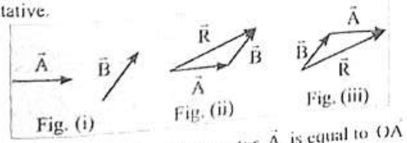
$\vec{A}$
$\vec{B}$
$\vec{A}$
$\vec{B}$

TRIANGLE LAW OF VECTOR ADDITION OF TWO VECTORS

The triangle law uses principles of plane geometry. This law states:

The vectors to be added are drawn in such a manner that the tail of a vector coincides the tip of the preceding vector (in tip to tail fashion); their resultant is defined by the vector drawn from the tail of the first vector to the tip of the second vector. The two vectors to be added and their resultant are coplanar.

Consider vectors $\vec{A}$ and $\vec{B}$ shown in the figure-(i). Using the triangle law, we obtain geometrical construction shown in the figure-(ii), where it is shown that two vectors and their sum $\vec{R} = \vec{A} + \vec{B}$ always make a closed triangle. If we change order of vector $\vec{A}$ and $\vec{B}$, then sum given by equation $\vec{R} = \vec{B} + \vec{A}$ remains unchanged as is shown in figure-(iii). Therefore, vector addition is commutative.

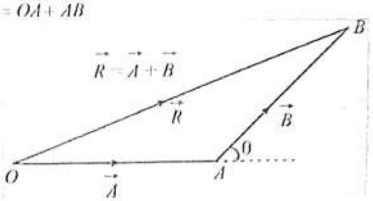


$\vec{A}$
$\vec{B}$
$\vec{R}$
$\vec{B}$
$\vec{A}$
$\vec{A}$
$\vec{B}$
$\vec{R}$

Fig. (i) Fig. (ii) Fig. (iii)

Consider a triangle OAB in which vector $\vec{A}$ is equal to $\vec{OA}$ and $\vec{B}$ is equal to $\vec{AB}$ then $\vec{R}$ is equal to $\vec{OB}$.

$$\vec{OB} = \vec{OA} + \vec{AB}$$



B
$\vec{R} = \vec{A} + \vec{B}$
$\vec{R}$
$\vec{B}$
θ
O
$\vec{A}$
A

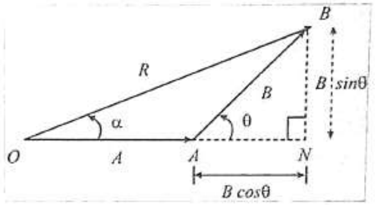
(*i*) **Magnitude of resultant vector**

In ΔABN, $\cos\theta = \frac{AN}{B}$ $\therefore$ $AN = B\cos\theta$

$\sin\theta = \frac{BN}{B}$ $\therefore$ $BN = B\sin\theta$

In ΔOBN, we have $OB^2 = ON^2 + BN^2$

$\Rightarrow R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$



B
R
B
$B\sin\theta$
α
θ
O
A
A
N
$B\cos\theta$

$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$

$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$

$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta \Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

(*ii*) **Direction of resultant vector:** If θ is angle between $\vec{A}$ and $\vec{B}$ and, $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN}$$

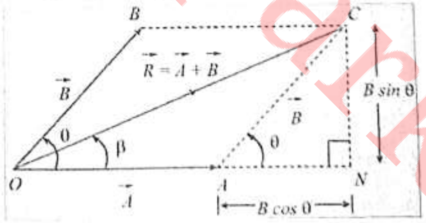
$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow \alpha = \tan^{-1}\left\{\frac{B\sin\theta}{A + B\cos\theta}\right\}$$

PARALLELOGRAM LAW OF VECTOR ADDITION OF TWO VECTORS

If two non zero vectors are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors.

(*i*) **Magnitude:** Since, $R^2 = ON^2 + CN^2$

$\Rightarrow R^2 = (OA + AN)^2 + CN^2 \Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$



B
C
$\vec{R} = \vec{A} + \vec{B}$
$\vec{B}$
$B\sin\theta$
$\vec{B}$
θ
β
θ
O
$\vec{A}$
A
N
$B\cos\theta$

$\therefore R = |\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Special cases: $R = A + B$ when $\theta = 0$

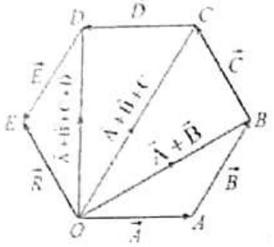
$R = A - B$ when $\theta = 180$

$R = \sqrt{A^2 + B^2}$ when $\theta = 90$

(*ii*) **Direction:** $\tan\beta = \frac{CN}{ON} = \frac{B\sin\theta}{A + B\cos\theta}$

POLYGON LAW OF VECTOR ADDITION

If a number of non zero vectors are represented by the $(n - 1)$ sides of an n-sided polygon then the resultant is given by the closing side or the n^{th} side of the polygon taken in opposite order. So,



$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$

$\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{OE}$

Note: Resultant of two unequal vectors can not be zero.

Resultant of three coplanar vectors may or may not be zero.

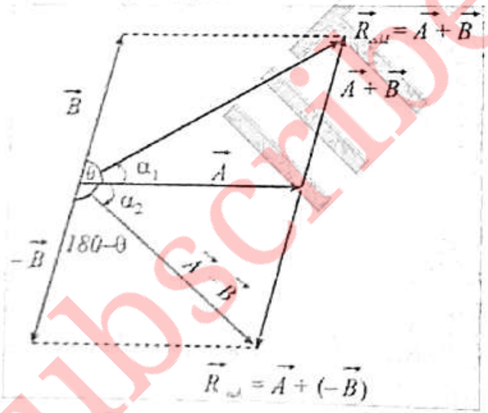
Resultant of three non coplanar vectors can not be zero.

SUBTRACTION OF VECTORS

Since $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ and $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \quad \vec{A} - \vec{B} = \sqrt{A^2 + B^2 + 2AB\cos(180^o - \theta)}$

Since, $\cos(180^\circ - \theta) = -\cos\theta$



$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$\tan\alpha_1 = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow \tan\alpha_2 = \frac{B\sin(180 - \theta)}{A + B\cos(180 - \theta)}$

But $\sin(180^\circ - \theta) = \sin\theta$ and $\cos(180^\circ - \theta) = -\cos\theta$

$\Rightarrow \tan\alpha_2 = \frac{B\sin\theta}{A - B\cos\theta}$

Train Your Brain

Example 1: There are two force vectors, one of 5 N and other of 12 N. At what angle should the two vectors be added to get resultant vector of 17 N, 7 N and 13 N respectively?

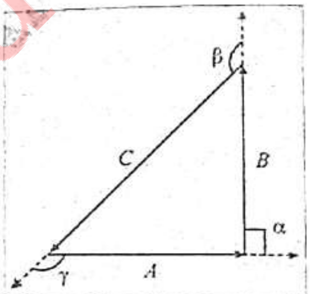
(*a*) 0°, 180° and 90°
(*b*) 0°, 90° and 180°
(*c*) 0°, 90° and 90°
(*d*) 180°, 0° and 90°

Sol. For 17 N, both the vectors should be parallel i.e. angle between them should be zero. For 7 N, both the vectors should be antiparallel i.e. angle between them should be 180°. For 13 N both the vectors should be perpendicular to each other i.e. angle between them should be 90°

Example 2: Given that $\vec{A} + \vec{B} + \vec{C} = 0$. Out of three vectors two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Then the angles between vectors are given by

(*a*) 30°, 60°, 90°
(*b*) 45°, 45°, 90°
(*c*) 45°, 60°, 90°
(*d*) 90°, 135°, 135°

Sol. From polygon law, three vectors having summation zero should form a closed polygon. (Triangle) since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude. i.e. the triangle should be right angled triangle



Angle between A and B, $\alpha = 90^\circ$

Angle between B and C, $\beta = 135^\circ$

Angle between A and C, $\gamma = 135^\circ$

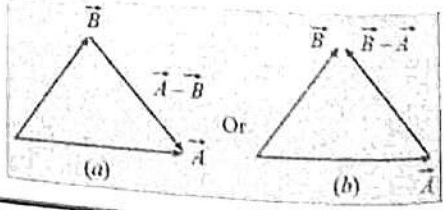
Example 3: Two vectors of 10 units and 5 units make an angle of 120° with each other. Find the magnitude and angle of resultant with the vector of 10 unit magnitude.

Sol. $|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

$= \sqrt{100 + 25 + 2\times10\times5(-1/2)} = 5\sqrt{3}$

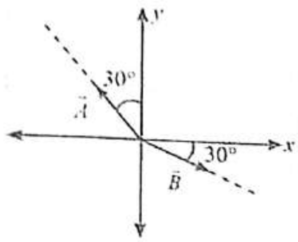
$\tan\alpha = \frac{5\sin120^\circ}{10 + 5\cos120^\circ} = \frac{5\sqrt{3}}{20 - 5} = \frac{5\sqrt{3}}{5\times3} = \frac{1}{\sqrt{3}} \Rightarrow \alpha = 30^\circ$

Note: $\vec{A} - \vec{B}$ or $\vec{B} - \vec{A}$ can also be found by making triangles as shown in figure. (*a*) and (*b*)

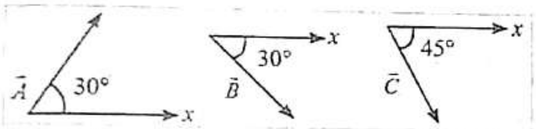


Concept Application

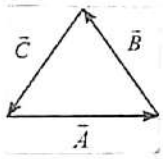
1. A vector $\vec{A}$ makes an angle 30° with the y-axis in anticlockwise direction. Another vector $\vec{B}$ makes an angle 30° with the x-axis in clockwise direction. Find angle between vectors $\vec{A}$ and $\vec{B}$.



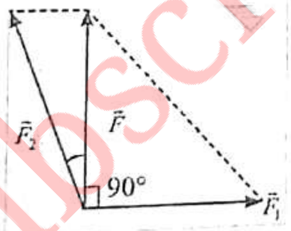
2. Three vectors $\vec{A}, \vec{B}, \vec{C}$ are shown in the figure. Find angle between
   (*a*) $\vec{A}$ and $\vec{B}$ (*b*) $\vec{B}$ and $\vec{C}$ (*c*) $\vec{A}$ and $\vec{C}$



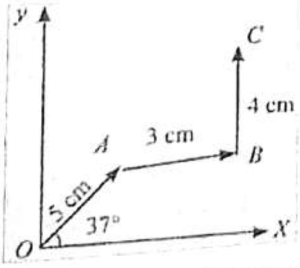
3. If $\vec{A}, \vec{B}, \vec{C}$ represent the three sides of an equilateral triangle taken in the same order then find the angle between
   (*a*) $\vec{A}$ and $\vec{B}$ (*b*) $\vec{B}$ and $\vec{C}$ (*c*) $\vec{A}$ and $\vec{C}$



4. The sum of magnitudes of two forces acting at a point is 16 N. If their resultant is normal to the smaller force and has a magnitude of 8 N. Then find the magnitude of the forces.



5. Find the resultant of the vectors shown in figure.



UNIT VECTOR AND ZERO VECTOR

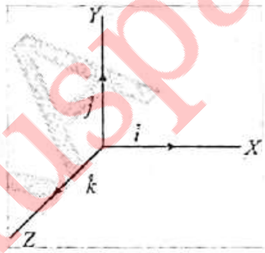
Unit vector is a vector which has a unit magnitude and points in a particular direction. Any vector then ($\vec{A}$) can be written as the product of unit vector ($\hat{A}$) in that direction and magnitude of the given vector.

$$\vec{A} = A\,\hat{A} \quad \text{or} \quad \hat{A} = \frac{\vec{A}}{A}$$

A unit vector has no dimensions and unit. Unit vectors along the positive *x*, *y* and *z*-axes of a rectangular coordinate system are denoted by $\hat{i}, \hat{j}$ and $\hat{k}$ respectively such that $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.

For example: If $\vec{A} = \hat{i} + \hat{j} - \hat{k}$, then unit vector parallel to

$$\vec{A} \text{ would be} = \frac{\vec{A}}{|A|} = \frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$$



A vector of zero magnitude is called a zero or a null vector. Its direction is indeterminate.

Train Your Brain

Example 4: If the sum of two unit vectors is a unit vector, then find the magnitude of their difference.

Sol. Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then the sum is $\vec{n}_s = \hat{n}_1 + \hat{n}_2$ or $n_s^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta$ $= 1 + 1 + 2\cos\theta$

Since it is given that n_s is also a unit vector, therefore $1 = 1 + 1 + 2\cos\theta$

or $\cos\theta = -\frac{1}{2}$ or $\theta = 120°$

Now the difference of the vectors is $\vec{n}_d = \hat{n}_1 - \hat{n}_2$

or $n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta = 1 + 1 - 2\cos(120°)$

$\therefore\ n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3 \Rightarrow n_d = \sqrt{3}$

Concept Application

6. When $\vec{A} = 3\hat{i} - 2\hat{j}$, $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$. Then unit vector along $(\vec{A} - \vec{B})$ would be
   (*a*) $\frac{1}{\sqrt{21}}(\hat{i} - 4\hat{j} + \hat{k})$ (*b*) $\frac{1}{\sqrt{21}}(2\hat{i} - 4\hat{j} + \hat{k})$
   (*c*) $\frac{1}{\sqrt{3}}(2\hat{i} - 4\hat{j} + \hat{k})$ (*d*) $\frac{1}{\sqrt{7}}(2\hat{i} - 4\hat{j} + \hat{k})$

7. When vector $\vec{A} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{B} = 3\hat{i} - 3\hat{j} + \hat{k}$. The unit vector parallel to $\vec{A} + \vec{B}$ would be

(a) $\frac{3\hat{i}}{5\sqrt{2}} - \frac{4\hat{i}}{5\sqrt{2}} + \frac{3\hat{k}}{5\sqrt{2}}$ (b) $\frac{\hat{i}}{2\sqrt{5}} - \frac{4\hat{j}}{2\sqrt{5}} + \frac{3\hat{k}}{5\sqrt{2}}$

(c) $\frac{3\hat{i}}{2\sqrt{5}} - \frac{4\hat{j}}{2\sqrt{5}} + \frac{3\hat{k}}{2\sqrt{5}}$ (d) None of these

8. $\vec{A} = 3\hat{i} - n\hat{j} + 2\hat{k}$. For what value of n, $\frac{\vec{A}}{5}$ may be unit vector

9. Two unit vectors are inclined at some angle so that their resultant is also a unit vector. The angle between the vectors may be

(a) 30° (b) 60° (c) 120° (d) 150°

10. $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$. A unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$ is

(a) $\frac{0}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{1}{\sqrt{22}}\hat{k}$

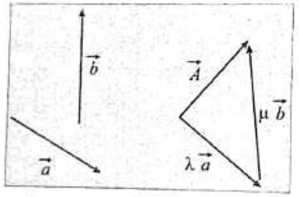
(b) $\frac{2}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$

(c) $\frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{3}{\sqrt{22}}\hat{k}$

(d) $\frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}k$

RESOLUTION OF VECTORS

If $\vec{a}$ and $\vec{b}$ be any two non-zero non parallel vectors in a plane and $\vec{A}$ be another vector in the same plane then $\vec{A}$ can be expressed as a sum of two vectors, one obtained by multiplying $\vec{a}$ by a real number and the other obtained by multiplying $\vec{b}$ by another real number.



$\vec{A} = \lambda\vec{a} + \mu\vec{b}$ (where λ and μ are real numbers)

We say that $\vec{A}$ has been resolved into two component vectors namely, $\lambda\vec{a}$ and $\mu\vec{b}$.

$\lambda\vec{a}$ and $\mu\vec{b}$ are along $\vec{a}$ and $\vec{b}$ respectively. Hence one can resolve a given vector into two component vectors along a set of two vectors – all the three lying in the same plane. Same process can be used to resolve a vector into as many components as needed.

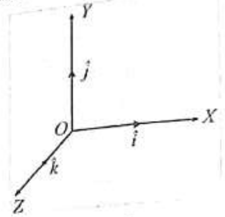
Similarly, in three dimensions, if $\vec{a}, \vec{b}$ and $\vec{c}$ are three non co-planer vectors then any vector can be expressed as a linear combinations of $\vec{a}, \vec{b}$ and $\vec{c}$

$\vec{A} = \lambda\vec{a} + \mu\vec{b} + \gamma\vec{c}$

where λ, μ and γ are real numbers

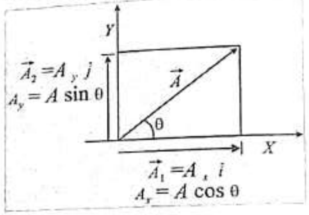
Rectangular Components of a Vector

It is convenient to resolve a general vector along axes of a rectangular coordinate system using vectors of unit magnitude, which we call as unit vectors. $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis as shown in figure below.



Resolution in two Dimensions

Consider a vector $\vec{A}$ that lies in xy plane as shown in figure.



Vector $\vec{A}$ can be expressed as sum of two vectors $\vec{A}_1$ and $\vec{A}_2$ which are along x and y axes respectively.

So, $\vec{A} = \vec{A}_1 + \vec{A}_2$

$\vec{A}_1 = A_x\hat{i}$, $\vec{A}_2 = A_y\hat{j}$

$\Rightarrow \vec{A} = A_x\hat{i} + A_y\hat{j}$

The quantities A_x and A_y are called x-and y-components of the vector $\vec{A}$.

A_x is itself not a vector but $A_x\hat{i}$ is a vector and so is $A_y\hat{j}$.

The values of A_x and A_y may be found as shown in the figure.

$A_x = A\cos\theta$ and $A_y = A\sin\theta$

Its clear from above equation that a component of a vector can be positive, negative or zero depending on the value of θ. A vector $\vec{A}$ can be specified in a plane by two ways:

(a) Its magnitude A and the direction θ it makes with the x-axis/y-axis (or x, y and z axes in 3-D); or

(b) its components A_x and A_y

$$A = \sqrt{A_x^2 + A_y^2},\ \theta = \tan^{-1}\frac{A_y}{A_x}$$

Note: If $A = A_x \Rightarrow A_y = 0$ and if $A = A_y \Rightarrow A_x = 0$ i.e., component of a vector perpendicular to itself is always zero.

Two or more vectors can be added (or subtracted) by expressing them in their components, as illustrated below. The rectangular components of vectors $\vec{A}$ and $\vec{B}$ and those of the sum $\vec{C} = \vec{A} + \vec{B}$ are shown in figure.

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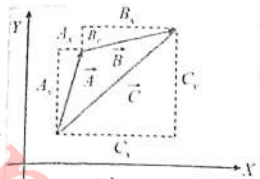
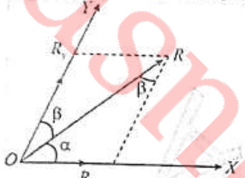


(*a*)

We see that $\vec{C} = \vec{A} + \vec{B}$ is equivalent to both
$C_x = A_x + B_x$ and $C_y = A_y + B_y$

Resolution of vectors may also be done along inclined axes.



(*b*)

Vector $\vec{R}$ has been resolved along two axes x and y not perpendicular to each other. Applying sine law in the triangle, we have

$$\frac{R}{\sin[180° - (\alpha + \beta)]} = \frac{R_x}{\sin\beta} = \frac{R_y}{\sin\alpha}$$

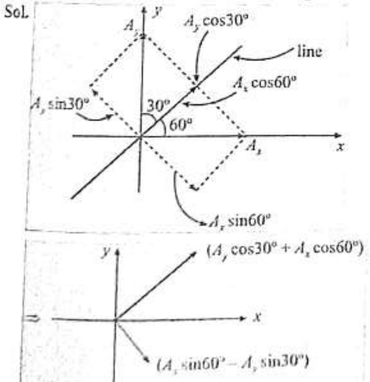
$$\text{or } R_x = \frac{R\sin\beta}{\sin(\alpha+\beta)} \text{ and } R_y = \frac{R\sin\alpha}{\sin(\alpha+\beta)}$$

If $\alpha + \beta = 90°$, $R_x = R \sin\beta$ and $R_y = R\sin\alpha$

Train Your Brain

Example 5: Resolve the vector $\vec{A} = A_x\hat{i} + A_y\hat{j}$ along and perpendicular to the line which makes angle 60° with x-axis.

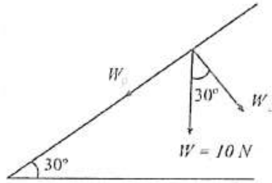
Sol.



so the component along the line $= |A_y\cos30° + A_x\cos60°|$
and perpendicular to the line $= |A_x\sin60° - A_y\sin30°|$

Example 6: Resolve a weight of 10 N in two directions which are parallel and perpendicular to a slope inclined at 30° to the horizontal.

Sol.



Component perpendicular to the plane

$$W_{\perp} = W\cos 30°$$

$$= (10)\frac{\sqrt{3}}{2} = 5\sqrt{3}\text{ N}$$

and component parallel to the plane

$$W_{\parallel} = W\sin 30° = (10)\left(\frac{1}{2}\right) = 5\text{ N}$$

Example 7: Resolve horizontally and vertically a force $F = 8$ N which makes an angle of 45° with the horizontal.

Sol. Horizontal component of $\vec{F}$ is

$$F_H = F\cos 45° = (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\text{ N}$$

and vertical component of $\vec{F}$ is

$$F_V = F\sin 45° = (8)\left(\frac{1}{\sqrt{2}}\right) = 4\sqrt{2}\text{ N}$$

Concept Application

11. If two forces of 5 N each are simultaneously acting along x and y-axes, the magnitude and direction of resultant from x-axis is

(*a*) $5\sqrt{2}, \pi/3$
(*b*) $5\sqrt{2}, \pi/4$
(*c*) $-5\sqrt{2}, \pi/3$
(*d*) $-5\sqrt{2}, \pi/4$

12. The x-component of a force of 50 N is 30 N, then what will be the y-component of the same force?

(*a*) 20 N
(*b*) 30 N
(*c*) 40 N
(*d*) 150 N

13. A vector $\vec{p}$, when added to the resultant of the vectors $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ gives a unit vector along +y-axis, find the vector $\vec{p}$.

(*a*) $-5\hat{i} + 2\hat{j} - 4\hat{k}$
(*b*) $-3\hat{i} + 5\hat{j} - 3\hat{k}$
(*c*) $-5\hat{i} - 4\hat{k}$
(*d*) $-5\hat{i} + 6\hat{j} + 3\hat{k}$

14. The process of splitting a vector into two/more vectors in such a way that their combined effect is same as that of the given vector, may be called as

(*a*) Resolution of vector
(*b*) Unit vector
(*c*) Uniqueness of resolution
(*d*) None of these

15. A child pulls a rope attached to a stone with a force of 60N. The rope makes an angle of 60° to the horizontal ground. What is the effective force along the horizontal direction.

(*a*) 45.96 N (*b*) 40.3 N (*c*) 28 N (*d*) 30 N

PROCEDURE TO SOLVE THE VECTOR EQUATION IN TWO DIMENSIONS

$\vec{A} = \vec{B} - \vec{C}$ ...(i)

There are six variables in this equation which are A_x, A_y, B_x, B_y, C_x and C_y.

Two equations can be formed from the above vector equation, one from x-component and other from y-component. So, we can solve these equations if we know the value of four variables.

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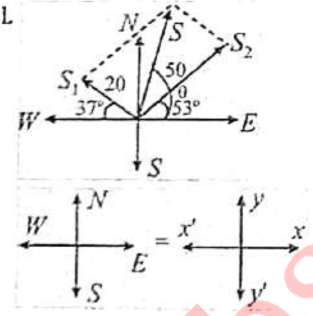
Example 8: Find the net displacement of a particle from its starting point if it undergoes two successive displacement given by $\vec{S}_1 : 20\text{m}$, 37° North of West and $\vec{S}_2 : 50\text{m}$, 53° North of East.

Sol.



$\vec{S} = \vec{S}_1 + \vec{S}_2$

$S_x = S_{1x} + S_{2x} \Rightarrow S_y = S_{1y} + S_{2y}$

$S_x = -20\cos 37° + 50\cos 53° = 14\text{m}$

$S_y = 20\sin 37° + 50\sin 53° = 52\text{m}$

$$S = \sqrt{S_x^2 + S_y^2} = \sqrt{(14)^2 + (52)^2} = 53.85\text{m}$$

Angle from west-east axis (x-axis)

$$\tan\theta = \frac{S_y}{S_x} = \frac{52}{14} = \frac{26}{7}$$

$$\theta = \tan^{-1}\left(\frac{26}{7}\right)$$

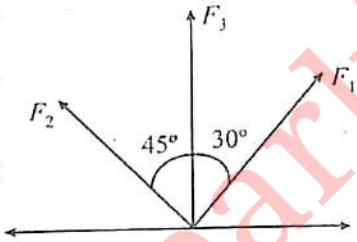
Example 9: Find the magnitude of F_1 and F_2 if F_1 and F_2 make angles of 30° and 45° with F_3 and magnitude of F_3 is 10 N. (given $\vec{F}_1 + \vec{F}_2 = \vec{F}_3$)

Sol. $|\vec{F}_3| = F_1\cos 30° + F_2\cos 45°$

and $F_2\sin 45° = F_1\sin 30°$

$$\Rightarrow 10 = \frac{\sqrt{3}F_1}{2} + \frac{F_2}{\sqrt{2}} \quad ...(i)$$

$$\frac{F_2}{\sqrt{2}} = \frac{F_1}{2} \quad ...(ii)$$

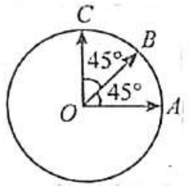


Solving (i) and (ii)

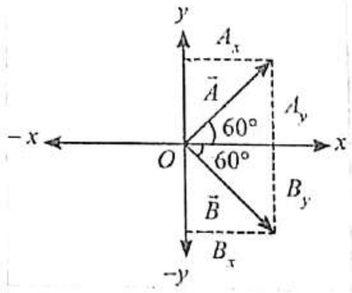
$$\Rightarrow F_1 = \frac{20}{\sqrt{3}+1} \text{ and } F_2 = \frac{10\sqrt{2}}{\sqrt{3}+1}$$

Concept Application

16. Find the resultant of the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ as shown in figure. $[OA = r]$



17. Vector $\vec{A}$ is 2 cm long and is 60° above the x-axis in the first quadrant, vector $\vec{B}$ is 2 cm long and is 60° below the x-axis in the fourth quadrant. Find $\vec{A} + \vec{B}$.



18. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$. Find a vector having the same magnitude as $\vec{B}$ but is parallel to $\vec{A}$.

19. The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$ and equal to half of the magnitude of $\vec{B}$. Find angle between $\vec{A}$ and $\vec{B}$?

20. A car moves 40 m due east, turns towards north and moves 30 m then turns 45° east of north and moves $20\sqrt{2}$m. The net displacement of car is (East is taken positive x-axis, North as positive y-axis.

(*a*) $50\hat{i}+60\hat{j}$ (*b*) $60\hat{i}+50\hat{j}$

(*c*) $30\hat{i}+40\hat{j}$ (*d*) $40\hat{i}+30\hat{j}$

21. An aeroplane is heading north east at a speed of 141.4 ms^{-1}. The northward component of its velocity is

(*a*) 141.4 ms^{-1} (*b*) 100 ms^{-1}

(*c*) zero (*d*) 50 ms^{-1}

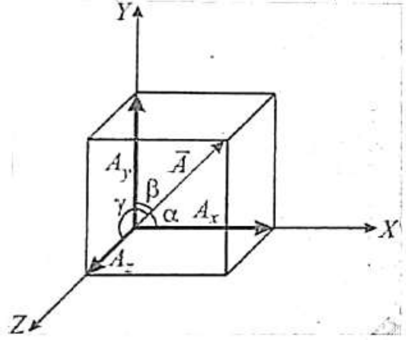
RECTANGULAR COMPONENTS IN THREE DIMENSION

A vector in three dimensions can be expressed as

$$\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$$

where A_x, A_y and A_z are components of the vector along X, Y and Z axes respectively.

The magnitude of the vector is given by $|\vec{A}| = \sqrt{A_x^2+A_y^2+A_z^2}$



If the vector $\vec{A}$ makes angles α, β and γ with X, Y and Z axes respectively, then

$A_x = A\cos\alpha$ = Projection of a $\vec{A}$ on X-axis

$A_y = A\cos\beta$ = Projection of a $\vec{A}$ on Y-axis

$A_z = A\cos\gamma$ = Projection of a $\vec{A}$ on Z-axis

where cosα, cosβ and cosγ are called direction cosines. Now

$$A_x^2+A_y^2+A_z^2=A^2$$

$$\therefore A^2\cos^2\alpha + A^2\cos^2\beta + A^2\cos^2\gamma = A^2$$

$$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Therefore sum of squares of direction cosines is 1.

SCALAR PRODUCT OF TWO VECTORS

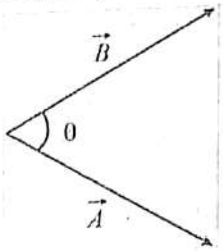
Definition: The scalar product (or dot product) of two vectors is defined as the product of the magnitude of two vectors with cosine of angle between them.

Thus if there are two vectors $\vec{A}$ and $\vec{B}$ having angle θ between them, then their scalar product, written as $\vec{A}.\vec{B}$, is defined as

$$\vec{A}.\vec{B} = AB\cos\theta$$

Properties:

(*i*) It is always a scalar which is positive if the angle between the vectors is acute (i.e., < 90°) and negative if angle between them is obtuse (i.e. 90°< θ < 180°).



(*ii*) It is commutative, i.e. $\vec{A}.\vec{B}=\vec{B}.\vec{A}$

(*iii*) It is distributive, i.e. $\vec{A}.(\vec{B}+\vec{C})=\vec{A}.\vec{B}+\vec{A}.\vec{C}$

(*iv*) As by definition $\vec{A}.\vec{B}=AB\cos\theta$

The angle between the two vectors is given by

$$\theta=\cos^{-1}\left[\frac{\vec{A}.\vec{B}}{AB}\right]$$

(*v*) Scalar product of two vectors will be maximum when $\cos\theta=1$, i.e. $\theta=0°$, i.e., vectors are parallel $(\vec{A}.\vec{B})_{\max}=AB$

(*vi*) Scalar product of two vectors will be zero when $\cos\theta=0$, i.e. $\theta=90°$, $(\vec{A}.\vec{B})_{\min}=0$

i.e., if the scalar product of two nonzero vectors vanishes then the vectors are orthogonal.

(*vii*) The scalar product of a vector with itself is termed as self dot product and is given by

$$\vec{A}.\vec{A}=(\vec{A})^2=AA\cos 0°=A^2$$

i.e., $A=\sqrt{\vec{A}.\vec{A}}$

(*viii*) In case of unit vector $\hat{n}$

$$\hat{n}.\hat{n}=1\times1\times\cos 0=1$$

so $\hat{n}.\hat{n}=\hat{i}.\hat{i}=\hat{j}.\hat{j}=\hat{k}.\hat{k}=1$

(*ix*) In case of orthogonal unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ and

$$\hat{i}.\hat{j}=\hat{j}.\hat{k}=\hat{k}.\hat{i}=1\times1\cos 90=0$$

(*x*) In terms of components

$$\vec{A}.\vec{B}=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}).(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$=[A_xB_x+A_yB_y+A_zB_z]$$

Application:

(*i*) There are physical quantities which are expressed with the help of dot product. For example:

Work *W*: In physics, for constant force, work done on a body is defined as, $W=\vec{F}.\vec{S}=FS\cos\theta$

i.e. work is the scalar product of force with displacement.

Power *P*: As $W=\vec{F}.\vec{S}$ or $\dfrac{dW}{dt}=\vec{F}.\dfrac{d\vec{S}}{dt}=\vec{F}.\vec{V}$

$$\left[\text{As}\ \frac{dW}{dt}=P\ \text{and}\ \frac{d\vec{S}}{dt}=\vec{V}\right]$$

i.e., power is the scalar product of force with velocity.

(*ii*) To find angle between 2 vectors.

(*iii*) To find projection of a given vector on another vector.

(*iv*) To check the orthogonal condition between vectors.

Train Your Brain

Example 10: $\vec{A} = 2\hat{i} + 4\hat{j} + 4\hat{k}$ and $\vec{B} = 4\hat{i} + 2\hat{j} - 4\hat{k}$ are two vectors. The angle between them will be

(*a*) 0° (*b*) 45° (*c*) 60° (*d*) 90°

Sol. $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|} = \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{|\vec{A}|.|\vec{B}|}$

$= \dfrac{2\times4+4\times2-4\times4}{|\vec{A}|.|\vec{B}|} = 0$

$\therefore\ \theta = \cos^{-1}(0^\circ)$

$\Rightarrow\ \theta = 90^\circ$

Example 11: If vector $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -4\hat{i} - 6\hat{j} - \lambda\hat{k}$ are perpendicular to each other then value of λ will be

(*a*) 25 (*b*) 26 (*c*) –26 (*d*) –25

Sol. If $\vec{A}$ and $\vec{B}$ are perpendicular to each other then

$\vec{A}.\vec{B} = 0 \Rightarrow a_1b_1 + a_2b_2 + a_3b_3 = 0$

So. $2(-4) + 3(-6) + (-1)(-\lambda) = 0 \Rightarrow \lambda = +26$

Example 12: A body, acted upon by a force of 50 N, is displaced through a distance 10 meter in a direction making an angle of 60° with the force. The work done by the force will be

(*a*) 200 J (*b*) 100 J (*c*) 300 J (*d*) 250 J

Sol. $W = \vec{F}.\vec{S} = FS\cos\theta$

$= 50\times10\times\cos60^\circ = 50\times10\times\dfrac{1}{2} = 250\,\text{J}.$

Example 13: A particle moves from position $3\hat{i} + 2\hat{j} - 6\hat{k}$ to $14\hat{i} + 13\hat{j} + 9\hat{k}$ due to a uniform force of $4\hat{i} + \hat{j} + 3\hat{k}\,N$. If the displacement is in meters then work done will be

(*a*) 100 J (*b*) 200 J

(*c*) 300 J (*d*) 250 J

Sol. $S = \vec{r_2} - \vec{r_1}$

$W = \vec{F}.\vec{S} = (4\hat{i} + \hat{j} + 3\hat{k}).(11\hat{i} + 11\hat{j} + 15\hat{k})$

$= (4\times11 + 1\times11 + 3\times15) = 100\,\text{J}.$

Example 14: If $\vec{a}_1$ and $\vec{a}_2$ are two non collinear unit vectors and if $|\vec{a}_1 + \vec{a}_2| = \sqrt{3}$, then find the value of $(\vec{a}_1 - \vec{a}_2).(2\vec{a}_1 + \vec{a}_2)$.

Sol. $a_1 = a_2 = 1$ and $a_1^2 + a_2^2 + 2a_1a_2\cos\theta = 3$

Or $1 + 1 + 2\cos\theta = 3$ or $\cos\theta = \dfrac{1}{2}$

Now $(\vec{a}_1 - \vec{a}_2).(2\vec{a}_1 + \vec{a}_2) = 2a_1^2 - a_2^2 - a_1a_2\cos\theta$

$= 2 - 1 - \dfrac{1}{2} = \dfrac{1}{2}$

Concept Application

22. If $\vec{A} = (2\hat{i} + 3\hat{j})$ and $\vec{B} = (\hat{i} - \hat{j})$ then component of $\vec{A}$ perpendicular to vector $\vec{B}$ and in the same plane is

(*a*) $\dfrac{5}{2}(\hat{i} + \hat{j})$ (*b*) $\dfrac{5}{\sqrt{2}}(\hat{i} + \hat{j})$

(*c*) $\dfrac{\sqrt{5}}{2}(\hat{i} + \hat{j})$ (*d*) $\dfrac{\sqrt{5}}{2}(\hat{i} + \hat{k})$

23. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is

(*a*) 120° (*b*) 90° (*c*) 60° (*d*) 45°

24. If $\vec{A} = 9\hat{i} - 7\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j} - 6\hat{k}$ then the value of $(\vec{A} + \vec{B}).(\vec{A} - \vec{B})$

(*a*) 206 (*b*) 128 (*c*) 106 (*d*) –17

25. If $\vec{A} = 5\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} + 2\hat{k}$, then component of $\vec{B}$ along $\vec{A}$ is

(*a*) $\dfrac{\sqrt{14}}{38}$ (*b*) $\dfrac{28}{\sqrt{38}}$ (*c*) $\dfrac{\sqrt{28}}{38}$ (*d*) $\dfrac{14}{\sqrt{38}}$

26. The angle between the two vectors $-2\hat{i} + 3\hat{j} - \hat{k}$ and $\hat{i} + 2\hat{j} + 4\hat{k}$ is

(*a*) 0° (*b*) 90° (*c*) 180° (*d*) 45°

27. If the vectors $\vec{A} = a\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = a\hat{i} - a\hat{j} + \hat{k}$ are perpendicular to each other then the positive value of 'a' is

(*a*) Zero (*b*) 1 (*c*) 2 (*d*) 3

VECTOR PRODUCT OF TWO VECTORS

Definition

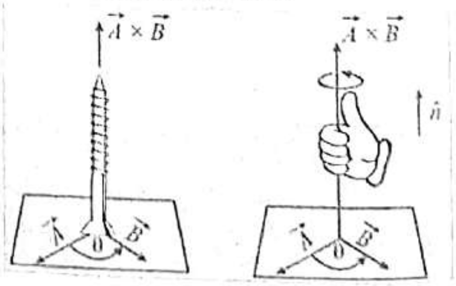
The vector product or cross product of two vectors is defined as a vector having a magnitude equal to the product of the magnitudes of two vectors with the sine of angle between them, and direction perpendicular to the plane containing the two vectors in accordance with right hand screw rule.

$\vec{C} = \vec{A} \times \vec{B}$

Thus, if $\vec{A}$ and $\vec{B}$ are two vectors, then their vector product written as $\vec{A} \times \vec{B}$, is a vector $\vec{C}$ defined by

$\vec{C} = \vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$

where, $\hat{n}$ is a unit vector along $\vec{A} \times \vec{B}$.



The direction of $\vec{A}\times\vec{B}$ i.e. $\vec{C}$ is perpendicular to the plane containing vectors $\vec{A}$ and $\vec{B}$ and is in the sense of advancing of a right handed screw rotated from $\vec{A}$ (first vector) to $\vec{B}$ (second vector) through the smaller angle between them. Thus, if a right handed screw whose axis is perpendicular to the plane framed by $\vec{A}$ and $\vec{B}$ is rotated from $\vec{A}$ to $\vec{B}$ through the smaller angle between them, then the direction of advancement of the screw gives the direction of $\vec{A}\times\vec{B}$ i.e. $\vec{C}$.

Properties:

(*i*) Vector product of any two vectors is always a vector perpendicular to the plane containing these two vectors, i.e., orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, though the vectors $\vec{A}$ and $\vec{B}$ may or may not be orthogonal.

(*ii*) Vector product of two vectors is not commutative,

i.e., $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$ [but $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$]

Here it is worthy to note that $|\vec{A}\times\vec{B}|=|\vec{B}\times\vec{A}|=AB\sin\theta$

i.e., in case of vector $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ magnitudes are equal but directions are opposite.

(*iii*) The vector product is distributive when the order of the vectors is strictly maintained, i.e.

$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

(*iv*) As by definition of vector product of two vectors $\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}$

So $|\vec{A}\times\vec{B}|=AB\sin\theta$ i.e. $\theta=\sin^{-1}\left[\frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}\right]$

(*v*) The vector product of two vectors will be maximum when $\sin\theta=1$, i.e., $\theta=90°$

$[\vec{A}\times\vec{B}]_{\max}=AB\,\hat{n}$

Thus vector product is maximum if the vectors are orthogonal.

(*vi*) The vector product of two non-zero vectors will be minimum when $|\sin\theta|=0$, i.e., $\theta=0^{\circ}$ or 180°

$[\vec{A}\times\vec{B}]_{\min}=0$

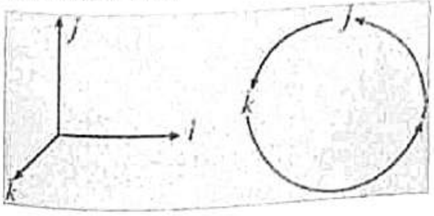
i.e. if the vector product of two non-zero vectors vanishes, the vectors are collinear.

(*vii*) The self cross product, i.e., product of a vector by itself vanishes, i.e., null vector. $\vec{A}\times\vec{A}=AA\sin 0^{\circ}\,\hat{n}=\vec{0}$

(*viii*) In case of unit vector $\hat{n}\times\hat{n}=\vec{0}$.

So that $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=\vec{0}$

(*ix*) The orthogonal unit vectors $\hat{i},\hat{j},\hat{k}$ are in accordance with right hand screw rule.



$\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{k}=\hat{i}$ and $\hat{k}\times\hat{i}=\hat{j}$

And as cross product is not commutative,

$\hat{j}\times\hat{i}=-\hat{k}$, $\hat{k}\times\hat{j}=-\hat{i}$ and $\hat{i}\times\hat{k}=-\hat{j}$

(*x*) In terms of components $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$

$=\hat{i}(A_yB_z-A_zB_y)+\hat{j}(A_zB_x-A_xB_z)+\hat{k}(A_xB_y-A_yB_x)$

Application

- Since vector product of two vectors is a vector, some of the vector physical quantities (particularly representing rotational effects) like torque, angular momentum, velocity and force on a moving charge in a magnetic field can be expressed as the vector product of two vectors. It is well - established in physics that:

  (*i*) Torque $\vec{\tau}=\vec{r}\times\vec{F}$

  (*ii*) Angular momentum $\vec{L}=\vec{r}\times\vec{p}$

  (*iii*) Velocity $\vec{v}=\vec{\omega}\times\vec{r}$

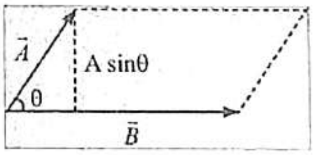
  (*iv*) Force on a charged particle q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F}=q(\vec{v}\times\vec{B})$

  (*v*) Torque on a dipole in a field $\vec{\tau}_E=\vec{p}\times\vec{E}$

- To find the unit vector perpendicular to two given vectors $\vec{A}$ and $\vec{B}$,

  $(\hat{n})=\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

- If $\vec{A}$ and $\vec{B}$ represent two sides of a parallelogram, then $|\vec{A}\times\vec{B}|=$ area of the parallelogram.



Area = Base × height

$= B\times A\sin\theta$

$=|\vec{A}\times\vec{B}|$

Train Your Brain

Example 15: If two vectors $2\hat{i}+3\hat{j}-\hat{k}$ and $-4\hat{i}-6\hat{j}+\lambda\hat{k}$ are parallel to each other then value of λ is

(*a*) 0 (*b*) 2 (*c*) 3 (*d*) 4

Sol. Let $\vec{A}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{B}=-4\hat{i}-6\hat{j}+\lambda\hat{k}$

$\vec{A}$ and $\vec{B}$ are parallel to each other $\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}$

i.e. $\frac{2}{-4}=\frac{3}{-6}=\frac{-1}{\lambda}$

$\Rightarrow \lambda=2$

Example 16: If $\vec{A}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{B}=2\hat{i}-2\hat{j}+4\hat{k}$ then value of $|\vec{A}\times\vec{B}|$ will be

(a) $8\sqrt{2}$ (b) $8\sqrt{3}$ (c) $8\sqrt{5}$ (d) $5\sqrt{8}$

Sol. $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 2\\ 2 & -2 & 4\end{vmatrix}$

$=(1\times4-2\times-2)\hat{i}+(2\times2-4\times3)\hat{j}+(3\times(-2)-1\times2).$

$=8\hat{i}-8\hat{j}-8\hat{k}$

$\therefore$ Magnitude of $\vec{A}\times\vec{B}$

$|\vec{A}\times\vec{B}|=\sqrt{(8)^2+(-8)^2+(-8)^2}$

$=8\sqrt{3}$

Example 17: In the above example a unit vector, perpendicular to both $\vec{A}$ and $\vec{B}$ will be

(a) $+\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$ (b) $-\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

(c) Both (a) and (b) (d) None of these

Sol. $\hat{n}=\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}=\frac{8\hat{i}-8\hat{j}-8\hat{k}}{8\sqrt{3}}=\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

There are two unit vectors perpendicular to both $\vec{A}$ and $\vec{B}$ they are $\hat{n}=\pm\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

Example 18: The angle between the vectors $\vec{A}$ and $\vec{B}$ is 0. The value of the triple product $\vec{A}.(\vec{B}\times\vec{A})$ is

(a) A^2B (b) Zero

(c) $A^2B\sin\theta$ (d) $A^2B\cos\theta$

Sol. Let $\vec{A}.(\vec{B}\times\vec{A})=\vec{A}.\vec{C}$

Here $\vec{C}=\vec{B}\times\vec{A}$ is perpendicular to both vectors $\vec{A}$ and $\vec{B}$.

$\therefore\ \vec{A}.\vec{C}=0$

Concept Application

28. Find the values of x and y for which vectors $\vec{A}=6\hat{i}+x\hat{j}-2\hat{k}$ and $\vec{B}=5\hat{i}-6\hat{j}-y\hat{k}$ may be parallel

(a) $x=0,\ y=\frac{2}{3}$ (b) $x=\frac{-36}{5},\ y=\frac{5}{3}$

(c) $x=\frac{-15}{3},\ y=\frac{23}{5}$ (d) $x=\frac{36}{3},\ y=\frac{15}{4}$

29. Two sides of a triangle are given by $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, then area of triangle is

(a) $\frac{\sqrt{26}}{2}$ (b) $\sqrt{\frac{26}{2}}$ (c) $\sqrt{46}$ (d) 26

30. A vector $\vec{C}$ of magnitude 5 is perpendicular to both vector $\vec{A}=2\hat{i}+2\hat{j}$ and $\vec{B}=\hat{i}-\hat{k}$. Then $\vec{C}$ is

(a) $\frac{-5\hat{i}+5\hat{j}-5\hat{k}}{\sqrt{3}}$ (b) $\frac{5\hat{i}+5\hat{j}+5\hat{k}}{\sqrt{3}}$

(c) $\frac{5\hat{i}+5\hat{j}}{\sqrt{2}}$ (d) $\frac{5\hat{i}-5\hat{k}}{\sqrt{2}}$

31. Find the unit vector perpendicular to both $\vec{A}=3\hat{i}+2\hat{j}-\hat{k}$ and $\vec{B}=\hat{i}-\hat{j}+\hat{k}$

32. If $\vec{F}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{r}=2\hat{i}-\hat{j}+\hat{k}$ find $\vec{r}\times\vec{F}$.

33. The adjacent sides of a parallelogram are $\vec{A}=2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{B}=-2\hat{i}+4\hat{j}-\hat{k}$. What is the area of the parallelogram?

(a) 4 units (b) 7 units

(c) $\sqrt{5}$ units (d) $\sqrt{8}$ units

34. What is the condition for the vectors $2\hat{i}+3\hat{j}-4\hat{k}$ and $3\hat{i}-a\hat{j}+b\hat{k}$ to be parallel?

(a) $a=-9/2,\ b=-6$ (b) $a=-6,\ b=-9/2$

(c) $a=4,\ b=5$ (d) $\sqrt{8}$ units

MOTION IN TWO AND THREE DIMENSIONS

When a particle is moving in space then its motion can be resolved in three independent motions in mutually perpendicular directions. Let these directions be x, y and z axis. The motion in these three directions is governed only by velocity and acceleration in that particular direction and is totally independent of the velocities and acceleration in other directions.

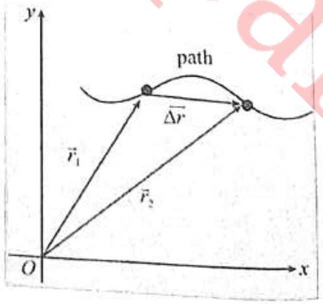
Lets say a particle is moving in space. The vector

$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ ...(i)

gives position of particle in space.

Velocity

Differentiating $\vec{r}$ w.r.t. time (t) gives us velocity vector of particle at that time.



$$\vec{v} = \frac{d\vec{r}}{dt} \quad ...(ii)$$

To write equation in a unit vector form, we substitute $\vec{r}$ from equation (i)

$$\vec{v} = \frac{d\left(x\hat{i} + y\hat{j} + z\hat{k}\right)}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

This equation can be simplified by writing it as

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \quad ...(iii)$$

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, v_z = \frac{dz}{dt} \quad ...(iv)$$

where, v_x, v_y and v_z are the components of $\vec{v}$ in x, y and z axes.

Acceleration

Similarly, if we differentiate $\vec{v}$ w.r.t. time we get acceleration of particle

$$\vec{a} = \frac{d\vec{v}}{dt} \quad ...(v)$$

If the velocity changes in either magnitude or direction (or both), the particle must have an acceleration.

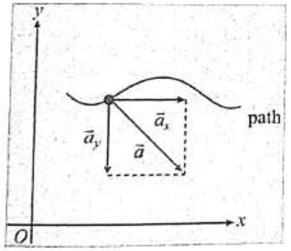
We can write equation (v) in a vector form by substituting $\vec{v}$ from equation (iii) to obtain

$$\vec{a} = \frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

We can rewrite this as

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \quad ...(vi)$$

where the components of $\vec{a}$ are



$$a_x = \frac{dv_x}{dt}; a_y = \frac{dv_y}{dt}; a_z = \frac{dv_z}{dt} \quad ...(vii)$$

Thus, we can find the components of $\vec{a}$ by differentiating the components of $\vec{v}$.

Figure shows an acceleration vector $\vec{a}$ and its components for a particle moving in two dimensions.

Now, collecting equations of motion related to x and y axis separately

x-axis	**y-axis**
$v_x = \frac{dx}{dt}$	$v_y = \frac{dy}{dt}$
$a_x = \frac{dv_x}{dt}$	$a_y = \frac{dv_y}{dt}$

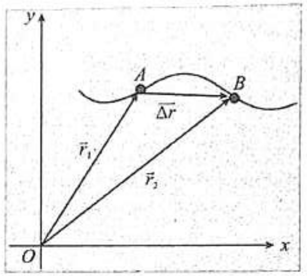
Thus we can see that motion in plane is composed of two straight line motions. These motions are completely independent of each other. Only thing connecting them is fact that they are occurring simultaneously.

Velocity is Along Tangent of Path

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$



$\Delta\vec{r}$ will be along tangent at point A when $\Delta\vec{r}$ tends to 0.

The result is the same in three dimensions also.

Problem solving strategy

For solving any problem of two dimensional motion, following is standard algorithm.

1. Break the motion in two separate one dimensional motions.
2. Solve both motions separately.
3. Connect the final equations thoroughly using 'time' (because it is the only common thing).

PROJECTILE MOTION

Projectile motion is the motion of an object when it is thrown into the air at some angle under the action of gravity. The path followed by the particle is a curved path.

The particle which is projected is known as projectile. The path followed by the particle is called its trajectory.

It consists of two independent motions, a horizontal motion at constant speed and a vertical motion under acceleration due to gravity.

In order to deal with problems in projectile motion, one has to choose a coordinate system. Let's take horizontal as X-axis and vertical upward direction as Y-axis, then

$a_x = 0$ and $a_y = -g$, since there is only one force 'mg' downward (ignore air resistance).

Equations along x-axis	**Equations along y-axis**
$v_x = u_x$ (constant)	$v_y = u_y - gt$
$S_x = u_x t$	$S_y = u_y t - \frac{1}{2}gt^2$
	$v_y^2 = u_y^2 - 2g(S_y)$

If an object is dropped from rest or projected up or down, it follows straight line path. If its initial velocity is not along the line of force it follows parabolic path which is proved mathematically in this chapter later on.

PROJECTILE THROWN FROM THE GROUND LEVEL

Consider the motion of a bullet which is fired from a gun so that its initial velocity $\vec{u}$ makes an angle θ with the horizontal direction. Let us take X-axis along ground and Y-axis along vertically upward direction.

$\vec{u}$ can be resolved as

$u_x = u\cos\theta$ (along horizontal)

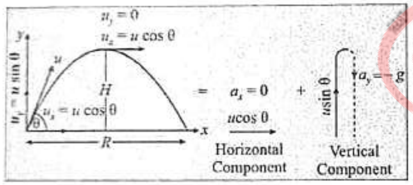
and $u_y = u\sin\theta$ (along vertical)

Motion of bullet can be resolved into horizontal and vertical motions.

(i) In horizontal direction there is no acceleration so it moves with constant velocity $v_x = u_x = u\cos\theta$

So displacement along X-direction in time t is $x = u_x\, t$ or

$$x = (u\cos\theta)\,t \text{ or } t = \frac{x}{u\cos\theta}. \quad \text{...(i)}$$



The motion in the vertical direction is the same as that of a ball thrown upward with an initial velocity $u_y = u\sin\theta$ and acceleration $a_y = -g$ (downward).

So, at time t vertical component of velocity

$$v_y = u_y - gt$$

$$= u\sin\theta - gt \quad \text{...(ii)}$$

Displacement along Y direction

$$y = (u\sin\theta)\,t - \frac{1}{2}gt^2 \quad \text{...(iii)}$$

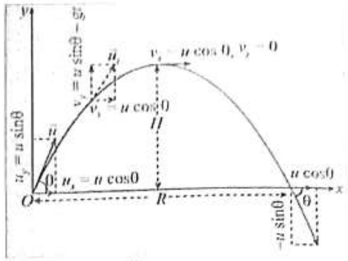
Substituting the value of t from eqn. (i) in eqn. (iii)

we get $y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$

or $\boxed{y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2}$ **Equation of trajectory**

- This is equation of parabola.
- The trajectory of projectile is parabolic.
- The projectile will rise to maximum height H (where $v_x = u\cos\theta$, $v_y = 0$) and then move down again to reach the ground at a distance R from origin.

Setting $x = R$ and $y = 0$ (when projectile reaches ground again)



$$0 = R\tan\theta - \frac{g}{2u^2\cos^2\theta}\cdot R^2$$

We get $R = \frac{2u^2\cos^2\theta}{g}\times\frac{\sin\theta}{\cos\theta}$

or $R = \frac{2u^2}{g}\cdot\sin\theta\cos\theta$

or Range, $R = \frac{u^2\sin 2\theta}{g}$

If time for upward journey is t, at highest point $v_y = 0$, so

$0 = (u\sin\theta) - gt$ (using $v_y = u_y - gt$)

or $t = \frac{u\sin\theta}{g}$

$\therefore$ $T = 2t$ (it will take same time for downward journey)

$\therefore$ $\boxed{T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}}$ **Time of flight**

At the highest point $y = H$ and $v_y = 0$

So $\boxed{H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}}$ **Maximum Height**

[using $v_y^2 = u_y^2 - 2gy$]

We can also determine R as follows

$x = u_x\, t$

so $R = u_x \cdot T$

$= (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right)$

or $\boxed{R = \frac{u^2\sin 2\theta}{g} = \frac{2u_xu_y}{g}}$ **Horizontal Range**

Velocity at time t

$\vec{v}_t = v_{xt}\hat{i} + v_{yt}\hat{j} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

$v = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$

If angle which the direction of motion makes at an instant is ϕ, then

$$\tan\phi = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

$\tan\phi$ is positive during its upward motion i.e. before reaching highest point and after that $\tan\phi$ is negative.

Note:

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Note:

(*i*) Alternative equation of trajectory $\left[y = x\tan\theta\left(1 - \frac{x}{R}\right)\right]$ where $R = \frac{2u^2\sin\theta\cos\theta}{g}$

(*ii*) Vertical component of velocity $v_y = 0$, when particle is at the highest point of trajectory.

(*iii*) Linear momentum at highest point is $mu\cos\theta$ and is in horizontal direction.

(*iv*) Vertical component of velocity is +ve when particle is moving up.

(*v*) Vertical component of velocity is –ve when particle is moving down.

(*vi*) Resultant velocity of particle at time t, $v = \sqrt{v_x^2 + v_y^2}$ and at an angle $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$.

(*vii*) Displacement from origin, $s = \sqrt{x^2 + y^2}$

(*viii*) Maximum height and time of flight depend only on vertical component of initial velocity

Train Your Brain

Example 19: A particle is projected with 20 m/s at an angle of 60° with the horizontal. At what time is it moving at an angle of 45° with the horizontal during its downward journey?

(*a*) $(\sqrt{3}-1)$s (*b*) $(\sqrt{3}+1)$s

(*c*) $\sqrt{3}$ s (*d*) $\frac{1}{\sqrt{3}}$ s

Sol. $u_x = 20\cos 60° = 10$ m/s

and $u_y = 20\sin 60° = 10\sqrt{3}$ m/s

At required instant, $\tan\phi = -1$

i.e. $\frac{u_y - gt}{u_x} = -1$

i.e. $\frac{10\sqrt{3} - 10t}{10} = -1$

on solving, we get $t = \left(\sqrt{3}+1\right)$sec

Example 20: A particle is projected in the *X-Y* plane with *Y*-axis along vertical. After 2*s* of projection the velocity of the particle makes an angle 45° with the *X* - axis and after 4*s* of projection, it moves horizontally. Find the velocity of projection.

(*a*) 40 ms^{-1} (*b*) 20 ms^{-1}

(*c*) $50\sqrt{2}$ ms^{-1} (*d*) $20\sqrt{5}$ ms^{-1}

Sol. At $t = 2$ sec, $\tan\phi = \frac{u_y - 10(2)}{u_x} = 1$ ($\because \phi = 45°$)

$\Rightarrow u_y - 20 = u_x$ ...(i)

Also, $\frac{1}{2}$ (time of flight) = 4 sec

$\Rightarrow \frac{1}{2}\left(\frac{2u_y}{g}\right) = 4$

$\Rightarrow u_y = 40$ m/s

From equation (i), $u_x = 20$ m/s

$\Rightarrow u = \sqrt{u_x^2 + u_y^2} = 20\sqrt{5}$ m/s

Concept Application

35. A projectile is projected in the upward direction making an angle of 60° with horizontal direction with a velocity of 150 m/sec. Then the time after which its inclination with the horizontal is 45° is-

(*a*) 15 sec (*b*) 10.98 sec

(*c*) 5.49 sec (*d*) 2.745 sec

36. One second after projection, a stone moves at an angle of 45° with horizontal. Two seconds after projection, it moves horizontally. Its angle of projection is ($g = 10$ m/s^2)

(*a*) $\tan^{-1}(\sqrt{3})$ (*b*) $\tan^{-1}(4)$

(*c*) $\tan^{-1}(3)$ (*d*) $\tan^{-1}(2)$

Special Points

1. **The three basic equations of motion are,**

$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as$

For projectile motion :

$T = \frac{2u\sin\theta}{g} \qquad R = \frac{u^2\sin 2\theta}{g} \qquad H = \frac{u^2\sin^2\theta}{2g}$

2. **In case of projectile motion,**

The horizontal component of velocity, acceleration (*g*) and mechanical energy remain constant.

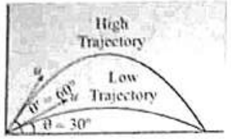
The vertical component of velocity, speed, velocity, momentum, kinetic energy and potential energy all change. Velocity and Kinetic energy are maximum at the point of projection, while minimum (but not zero) at the highest point.

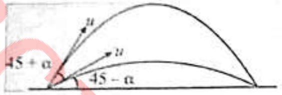
3. **If angle of projection, θ is changed as**

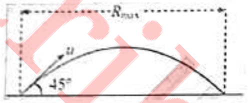
$\theta \longrightarrow \theta' = (90 - \theta)$,

then range

$R' = \frac{u^2\sin 2\theta'}{g} = \frac{u^2\sin 2(90-\theta)}{g} = \frac{u^2\sin 2\theta}{g} = R$







So a projectile has same range for angles of projection θ and (90 − θ), but has different times of flight (T), maximum height (H) and trajectories

Range is also same for $\theta_1 = 45° - \alpha$ and $\theta_2 = 45° + \alpha$.

$$\left[R = \frac{u^2\cos 2\alpha}{g}\right]$$

Note: For same Range

$\alpha - \beta = 90°$

α and β are two angles of projection with same velocity.

4. **For maximum Range** $R = R_{max} \Rightarrow 2\theta = 90°$ for $\theta = 45°$

$$R_{max} = \frac{u^2}{g} \quad [\text{For } \sin 2\theta = 1 = \sin 90° \text{ or } \theta = 45°]$$

When range is maximum then maximum height reached is

$$H = \frac{u^2\sin^2 45°}{2g} \quad \text{or} \quad H = \frac{u^2}{4g}$$

hence maximum height reached $H = \frac{R_{max}}{4}$

If a body is projected from a place at some height from the surface of earth, then for the maximum range, the angle of projection should be slightly less than 45°. For javelin throw and discus throw, the athlete throws the projectile at an angle slightly less than 45° to the horizontal to achieve the maximum range.

5. **For height H to be maximum**

$$H = \frac{u^2\sin^2\theta}{2g} = \text{max} \quad \text{i.e. } \sin^2\theta = 1 \text{ (max) or } \theta = 90°$$

So that $H_{max} = \frac{u^2}{2g}$ when projected vertically upwards (i.e. at $\theta = 90°$)

In this case, Range $R = \frac{u^2\sin(2\times 90°)}{g} = \frac{u^2\sin 180°}{g} = 0$

$H_{max} = \frac{u^2}{2g}$ (For vertical projection) and $R_{max} = \frac{u^2}{g}$ (For oblique projection with same velocity)

so $H_{max} = \frac{R_{max}}{2}$

- **A person can throw a projectile to a maximum distance of (with $\theta = 45°$) $R_{max} = \frac{u^2}{g}$.**
- **The maximum height to which he can throw the projectile (with $\theta = 90°$) $H_{max} = \frac{R_{max}}{2}$**

6. **At highest point**

Maximum Potential energy $(PE)_H$ will be $= mgH$

$$= mg\cdot\frac{u^2\sin^2\theta}{2g} \text{ or } (PE)_H = \frac{1}{2}mu^2\sin^2\theta.$$

While Kinetic energy(K.E) will be minimum (but not zero) at the highest point as the vertical component of velocity is zero.

$$(KE)_H = \frac{1}{2}mv_H^2 = \frac{1}{2}m(u\cos\theta)^2 = \frac{1}{2}mu^2\cos^2\theta$$

$$\text{so, } (PE)_H + (KE)_H = \frac{1}{2}mu^2\sin^2\theta + \frac{1}{2}mu^2\cos^2\theta = \frac{1}{2}$$

mu^2 = Total Mechanical energy(M.E)

In projectile motion mechanical energy is conserved.

$$\left(\frac{PE}{KE}\right)_H = \frac{\frac{1}{2}mu^2\sin^2\theta}{\frac{1}{2}mu^2\cos^2\theta} = \tan^2\theta$$

Note: If $\theta = 45° \Rightarrow \tan^2\theta = 1$

$\Rightarrow$ PE = K.E. = $\frac{1}{2}$ M.E. at highest point i.e. if a body is projected at an angle θ = 45° to the horizontal then at highest point, half of its Kinetic energy(K.E) is Mechanical energy(K.E) and half is Potential energy(P.E).

7. **In case of projectile motion if range R is n times the maximum height H, i.e. $R = nH$**

$$\text{then } \frac{u^2\sin 2\theta}{g} = n\cdot\frac{u^2\sin^2\theta}{2g}$$

$$\text{or } 2\cos\theta = \frac{n.\sin\theta}{2}$$

$$\text{or } \tan\theta = \frac{4}{n} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{n}\right)$$

Train Your Brain

Example 21: A projectile is projected with kinetic energy *K*. If it has the maximum possible horizontal range, then its kinetic energy at the highest point will be:

(*a*) 0.25 *K* (*b*) 0.5 *K*
(*c*) 0.75 *K* (*d*) 0.0 *K*

Sol. When range is maximum, θ = 45°

Now $u_x = u\cos 45° = u/\sqrt{2}$

At the highest point, the net velocity of projectile is u_x

$\therefore$ K.E. = $(1/2)\,m\,(u/\sqrt{2})^2$

= $(1/2)(1/2\,mu^2) = (1/2)K = 0.5\,K$

Example 22: When the angle of elevation of a gun are 60° and 30° respectively, the height it shoots are h_1 and h_2 respectively. h_1/h_2 equals to-

(*a*) 3/1 (*b*) 1/3
(*c*) 1/2 (*d*) 2/1

Sol. For angle of elevation of 60°, we have maximum height

$$h_1 = \frac{u^2 \sin^2 60°}{2g} = \frac{3u^2}{8g}$$

For angle of elevation of 30°, we have maximum height

$$h_2 = \frac{u^2 \sin^2 30°}{2g} = \frac{u^2}{8g} \Rightarrow \frac{h_1}{h_2} = \frac{3}{1}$$

Example 23: A particle is projected from ground at an angle of 45° with initial velocity $20\sqrt{2}$ m/s. Find magnitude of average velocity in the time interval from $t = 0$ to $t = 3$ s.

(*a*) 10 ms^{-1} (*b*) $5\sqrt{17}$ ms^{-1}
(*c*) 30 ms^{-1} (*d*) $15\sqrt{17}$ ms^{-1}

Sol. $\Delta\vec{r} = (20 \times 3\hat{i}) + (20 \times 3 - \frac{1}{2} \times 10 \times 9)\hat{j} = 60\hat{i} + 15\hat{j}$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v}_{av} = 20\hat{i} + 5\hat{j}$$

$$\Rightarrow |\vec{v}_{avg}| = 5\sqrt{17}\ \text{ms}^{-1}$$

Example 24: The coach throws a baseball to a player with an initial speed of 20 m/s at an angle of 45° with the horizontal. At the moment the ball is thrown, the player is 50 m away from the coach. At what speed and in what direction must the player run to catch the ball at the same height at which it was released?

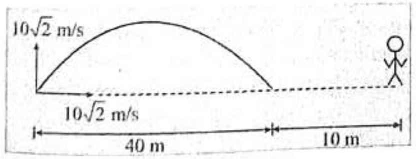
(*a*) $\frac{5}{\sqrt{2}}$ ms^{-1} away from the coach

(*b*) $20\sqrt{2}$ ms^{-1} away from the coach

(*c*) $\frac{5}{\sqrt{2}}$ ms^{-1} towards the coach

(*d*) $20\sqrt{2}$ ms^{-1} towards the coach

Sol.



Time of flight

$$S_y = 10\sqrt{2}t - \frac{1}{2} \times 10t^2 = 0$$

$$t = \frac{10\sqrt{2}}{5} = 2\sqrt{2}\ \text{sec}$$

Range

$$R = 10\sqrt{2} \times 2\sqrt{2} = 40\ m$$

Velocity of player should be,

$$v = \frac{10m}{2\sqrt{2}} = \frac{5}{\sqrt{2}}\ ms^{-1}$$

He should run towards the coach

Example 25: If T be the total time of flight of a projectile and H be the maximum height attained by it from the point of projection, then $\frac{H}{T}$ will be: (u = projection velocity and θ = projection angle)

(*a*) $\left(\frac{1}{2}\right)u\sin\theta$ (*b*) $\left(\frac{1}{4}\right)u\sin\theta$
(*c*) $u\sin\theta$ (*d*) $2u\sin\theta$

Sol. Total time of flight, $T = \frac{2u\sin\theta}{g}$

Maximum height attained $H = \frac{u^2\sin^2\theta}{2g}$

So, $\frac{H}{T} = \frac{u\sin\theta}{4}$

Example 26: If a baseball player can throw a ball at maximum distance = d over ground, the maximum vertical height to which he can throw it, will be: (Ball has same initial speed in each case)

(*a*) $\frac{d}{2}$ (*b*) d
(*c*) $2d$ (*d*) $\frac{d}{4}$

Sol. The range $R = \frac{u^2\sin 2\theta}{g}$

$$\therefore \text{ Maximum range } R_{\max} = d = \frac{u^2}{g} \qquad \text{...(i)}$$

Height $H = \frac{u^2\sin^2\theta}{2g}$

∴ Maximum height

$$H_{\max} = \frac{u^2}{2g} \qquad \text{...(ii)}$$

From (i) & (ii), $H_{\max} = \frac{d}{2}$

Example 27: A projectile can have the same range R for two angles of projections. If t_1 and t_2 be the times of flight in two cases, then:

(*a*) $t_1t_2 \propto R$ (*b*) $t_1t_2 \propto R^2$
(*c*) $t_1t_2 \propto 1/R$ (*d*) $t_1t_2 \propto 1/R^2$

Sol. $t_1 = \frac{2u\sin\theta}{g}$

$$t_2 = \frac{2u\sin(90° - \theta)}{g}$$

$$= \frac{2u\cos\theta}{g} \qquad [2\sin\theta\cos\theta \quad \sin 2\theta]$$

$$t_1t_2 = \frac{2}{g}\frac{u^2\sin 2\theta}{g} = \frac{2}{g}\cdot R$$

where R is the range. Hence $t_1t_2 \propto R$

Hence correct answer is (*a*).

Example 28: A ball projected with speed 'u' at angle of projection 15° has range R. The other angle of projection at which the range with same initial speed 'u' will be R, is:

(*a*) 45° (*b*) 35°

(*c*) 75° (*d*) 90°

Sol. Range of projectile, $R = \dfrac{u^2\sin 2\theta}{g}$

The range is same for two angles θ_1 and θ_2 provided $\theta_2 = 90° - \theta_1$

At an angle θ_1, range $R_1 = \dfrac{u^2\sin 2\theta_1}{g}$

At an angle of projection θ_2,

$$\text{range } R_2 = \frac{u^2\sin 2\theta_2}{g} = \frac{u^2\sin 2(90° - \theta_1)}{g} = \frac{u^2\sin 2\theta_1}{g}$$

$\Rightarrow R_1 = R_2$

$\therefore$ other angle $(\theta_2) = 90° - \theta_1 = 90° - 15° = 75°$

Concept Application

37. An arrow is shot into air has a range of 200 metres and its time of flight is 5 seconds. If $g = 10$ m/sec^2, then the horizontal component of velocity of the arrow is-

(*a*) 12.5 m/sec (*b*) 25.0 m/sec

(*c*) 31.25 m/sec (*d*) 40 m/sec

38. A ball is projected with a velocity 80 m/s making an angle of 30° with the horizontal. Its range will be-

(*a*) 433 m (*b*) 632 m

(*c*) 108 m (*d*) 566 m

39. At the top of the trajectory of a projectile the direction of its velocity and acceleration are-

(*a*) Parallel to each other

(*b*) Inclined at an angle of 45° to the horizontal

(*c*) Perpendicular to each other

(*d*) None of the above statement is correct

40. A player kicks up a ball at an angle θ with the horizontal. The horizontal range is maximum when θ equals-

(*a*) 30° (*b*) 45°

(*c*) 60° (*d*) 90°

41. The angle of projection of a body is 25°. The other angle for which the range is the same as the first one is equal to:

(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 65°

42. At which angle with the verticle should a body be projected so as to travel the maximum horizontal range, keeping the projection velocity constant?

(*a*) 60° (*b*) 30° (*c*) 45° (*d*) 75°

43. A body is thrown with a velocity of 9.8 m/s making an angle of 30° with the horizontal. It will hit the ground after a time

(*a*) 3 s (*b*) 2 s (*c*) 1.5 s (*d*) 1 s

44. The maximum range of a gun on a horizontal plane is 16 km. If $g = 10$ m/sec^2, the muzzle velocity of the shell must be:

(*a*) 400 m/sec (*b*) $160\sqrt{10}$ m/sec

(*c*) 1600 m/sec (*d*) $200\sqrt{2}$ m/sec

45. The kinetic energy of a projectile at the highest point is:

(*a*) Zero (*b*) Maximum

(*c*) Minimum (*d*) Equal to total energy

46. In a projectile motion, the velocity of the projectile is perpendicular to acceleration due to gravity

(*a*) For one instant only (*b*) Two times

(*c*) Three times (*d*) Four times

47. A cannon ball has a range R on a horizontal plane. If h and h' are the greatest heights in the two paths for which this is possible, then:

(*a*) $R = 4\sqrt{(hh')}$ (*b*) $R = \dfrac{4h}{h'}$

(*c*) $R = 4h\,h'$ (*d*) $R = \sqrt{(hh')}$

HORIZONTAL PROJECTION

In Horizontal Direction:

(*i*) Initial velocity $u_x = u$

(*ii*) Acceleration = 0

(*iii*) Horizontal velocity of particle remains same after time t, horizontal velocity = $v_x = u$

(*iv*) Range $x = ut$

In Vertical Direction:

(*i*) Initial velocity $u_y = 0$

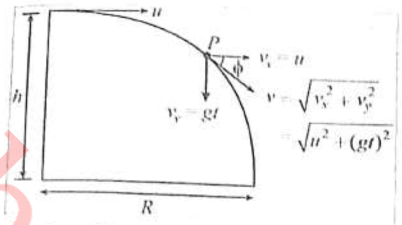
(*ii*) Acceleration = 'g' downwards

(*iii*) Velocity of particle after time $t = gt$ (downwards)

(*iv*) Displacement $y = (1/2)\,gt^2$ (downwards)

Velocity at a general point $P(x, y)$:

$$v = \sqrt{v_x^2 + v_y^2} \qquad \tan\phi = \left|\frac{v_y}{v_x}\right|$$



ϕ is angle made by v with horizontal in clockwise direction

Time of flight:

$$-h = u_y t - (1/2) gt^2$$

$$= 0 - \frac{1}{2} gt^2$$

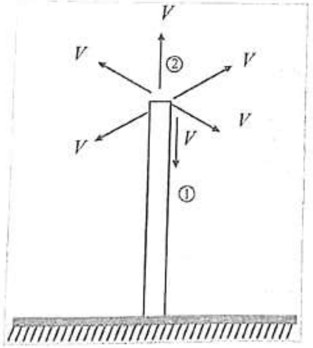
$$\Rightarrow t = \sqrt{\frac{2h}{g}}$$

Range:

$$R = u_x t = u \sqrt{\frac{2h}{g}}.$$

Note: If a projectile is projected with initial velocity u horizontally and another particle is dropped from same height at the same time, both the projectiles would strike the ground at the same instant and both will have same vertical components of velocity, but their net velocities would be different.

Relative motion of one projectile thrown horizontally from some height w.r.t. that of a particle dropped from same height at the same time would be in a horizontal straight line joining them.



All the particles thrown with same initial velocity from same height would strike the ground with same speed at different times irrespective of the direction of their initial velocities.

(*a*) Time would be least for the particle thrown with velocity V downwards i.e. particle (1).

(*b*) Time would be maximum for the particle thrown with velocity V vertically upwards i.e. particle (2).

Train Your Brain

Example 29: A bomb is dropped from an aeroplane flying horizontally with a velocity of 720 km/h at an altitude of 980 m. After what time, the bomb will hit the ground?

(*a*) 5 s	(*b*) $5\sqrt{2}$ s
(*c*) $10\sqrt{2}$ s	(*d*) 10 s

Sol. $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 980}{9.8}}$ s $= 10\sqrt{2}$ sec

Example 30: An aeroplane is in a level flight with a speed of 144 km/hr at an altitude of 1000 m. How far (horizontally) from a given target should a bomb be released so that it hits the target?

(*a*) 565.6 m	(*b*) 671.43 m
(*c*) 471.34 m	(*d*) 371.34 m

Sol. $y = (1/2)\, gt^2$ (downwards)

$\Rightarrow$ $1000 = (1/2) \times 10 \times t^2$

$\Rightarrow$ $t = 14.14$ sec

$$x = ut = \left(\frac{144 \times 5}{18}\right) \times 14.15 = 565.6 \text{ m}$$

Example 31: An aeroplane is flying in horizontal direction with a velocity 600 km/hr and at a height of 1960 m. When it is vertically above a point A on the ground, a body is dropped from it. The body strikes the ground at point B. The distance AB equals to

(*a*) 3.333 km	(*b*) 33.33 km
(*c*) 333.3 km	(*d*) 33.33 m

Sol. The velocity of plane in horizontal direction

$$= 600 \times \frac{5}{18}$$

$= (500/3)$ m/s

Initial velocity of body in vertical direction, $u_y = 0$

If t is time taken by the body to reach the earth, then from relation

$s = ut + (1/2)\, at^2$

we have

$h = (1/2)\, gt^2$

$$\Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20 \text{ sec}$$

$\therefore$ Distance traversed by body in horizontal direction.

$AB = v_x t$

$\Rightarrow$ $(500/3) \times 20 = 3.333$ km

Example 32: A horizontal stream of water leaves an opening on the side of a tank. If the opening is h metre above the ground and the stream hits the ground D metre away, then what is the speed of water as it leaves the tank in terms of g, h and D?

(a) $D\sqrt{\frac{2h}{g}}$ (b) $D\sqrt{\frac{g}{2h}}$

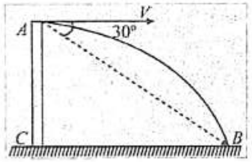
(c) $\sqrt{2gDh}$ (d) $\sqrt{gDh}$

Sol. The given problem is the problem of horizontal projectile. The stream of water shall follow the parabolic path.

Now, $t = \sqrt{\frac{2h}{g}}$; $v = \frac{D}{t} = D\sqrt{\frac{g}{2h}}$.

Concept Application

48. An object is thrown horizontally from a point 'A' from a tower and hits the ground 3s later at B. The line from 'A' to 'B' makes an angle of 30° with the horizontal. The initial velocity of the object is:
(take g = 10 m/s²)



(a) $15\sqrt{3}$ m/s (b) 15 m/s

(c) $10\sqrt{3}$ m/s (d) $25\sqrt{3}$ m/s

49. A body is projected horizontally from the top of a tower with initial velocity 18 ms^{-1}. It hits the ground at an angle of 45°. What is the vertical component of velocity when it strikes the ground?

(a) $18\sqrt{2}$ ms^{-1} (b) 18 ms^{-1}

(c) $9\sqrt{2}$ ms^{-1} (d) 9 ms^{-1}

50. A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find :
(take g = 9.8 m/s²)
(i) The time taken to reach the ground
(ii) The distance of the target from the hill
(iii) The velocity with which the particle hits the ground

PROJECTED FROM SOME HEIGHT AT SOME ANGLE

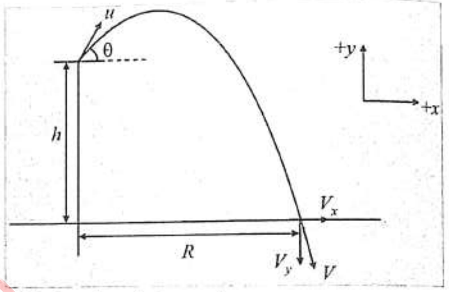
Case-I: When projected at some angle θ with the horizontal towards upward direction.

Let the time taken to strike the ground (time of flight) is t_1.

So, $\Delta y = -h$, $t = t_1$, $u_y = u \sin\theta$

$\therefore -h = (u \sin\theta)\, t_1 - \frac{1}{2} g t_1^2$

$\Rightarrow t_1^2 - \left(\frac{2u\sin\theta}{g}\right)t_1 - \frac{2h}{g} = 0$



Solution of quadratic equation

$\therefore t_1 = \frac{T + \sqrt{T^2 + 8h/g}}{2}$

$\left(\text{where } T = \frac{2u\sin\theta}{g}\right)$

Also $R = \Delta x = u_x t_1$ (putting values of t_1 and u_x we can find R whenever required).

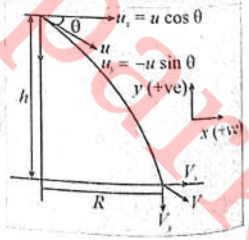
When it reaches ground $v_x = u\cos\theta$

and $v_y^2 = u_y^2 - 2g\,(\Delta y)$

$\Rightarrow v_y = \sqrt{u^2\sin^2\theta - 2g(-h)} = \sqrt{u^2\sin^2\theta + 2gh}$

$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$

Case-II: When projected at angle θ with horizontal towards downward direction



Here $u_y = -u\sin\theta$

Thus, if it takes time t_2 to strike the ground then

$-h = -(u\sin\theta)\, t_2 - \frac{1}{2} g\, t_2^2$

$\Rightarrow t_2^2 + \left(\frac{2u\sin\theta}{g}\right)t_2 - h = 0$

$\therefore t_2 = \frac{\sqrt{T^2 + 8h/g} - T}{2}$

Where $T = \frac{2u\sin\theta}{g}$

Also $R = u_x t_2$

Here on reaching ground, $v_x = u\cos 0$

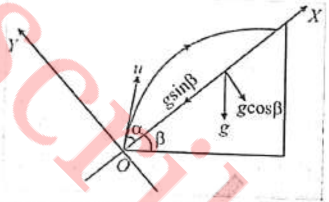
and $v_y = \sqrt{(u\sin 0)^2 - 2g(-h)}$ $\left[\text{Using } v_y^2 = u_y^2 + 2a_y(\Delta y)\right.$

$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$ $\left. a_y = -g;\ \Delta y = -h\right]$

PROJECTION ON AN INCLINED PLANE

Case-I: Particle is projected up the incline

Here α is angle of projection w.r.t. the inclined plane.



X and Y axis are taken along and perpendicular to the incline as shown in the diagram.

In this case: $a_x = -g\sin\beta$, $u_x = u\cos\alpha$

$a_y = -g\cos\beta$, $u_y = u\sin\alpha$

Time of flight (T): When the particle strikes the inclined plane, y becomes zero

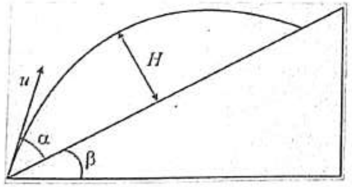
$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2$$

$$\Rightarrow T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

where $u_\perp$ and $g_\perp$ are component of u and g perpendicular to the incline.

Maximum distance from inclined plane (H):



When half of the time is elapsed y-coordinate is equal to maximum distance from the inclined plane of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2} g\cos\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u_\perp^2}{2g_\perp}$$

Range along the inclined plane (R):

When the particle strikes the inclined plane, x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2} g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \boxed{R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}}$$

Maximum Range on the Incline:

$$R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta} = \frac{u^2[\sin(2\alpha+\beta) - \sin\beta]}{g\cos^2\beta}$$

Range will be maximum if,

$\sin(2\alpha + \beta) = 1$

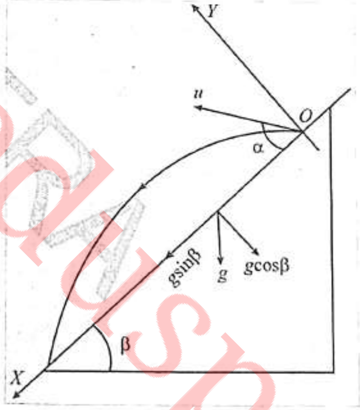
$$2\alpha + \beta = \frac{\pi}{2}$$

$$\alpha = \frac{1}{2}\left(\frac{\pi}{2} - \beta\right) = \frac{\pi}{4} - \frac{\beta}{2}$$

$$\text{Max. Range} = \frac{u^2[1-\sin\beta]}{g\cos^2\beta}$$

$$= \frac{u^2[1-\sin\beta]}{g(1-\sin^2\beta)} = \frac{u^2}{g[1+\sin\beta]}$$

Case-II: Particle is projected down the incline



In this case:

$a_x = g\sin\beta$ $u_x = u\cos\alpha$

$a_y = -g\cos\beta$ $u_y = u\sin\alpha$

Time of flight (*T*):

When the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2$$

$$\Rightarrow T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_y}{g_y}$$

Maximum distance (H):

When half of the time is elapsed y coordinate is equal to maximum distance of the projectile from the plane.

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\cos\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u_y^2}{2g_y}$$

Range along the inclined plane (R):

When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2}a_x t^2$$

$$\Rightarrow R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) + \frac{1}{2}g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\boxed{R = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}}$$

$$= \frac{u^2[\sin(2\alpha-\beta)+\sin\beta]}{g\cos^2\beta}$$

Range will be maximum

When $\sin(2\alpha-\beta) = 1$

$$2\alpha - \beta = \frac{\pi}{2}$$

$$\alpha = \frac{1}{2}\left(\frac{\pi}{2}+\beta\right) = \frac{\pi}{4} + \frac{\beta}{2}$$

$$\text{Maximum Range} = \frac{u^2[1+\sin\beta]}{g\cos^2\beta}$$

$$= \frac{u^2[1+\sin\beta]}{g(1-\sin\beta)(1+\sin\beta)} = \frac{u^2}{g(1-\sin\beta)}$$

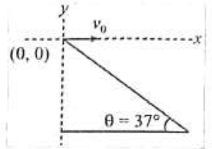
Table: Standard results for projectile motion on an inclined plane

	Up the Incline	Down the Incline
Range	$\frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$	$\frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$
Time of flight	$\frac{2u\sin\alpha}{g\cos\beta}$	$\frac{2u\sin\alpha}{g\cos\beta}$
Angle of projection for maximum range	$\frac{\pi}{4}-\frac{\beta}{2}$	$\frac{\pi}{4}+\frac{\beta}{2}$
Maximum Range	$\frac{u^2}{g(1+\sin\beta)}$	$\frac{u^2}{g(1-\sin\beta)}$

Here α is the angle of projection with the incline and β is the angle of incline from horizontal.

Train Your Brain

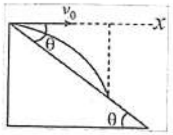
Example 33: A man standing on a hill top projects a stone horizontally with speed $v_0 = 4\sqrt{5}\ \text{ms}^{-1}$ as shown in figure. Taking the co-ordinate system as given in the figure. Find the co-ordinates of the point where the stone will hit the hill surface.



(*a*) (10, –10)
(*b*) (12, –9)
(*c*) (9, –12)
(*d*) (4, –3)

Sol. Range of the projectile on an inclined plane (down the plane) is,

$$R = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$$



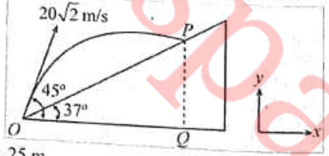
Here, $u = v_0$, $\alpha = \theta$ and $\beta = \theta$

$$\therefore R = \frac{2v_0^2\sin\theta}{g\cos^2\theta}$$

Now, $x = R\cos\theta = \frac{2v_0^2\tan\theta}{g} = \frac{2\times16\times5\times3}{10\times4} = 12\ \text{m}$

and $y = -R\sin\theta = -\frac{2v_0^2\tan^2\theta}{g} = \frac{-2\times16\times5\times9}{10\times16} = -9\ \text{m}$

Example 34: In the figure shown, find time of flight of the projectile along the inclined plane and range OP.



(*a*) 1*s*, 25 m
(*b*) 2*s*, 25 m
(*c*) 7*s*, 175 m
(*d*) 7*s*, 50 m

Sol. Horizontal component of initial velocity,

$$u_x = 20\sqrt{2}\cos45^\circ = 20\ \text{m/s}$$

Vertical component of initial velocity

$$u_y = 20\sqrt{2}\sin45^\circ = 20\ \text{m/s}$$

Let the particle strikes at P after time t, then horizontal displacement

$OQ = u_x t = 20t$

Displacement in vertical direction,

$QP = 20t - 5t^2$

In ΔOPQ,

$$\tan 37^\circ = \frac{QP}{OQ}$$

$$\text{or } \frac{3}{4} = \frac{20t - 5t^2}{20t}$$

Solving this equation, we get

$t = 1s$

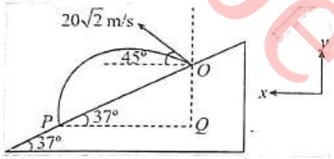
Range $OP = OQ \sec 37^\circ$

$$= (20t)\left(\frac{5}{4}\right)$$

Substituting $t = 1s$, we have

$OP = 25$ m

Example 35: In the shown figure, find time of flight of the projectile along the inclined plane and range OP.



(a) 1s, 25 m (b) 2s, 50 m

(c) 7s, 175 m (d) 7s, 50 m

Sol. Horizontal component of initial velocity,

$$u_x = 20\sqrt{2}\cos 45^\circ = 20 \text{ m/s}$$

Vertical component of initial velocity,

$$u_y = 20\sqrt{2}\sin 45^\circ = 20 \text{ m / s}$$

Let the particle strikes the inclined plane at P after time t, then horizontal displacement

$QP = u_x t = 20t$

Displacement in vertical direction,

$OQ = 5t^2 - 20t$

$$\text{In } \Delta OQP,\ \tan 37^\circ = \frac{OQ}{QP}$$

$$\frac{3}{4} = \frac{5t^2 - 20t}{20t}$$

Solving this equation, we get

$t = 7$s

Range, $OP = (PQ) \sec 37^\circ$

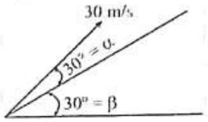
$$= (20t)\left(\frac{5}{4}\right)$$

Substituting the value of t, we get

$OP = 175$ m

Concept Application

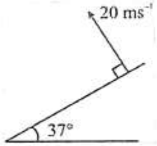
51. Find the range of the projectile along the inclined plane as shown in figure. ($g = 10$ m/s²)



(a) 120 m (b) 40 m

(c) 60 m (d) 200 m

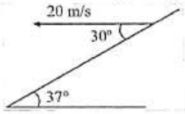
52. Find time of flight and range of the projectile along the inclined plane as shown in figure. ($g = 10$ m/s²)



(a) 5s, 45 m (b) 10s, 90 m

(c) 2.5s, 20 m (d) 15s, 150 m

53. Find time of flight and range of the projectile along the inclined plane as shown in figure. ($g = 10$ m/s²)



(a) 2s, 5 m (b) 10s, 60 m

(c) 2.5s, 12.5 m (d) 1.25s, 6.25 m

RELATIVE MOTION

A person travelling by train may ask his co-passenger "Has delhi come?". This statement seems to claim that Delhi is moving towards the passenger. We may say that his understanding of word is wrong as all of us know that 'actually' it is the train which is moving not the huge city Delhi.

If we observe from passenger's point of view we will realise relative to him all parts of train are at rest, which means relative to him train is at rest. Still when passenger gets down from train he is in a city, thus it must be the cities which are moving.

The point is that when a person standing on ground says that 'actually' it is the train (hence passengers sitting in train) which is moving and reaching to Delhi (which of course is stationary), he is also right. The difference here is of reference frame.

Reference Frame

Reference frame is an axis system from which motion is observed. A clock is attached to measure time. Reference frame can be stationary or moving. There are two types of reference frames: Inertial and non-inertial. the set of all frames which move at

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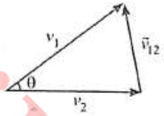
constant velocity with respect to each other are called as inertial frames. The frames which are accelerated with respect to inertial frame are non-inertial frames.

If the velocities of two bodies are known with respect to a common frame of reference, velocity of one body relative to other can be determined. Thus, if velocities of two bodies P and Q with respect to earth are $\vec{v}_P$ and $\vec{v}_Q$ respectively, then the velocity of Q relative to P is expressed as follows: $\vec{v}_{QP} = \vec{v}_Q - \vec{v}_P$

If two bodies have velocities $\vec{v}_1$ and $\vec{v}_2$ then the relative velocity of first body with respect to second body is

$$\vec{v}_{rel} = \vec{v}_{12} = \vec{v}_1 - \vec{v}_2 = \vec{v}_1 + (-\vec{v}_2)$$

$$v_{rel} = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$$

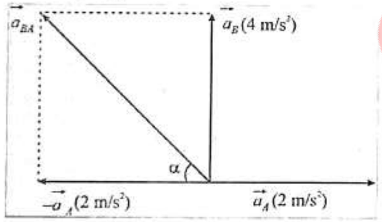


Train Your Brain

Example 36: Car A has an acceleration of 2 m/s² due east and car B, 4 m/s² due north. What is the acceleration of car B with respect to car A?

Sol. It is a two dimensional motion. Therefore,

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$



Here, $\vec{a}_B$ = acceleration of car B : 4 m/s² (due north)

and $\vec{a}_A$ = acceleration of car A : 2 m/s² (due east)

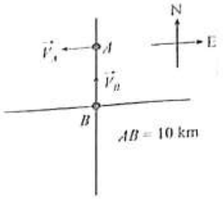
$$|\vec{a}_{BA}| = \sqrt{(4)^2 + (2)^2} = 2\sqrt{5} \text{ m/s}^2$$

$$\text{and } \alpha = \tan^{-1}\left(\frac{4}{2}\right) = \tan^{-1}(2)$$

Thus, $\vec{a}_{BA}$ is $2\sqrt{5}$ m/s² at an angle of $\alpha = \tan^{-1}(2)$ from west towards north.

Example 37: Two ships A and B are 10 km apart on a line running south to north. Ship A farther north is streaming west at 20 km/h and ship B is streaming north at 20km/h. What is their distance of closest approach and how long do they take to reach it?

Sol. Ships A and B are moving with same speed 20 km/h in the directions shown in figure. It is a two dimensional, two body problem with zero acceleration. Let us find $\vec{V}_{BA}$.

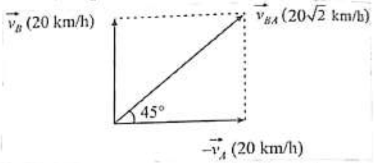


$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

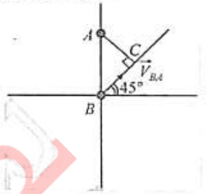
$$\text{Here, } |\vec{V}_{BA}| = \sqrt{(20)^2 + (20)^2}$$

$$= 20\sqrt{2} \text{ km/h}$$

i.e., $\vec{V}_{BA}$ is $20\sqrt{2}$ km/h at an angle of 45° from east towards north. Thus, the given problem can be simplified as:



In the frame of ship A, A is at rest and B is moving with $\vec{V}_{BA}$ in the direction shown in figure.



Therefore, the minimum distance between the two ships is

$$S_{\min} = AC = AB \sin 45°$$

$$= 10\left(\frac{1}{\sqrt{2}}\right) \text{km} = 5\sqrt{2} \text{ km}$$

and the time taken is

$$t = \frac{BC}{|\vec{V}_{BA}|} = \frac{5\sqrt{2}}{20\sqrt{2}} h \qquad (BC = AC = 5\sqrt{2} \text{ km})$$

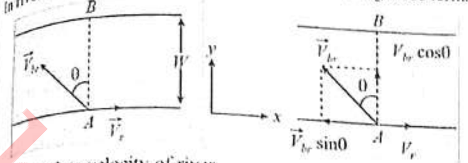
$$\Rightarrow t = \frac{1}{4} h = 15 \text{ min}$$

Concept Application

54. A particle is kept at rest at origin. Another particle starts from (5 m, 0) with a velocity of $-4\hat{i} + 3\hat{j}$ m/s. Find their closest distance of approach.

BOAT-RIVER PROBLEM

In river-boat problems we come across the following three terms:



$\vec{v}_r$ = absolute velocity of river

$\vec{v}_{br}$ = velocity of boatman with respect to river or velocity of boatman in still water.

$\vec{v}_b$ = absolute velocity of boatman.

Here, it is important to note that $\vec{V}_{br}$ is the velocity of boatman with which he steers and $\vec{V}_b$ is the actual velocity of boatman relative to ground.

Further, $\vec{v}_b = \vec{v}_{br} + \vec{v}_r$

Now, let us derive some standard results and their special cases.

A boatman starts from point A on one bank of a river with velocity $\vec{v}_{br}$ in the direction shown in figure. River is flowing along positive x-direction with velocity $\vec{v}_r$. Width of the river is W, then

$$\vec{v}_b = \vec{v}_{br} + \vec{v}_r$$

Therefore, $v_{bx} = v_{bx} + v_{brx} = v_r - v_{br}\sin\theta$

and $v_{by} = v_{ry} + v_{bry}$

$$= 0 + v_{br}\cos\theta = v_{br}\cos\theta$$

Now, time taken by the boatman to cross the river is

$$t = \frac{W}{v_{by}}$$

$$\text{or } t = \frac{W}{v_{br}\cos\theta}$$

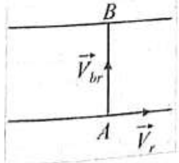
Further, displacement along X-axis when he reaches on the other bank (also called drift) is:

$x = V_{bx}t$

$$\text{or } x = (v_r - v_{br}\sin\theta)\frac{W}{v_{br}\cos\theta}$$

Three special cases are:

(*i*) When the boatman crosses the river in shortest interval of time.



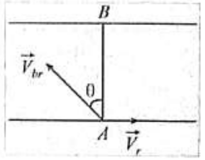
From Eq.(i) we can see that time (t) will be minimum when $\theta = 0°$, i.e., the boatman should steer his boat perpendicular to the river current.

Also, $t_{min} = \frac{W}{v_{br}}$ as $\cos\theta = 1$

and $x = v_r\frac{W}{v_{br}}$

(*ii*) When the boatman wants to reach point B, i.e., at a point just opposite from where he started.

In this case, the drift (x) should be zero.



$\therefore x = 0$

$$\text{or } (v_r - v_{br}\sin\theta)\frac{W}{v_{br}\cos\theta} = 0$$

or $v_r = v_{br}\sin\theta$

$$\text{or } \sin\theta = \frac{v_r}{v_{br}} \text{ or } \theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$$

Hence, to reach point B the boatman should row at an angle

$\theta = \sin^{-1}\left(\frac{V_r}{V_{br}}\right)$ upstream from AB.

Time 't' taken to cross the river in this case is

$$t = \frac{W}{v_{br}\cos\theta} = \frac{W}{v_{br}\sqrt{1-\sin^2\theta}} = \frac{W}{v_{br}\sqrt{1-\frac{v_r^2}{v_{br}^2}}} = \frac{W}{\sqrt{v_{br}^2 - v_r^2}}$$

Further, since sin θ can not be greater than 1.

So, if $v_r \geq v_{br}$, the boatman can never reach at point B. It can be realized practically that it is not possible to reach at B if river velocity (v_r) is too high.

(*iii*) **Shortest path:** Path length travelled by the boatman when he reaches the opposite shore is

$s = \sqrt{W^2 + x^2}$

Here, width of river (W) is constant. So for s to be minimum x (drift) should be minimum. Now two cases are possible.

Case-I: When $v_r < v_{br}$: In this case $x = 0$,

when $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ or $S_{min} = W$ at $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$

Case-II: When $v_r > v_{br}$: In this case x is minimum when

$$\frac{dx}{d\theta} = 0$$

$$\text{or } \frac{d}{d\theta}\left\{\frac{W}{v_{br}\cos\theta}(v_r - v_{br}\sin\theta)\right\} = 0$$

$$\text{or } -v_{br}\cos^2\theta - (v_r - v_{br}\sin\theta)(-\sin\theta) = 0$$

or $v_{br} + v_r \sin\theta = 0$

Or

$\theta = \sin^{-1}\left(\frac{v_{br}}{v_r}\right)$

Now, at this angle we can find v_{min} and then s_{min} which comes out to be

$s_{min} = d\left(\frac{v_r}{v_{br}}\right)$ at $\theta = \sin^{-1}\left(\frac{v_{br}}{v_r}\right)$

- If a man swims downstream in a river, then the time taken by the man to cover a distance d is $t_1 = \frac{d}{v_{br} + v_r}$

If a man swims upstream in a river, then the time taken by him to cover a distance d is

$t_2 = \frac{d}{v_{br} - v_r}$ So, $\frac{t_1}{t_2} = \frac{v_{br} - v_r}{v_{br} + v_r}$

Train Your Brain

Example 38: A 400 m wide river is flowing at a rate of 2.0 m/s. A boat is sailing with a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river.

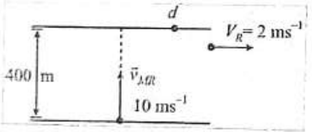
(*a*) Find the time taken by the boat to reach the opposite bank.

(*b*) How far from the point directly opposite to the starting point does the boat reach the opposite bank?

(*c*) In what direction does the boat actually move w.r.t. direction of river flow?

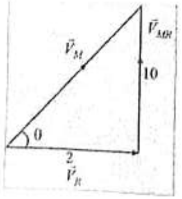
(*d*) If he can walk at 5 ms⁻¹ find the total time he will take to come directly opposite to the starting point.

Sol.



$\vec{v}_{MR}$ is velocity of man w.r.t. river i.e. in still river. So in ground reference frame

$|\vec{v}_M| = \sqrt{10^2 + 2^2} = \sqrt{104}$ m/s



(*a*) $\text{time} = \frac{400}{10} = 40$ sec

(*b*) drift $= 2 \times 40 = 80$ m

(*c*) $\tan\theta = \frac{10}{2} = 5$

$\theta = \tan^{-1}(5)$ w.r.t. river flow.

(*d*) Time taken to come to directly opposite end

$= 40 + \frac{80}{5} = 40 + 16 = 56$ s

Example 39: A man can swim at the rate of 5 km/h in still water. A 1 km wide river flows at the rate of 3 km/h. The man wishes to swim across the river directly opposite to the starting point.

(*a*) Along what direction must the man swim?

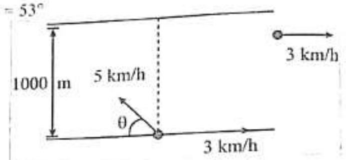
(*b*) What would be his resultant velocity (w.r.t. earth)?

(*c*) How much time will he take to cross the river?

Sol. $5\cos\theta = 3$

$\cos\theta = 3/5$

$\theta = 53°$



(*a*) so 180° – 53° = 127° w.r.t. river flow

(*b*) $5\sin\theta = 5 \times \frac{4}{5} = 4$ km/h

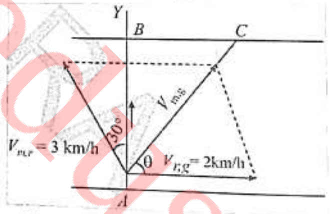
(*c*) $\text{time} = \frac{1\text{ km}}{4\text{ km/h}} = \frac{1}{4} \times 60\text{ min} = 15\text{ min}$

Example 40: A man can swim at a speed of 3 km/h in still water. He wants to cross a 500 m wide river flowing at 2 km/h. He keeps himself always at an angle of 120° with the river flow while swimming.

(*a*) Find the time he takes to cross the river.

(*b*) At what point on the opposite bank will he arrive?

Sol. The situation is shown in figure



Here $\vec{V}_{r,g}$ = velocity of the river with respect to the ground

$\vec{V}_{m,r}$ = velocity of the man with respect to the river

$\vec{V}_{m,g}$ = velocity of the man with respect to the ground.

(*a*) We have

$\vec{V}_{m,g} = \vec{V}_{m,r} + \vec{V}_{r,g}$ ...(i)

Hence, the velocity with respect to the ground is along *AC*.

Taking *y*-components of equation (i),

$V_{m,g}\sin\theta = 3\text{ km/h}\cos 30° + 2\text{ km/h}\cos 90° = \frac{3\sqrt{3}}{2}$ km/h

Time taken to cross the river

$= \frac{\text{displacement along the } Y\text{-axis}}{\text{velocity along the } Y\text{-axis}} = \frac{1/2\text{km}}{3\sqrt{3}/2\text{ km/h}}$

$= \frac{1}{3\sqrt{3}}$ h

(*b*) Taking *x*-components of equation (i),

$v_{r,g}\cos\theta = -3\text{km/h}\sin 30° + 2\text{ km/h} = \frac{1}{2}\text{km/h}$

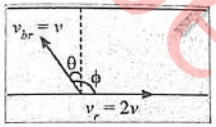
Displacement along the *X*-axis as the man crosses the river = (velocity along the *X*-axis) × (time)

$= \left(\frac{1\text{km}}{2h}\right)\times\left(\frac{1}{3\sqrt{3}}h\right) = \frac{1}{6\sqrt{3}}\text{ km}$

Example 41: A boat moves relative to water with a velocity *v* and river is flowing with 2*v*. At what angle the boat shall move with respect to the stream to have minimum drift?

(*a*) 30°
(*b*) 60°
(*c*) 90°
(*d*) 120°

Sol. Let boat move at angle θ to the normal as shown in figure, then time to cross the river $= \frac{1}{v\cos\theta}$, drift $x = (2v - v\sin\theta)\frac{1}{v\cos\theta}$



For *x* to be minimum, $\frac{dx}{d\theta} = 0$

$\Rightarrow 2\sec\theta\tan\theta - \sec^2\theta = 0$

or $\sin\theta = 1/2$ or $\theta = 30°$ and $\phi = 90° + 30° = 120°$

Concept Application

55. A boat, which has a speed of 5 km/h in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/h is can be

(*a*) 1
(*b*) 3
(*c*) 4
(*d*) 2

56. A boat can go across a lake and return in time T_0 at a speed *V*. On a rough day there is uniform current at speed *v* to help the onward journey and impede the return journey. If the time taken to go across and return on the rough day be *T*, then $\left(\frac{T}{T_0}\right)^2$ is

(*a*) $1 - \frac{v^2}{V^2}$
(*b*) $\frac{1}{1 - \frac{v^2}{V^2}}$
(*c*) $1 + \frac{v^2}{V^2}$
(*d*) $\frac{1}{1 + \frac{v^2}{V^2}}$

RAIN MAN PROBLEMS

In these type of problems we again come across three terms $\vec{v}_r$, $\vec{v}_m$ and $\vec{v}_{rm}$. Here,

$\vec{v}_r$ = velocity of rain

$\vec{v}_m$ = velocity of man (it may be velocity of cyclist or velocity of motorist also)

and $\vec{v}_{rm}$ = velocity of rain with respect to man.

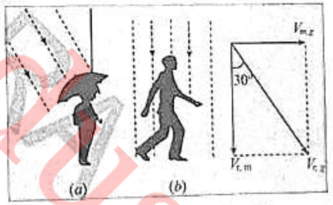
Following is the relation between the three quantities,

$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$

Train Your Brain

Example 42: A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that raindrops are hitting his head vertically. Find the speed of raindrops with respect to (*a*) the road, (*b*) the moving man.

Sol. When the man is at rest with respect to the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground. The situation when the man runs is shown in the figure (*b*).



Here $\vec{v}_{r,g}$ = velocity of the rain with respect to the ground

$\vec{v}_{m,g}$ = velocity of the man with respect to the ground

and $\vec{v}_{r,m}$ = velocity of the rain with respect to the man.

We have, $\vec{v}_{r,g} = \vec{v}_{r,m} + \vec{v}_{m,g}$ ...(i)

Taking horizontal components, equation (i) gives

$v_{r,g}\sin 30° = V_{m,g} = 10\text{ km/h}$

or, $V_{r,g} = \frac{10\text{ km/h}}{\sin 30°} = 20\text{ km/h}$

Taking vertical components, equation (i) gives

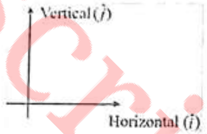
$v_{r,g}\cos 30° = v_{r,m}$ or, $v_{r,m} = (20\text{ km/h})\frac{\sqrt{3}}{2} = 10\sqrt{3}\text{ km/h}$

Example 43: To a man, walking at the rate of 3 km/h, the rain appears to fall vertically. When he increases his speed to 6 km/h it appears to meet him at an angle of 45° with vertical. Find the speed of rain.

Sol. Let $\hat{i}$ and $\hat{j}$ be the unit vectors in horizontal and vertical directions respectively.

Let velocity of rain

$$\vec{v}_r = a\hat{i} + b\hat{j} \qquad \text{...(i)}$$



Then speed of rain will be

$$|\vec{v}_r| = \sqrt{a^2 + b^2} \qquad \text{...(ii)}$$

In the first case $\vec{v}_m$ = velocity of man = $3\hat{i}$

$\therefore \quad \vec{v}_{rm} = \vec{v}_r - \vec{v}_m = (a-3)\hat{i} + b\hat{j}$

It seems to be in vertical direction. Hence,

$a - 3 = 0$ or $a = 3$

In the second case $\vec{v}_m = 6\hat{i}$

$\therefore \quad \vec{v}_{rm} = (a-6)\hat{i} + b\hat{j} = -3\hat{i} + b\hat{j}$

This seems to be at 45° with vertical.

Hence, $|b| = 3$

Therefore, from Eq. (ii) speed of rain is

$$|\vec{v}_r| = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2} \text{ km/h}$$

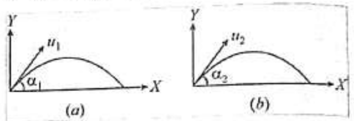
Concept Application

57. It is raining vertically downwards with a velocity of 3 km h^{-1}. A man walks in the rain with a velocity of 4 kmh^{-1}. The rain drops will fall on the man with a relative velocity of:
    (*a*) 1 kmh^{-1} (*b*) 3 kmh^{-1}
    (*c*) 4 kmh^{-1} (*d*) 5 kmh^{-1}
58. A man walks in rain with a velocity of 5 kmh^{-1}. The rain drops strike at him at an angle of 45° with the horizontal. Velocity of rain if it is falling vertically downward
    (*a*) 5 kmh^{-1} (*b*) 4 kmh^{-1}
    (*c*) 3 kmh^{-1} (*d*) 1 kmh^{-1}
59. Raindrops are falling vertically with a velocity of 10 m/s. To a cyclist moving on a straight road the raindrops appear to be coming with a velocity of 20 m/s. The velocity of cyclist is
    (*a*) 10 m/s (*b*) $10\sqrt{3}$ m/s
    (*c*) 20 m/s (*d*) $20\sqrt{3}$ m/s

RELATIVE MOTION BETWEEN TWO PROJECTILES

Let us now discuss the relative motion between two projectiles or the path of one projectile as observed by the other. Suppose that two particles are projected from the ground with speeds u_1 and u_2 at angles α_1 and α_2 as shown in figure (*a*) and (*b*). Acceleration of both the particles is g downwards. So, relative acceleration between them is zero because

$$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2 = (-g) - (-g) = 0$$



So, the relative motion between the two projectiles is uniform and the path of one projectile as seen by other will be a straight line.

Now,

$u_{1x} = u_1 \cos\alpha_1$ and $u_{2x} = u_2 \cos\alpha_2$

$u_{1y} = u_1 \sin\alpha_1$ and $u_{2y} = u_2 \sin\alpha_2$

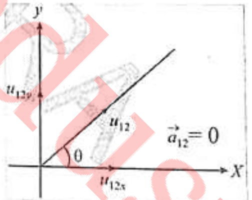
Therefore, $u_{12x} = u_{1x} - u_{2x} = u_1 \cos\alpha_1 - u_2 \cos\alpha_2$

and $u_{12y} = u_{1y} - u_{2y} = u_1 \sin\alpha_1 - u_2 \sin\alpha_2$

u_{12x} and u_{12y} are the x and y components of initial relative velocities of 1 with respect to 2.

Hence, relative motion of 1 with respect to 2 is a straight line.

at an angle $\theta = \tan^{-1}\left(\dfrac{u_{12y}}{u_{12x}}\right)$ with positive x-axis.



Now, if $u_{12x} = 0$ or $u_1 \cos\alpha_1 = u_2 \cos\alpha_2$, the relative motion is along y-axis or in vertical direction (as $\theta = 90°$). Similarly, if $u_{12y} = 0$ or $u_1 \sin\alpha_1 = u_2 \sin\alpha_2$, the relative motion is along x-axis or in horizontal direction (as $\theta = 0°$).

Note: Since, relative acceleration between two projectiles is zero and relative motion between them is uniform, the condition of collision of two particles in air is that relative velocity of one with respect to the other should be along line joining them, i.e., if two projecticles A and B collide in mid air, then $\vec{V}_{AB}$ should be along $\overrightarrow{AB}$ or $\vec{V}_{BA}$ along $\overrightarrow{BA}$.

Condition for collision of two projectiles

Consider the situation shown in the figure. For projectiles to collide, direction of velocity of A with respect to B has to be along line AB.

---

h

Here, v_{AB}

$v_{ABy} = u_1$

Let, dire

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Sol. $\vec{v}_B$

v_{se}

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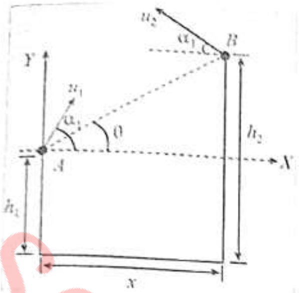
Motion



Here, $v_{(B)x} = u_1 \cos\alpha_1 + u_2 \cos\alpha_2$

$v_{(B)y} = u_1 \sin\alpha_1 - u_2 \sin\alpha_2$

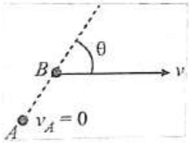
Let direction of velocity vector of A(wrt B) is making an angle β with −ve X-axis, which is given by

$$\tan\beta = \frac{v_{(B)y}}{v_{(B)x}} = \frac{u_1 \sin\alpha_1 - u_2 \sin\alpha_2}{u_1 \cos\alpha_1 + u_2 \cos\alpha_2}$$

For collision to take place, $\tan\beta = \tan\theta = \frac{h_2 - h_1}{x}$

Train Your Brain

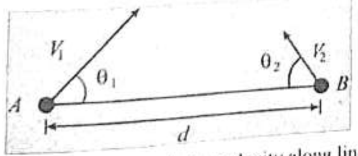
Example 44: Particle A is at rest and particle B is moving with constant velocity v as shown in the diagram at $t = 0$. Find their velocity of separation.



Sol. $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

v_{sep} = component of v_{BA} along line $AB = v\cos\theta$

Example 45: Two particles A and B are moving with constant velocities v_1 and v_2. At $t = 0$, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B. Find their velocity of approach.



Sol. Velocity of approach is relative velocity along line AB

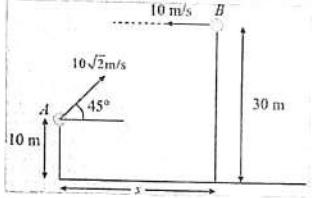
$v_{approach} = v_1\cos\theta_1 + v_2\cos\theta_2$

Concept Application

60. For two particles A and B, given that $\vec{r}_A = 2\hat{i} + 3\hat{j}$, $\vec{r}_B = 6\hat{i} + 7\hat{j}$, $\vec{v}_A = -3\hat{i} - \hat{j}$ and $\vec{v}_B = x\hat{i} - 5\hat{j}$. What is the value of x if they collide?

(a) 1 (b) −1 (c) 2 (d) −2

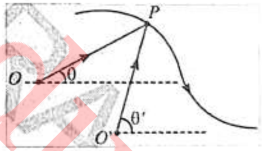
61. Two projectiles are projected simultaneously from two towers as shown in figure. If the projectiles collide in the air, then find the distance "s" between the towers.



KINEMATICS OF CIRCULAR MOTION

Angular Position

Consider a particle moving on any arbitrary path. The angle made by position vector from some origin with reference to some line is called the angular position of the particle. It depends on choice of origin as well as that of reference line

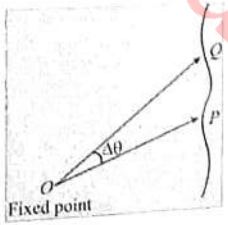


θ : angular position w.r.t. O.

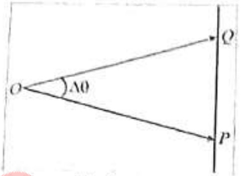
θ' : angular position w.r.t. O'.

Angular Displacement

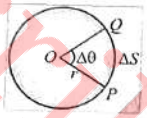
Difference between the two angular positions of the particle moving along any arbitrary path w.r.t. some fixed point is called angular displacement. It does not depend of the choice of reference line.



(a) Particle moving in an arbitrary path



(b) Particle moving in straight line



(c) Particle moving in circular path

(i) Finite angular displacement is not a vector but infinitesimaly small angular displacement is a vector quantity.

(ii) Its direction is perpendicular to plane of rotation and given by right hand screw rule.

Note: Clockwise angular displacement is taken as negative and anticlockwise displacement as positive.

(iii) For circular motion $\Delta S = r \times \Delta\theta$

(iv) Its unit is radian (in M.K.S)

Note: Always change degree into radian, if it occurs in numerical problems.

Note: 1 radian = $\frac{360°}{2\pi} \Rightarrow \pi$ radians = 180°

(v) It is a dimensionless quantity

i.e. dimensions $[M^0L^0T^0]$

Important Points

- It is dimensionless and its unit SI unit is radian while other units are degree or revolutions. 2π rad = 360° = 1 rev.
- Infinitely small angular displacement is a vector quantity but finite angular displacement is not because the addition of the small angular displacements is cummutative while for large angular displacement is not.
- $d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1$ but $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$
- Direction of small angular displacement is decided by right hand thumb rule. When the fingers are directed along the motion of the point then thumb will represents the direction of angular displacement. So, it is an axial vector.
- Angular displacement can be different for different observers.

Angular Velocity

It is defined as the rate of change of angular position of a particle moving in any path.

(i) It is a vector quantity.

(ii) Its direction is same as that of angular displacement i.e. perpendicular to plane of rotation.

Note: If the particle is revolving in the clockwise direction then the direction of angular velocity is perpendicular to the plane and inwards. Whereas in case of anticlockwise direction the direction will be perpendicular to the plane and outwards.

(iii) Its unit is radian/sec

(iv) Its dimensions are $[M^0L^0T^{-1}]$

Average Angular Speed

It is defined as the ratio of total angular displacement and the time taken to do so.

$$\omega_{av} = \frac{\text{Total angular displacement}}{\text{Total time taken}} = \frac{\Delta\theta}{\Delta t}$$

It is a scalar quantity because finite angular displacement is a scalar quantity.

Instantaneous Angular Velocity

The instantaneous angular velocity is the rate of change of angular position at some particular instant.

Instantaneous angular velocity

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} = \vec{\omega} = \frac{d\vec{\theta}}{dt}$$

Note: Instantaneous angular velocity can also be called as simply angular velocity.

Important Points

- It is an axial vector with dimensions $[T^{-1}]$ and SI unit rad/s.
- For a rigid body, since all points will rotate through same angle in same time, angular velocity is a characteristic of the body as a whole, e.g., angular velocity of all points of earth about its own axis is $(2\pi/24)$ rad /hr.
- If a body makes 'n' rotations in 't' seconds then angular velocity in radian per second will be

$$\omega_{av} = \frac{2\pi n}{t}$$

If T is the period and 'f' the frequency of uniform circular motion, then

$$\omega_{av} = \frac{2\pi \times 1}{T} = 2\pi f$$

Relation Between Linear Velocity and Angular Velocity in Circular Motion

We have $\omega = \frac{d\theta}{dt} = \frac{d\theta}{ds} \cdot \frac{ds}{dt} = \frac{1}{r} \cdot v$

$\left[\because d\theta = \frac{ds}{dr}, \text{angle} = \frac{arc}{radius} \text{ and } v = \frac{ds}{dt} = \text{linear velocity}\right]$

$\Rightarrow v = r\omega$

In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$

Note:
(i) When a
velocity
(ii) When a
has two
which
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(iii) ω =

Relative A

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ω_{PO} =

Definition

Relative
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'A' with r
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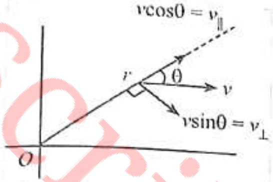
Note:

(i) When a particle moves along a curved path, its linear velocity at a point is along the tangent drawn at that point.

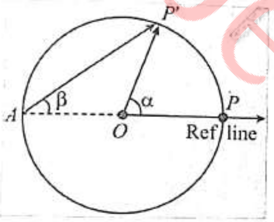
(ii) When a particle moves along curved path, its velocity has two components. One along the position vector, which increases or decreases the distance of particle from origin and another one perpendicular to the position vector, which makes the particle to revolve about the point of observation.

(iii) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{v\sin\theta}{r} = \frac{v_\perp}{r}$



Relative Angular Velocity

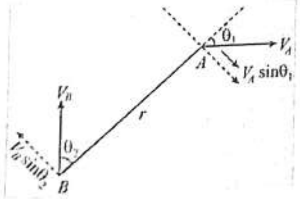
Angular velocity is defined with respect to the point from which the position vector of the moving particle is drawn. Here angular velocity of the particle w.r.t. '*O*' and '*A*' will be different



$$\omega_{PO} = \frac{d\alpha}{dt} = \omega \text{ (say)}; \ \omega_{PA} = \frac{d\beta}{dt} = \frac{\omega}{2} \qquad \left[\because \beta = \frac{\alpha}{2}\right]$$

Definition

Relative angular velocity of a particle '*A*' with respect to the other particle '*B*' is the angular velocity of the position vector of '*A*' with respect to '*B*'. It means that is the rate at which position vector of '*A*' with respect to '*B*' rotates at that instant.



$$\omega_{AB} = \frac{(V_{AB})_\perp}{r_{AB}}$$

here $(V_{AB})_\perp$ = Relative velocity ⊥ to position vector *AB*

$$= \frac{\text{Component relative velocity of } A \text{ w.r.t. } B \text{ perpendicular to line } AB}{\text{Seperation between } A \text{ and } B}$$

$$(V_{AB})_\perp = V_A\sin\theta_1 + V_B\sin\theta_2$$

$$r_{AB} = r$$

$$\omega_{AB} = \frac{V_A\sin\theta_1 + V_B\sin\theta_2}{r}$$

Train Your Brain

Example 46: If θ depends on time *t* in following way θ = 2*t*² + 3, then

(i) Find out average angular speed upto 3 sec.

(ii) angular velocity at 3 sec.

Sol. (i) $\omega_{avg} = \frac{\text{Total angular displacement}}{\text{total time}} = \frac{\theta_f - \theta_i}{t_2 - t_1}$

$$\theta_f = 2(3)^2 + 3 = 21 \text{ rad}$$

$$\theta_i = 2(0) + 3 = 3 \text{ rad}$$

$$\text{So, } \omega_{avg} = \frac{21-3}{3} = 6 \text{ rad/sec}$$

(ii) $\omega_{instantaneous} = \frac{d\theta}{dt} = 4t$

$$\omega_{at\ t = 3\ sec} = 4 \times 3 = 12 \text{ rad/sec}$$

Example 47: A particle revolving in a circular path completes first one third of circumference in 2 sec, while next one third in 1 sec. The average angular speed of particle will be: (in rad/sec)

(*a*) $\frac{2\pi}{3}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{4\pi}{3}$ (*d*) $\frac{5\pi}{3}$

Sol. We have $\omega_{av} = \frac{\text{Total angular displacement}}{\text{Total time}}$

$$= \frac{2\pi}{2+1} = \frac{2\pi}{3} \text{ rad s}^{-1}$$

Example 48: The ratio of angular speeds of minute hand and hour hand of a watch is

(*a*) 1 : 12 (*b*) 6 : 1

(*c*) 12 : 1 (*d*) 1 : 6

Sol. Angular speed of hour hand,

$$\omega_1 = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{12 \times 60} \text{ rad/min}$$

angular speed of minute hand,

$$\omega_2 = \frac{2\pi}{60} \text{ rad/min} \Rightarrow \frac{\omega_2}{\omega_1} = \frac{12}{1}$$

Example 49: The angular displacement of a particle is given by $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, where ω_0 and α are constant and ω_0 = 1 rad/sec, α = 1.5 rad/sec². The angular velocity at time, *t* = 2 sec will be (in rad/sec)

(*a*) 1 (*b*) 5 (*c*) 3 (*d*) 4

Sol. We have $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$\Rightarrow \frac{d\theta}{dt} = \omega_0 + \alpha t$$

This is angular velocity at time t. Now angular velocity at $t = 2$ sec will be

$$\omega = \left(\frac{d\theta}{dt}\right)_{t=2\,sec} = \omega_0 + 2\alpha$$

$$= 1 + 2 \times 1.5 = 4 \text{ rad/sec}$$

Example 50: A particle moves in a circle of radius 20 cm with a linear speed of 10m/s. The angular velocity will be

(*a*) 50 rad/s (*b*) 100 rad/s

(*c*) 25 rad/s (*d*) 75 rad/s

Sol. The angular velocity is

$$\omega = \frac{v}{r}$$

Hence $v = 10$ m/s, $r = 20$ cm $= 0.2$ m.

$\therefore \omega = 50$ rad/s

Example 51: Two particles move on a circular path (one just inside and the other just outside) with angular velocities ω and 5ω starting from the same point. Then, which of the following is incorrect?

(*a*) They cross each other at regular intervals of time $\frac{2\pi}{4\omega}$ when their angular velocities are oppositely directed.

(*b*) They cross each other at points on the path subtending an angle of 60° at the centre if their angular velocities are oppositely directed.

(*c*) They cross at intervals of time $\frac{\pi}{3\omega}$ if their angular velocities are oppositely directed.

(*d*) They cross each other at points on the path subtending 90° at the centre if their angular velocities are in the same sense.

Sol. If the angular velocities are oppositely directed, they meet at intervals of

$$\text{time } t = \frac{2\pi}{\omega_2 + \omega_1} = \frac{2\pi}{6\omega} = \frac{\pi}{3\omega}$$

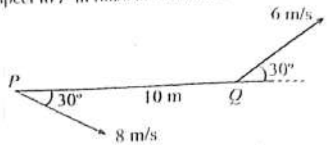
Angle subtended at the centre by the crossing points

$$\theta = \omega t = \frac{\pi}{3} = 60^\circ$$

When their angular velocities are in the same direction,

$$t' = \frac{2\pi}{\omega_2 - \omega_1} = \frac{2\pi}{4\omega} = \frac{\pi}{2\omega} \text{ and } \theta' = \frac{\pi}{2\omega} \times \omega = \frac{\pi}{2}$$

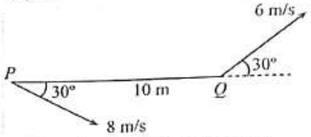
Example 52: Two moving particles P and Q are 10 m apart at a certain instant. The velocity of P is 8 m/s making 30° with the line joining P and Q and that of Q is 6 m/s making 30° with PQ in the figure. Then the angular velocity of Q with respect to P in rad/s at that instant is



(*a*) 0 (*b*) 0.1

(*c*) 0.4 (*d*) 0.7

Sol.



Angular velocity of Q relative to P

$$= \frac{\text{Projection of } V_{QP} \text{ perpendicular to the line } PQ}{\text{Separation between } P \text{ and } Q}$$

$$\frac{V_Q \sin\theta_2 - V_P \sin\theta_1}{PQ} = \frac{6\sin 30^\circ - (-8\sin 30^\circ)}{10}$$

$= 0.7$ rad/s

Concept Application

62. Two racing cars of masses m_1 and m_2 are moving in circles of radii r and 2r respectively and their angular speeds are equal. The ratio of the time taken by cars to complete one revolution is:

(*a*) $m_1 : m_2$ (*b*) 1 : 2

(*c*) 1 : 1 (*d*) $m_1 : 2m_2$

ANGULAR ACCELERATION

The rate of change of angular velocity is defined as angular acceleration.

If Δω be change in angular velocity in time Δ*t*, then angular acceleration

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

(*i*) It is a vector quantity.

(*ii*) Its direction is that of change in angular velocity

(*iii*) Unit : rad/sec^2

(*iv*) Dimension: $M^0L^0T^{-2}$

Important Points

- It is an axial vector with dimensions $[T^{-1}]$ and unit rad/s^2.
- As $\omega = \frac{d\theta}{dt}, \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$.

  i.e. second derivative of angular displacement w.r.t time gives angular acceleration.
- α is a axial vector and direction of α is along change of angular velocity.

  Direction of α is along ω if ω increases and it is opposite to ω if ω decreases (only if plane of motion of particle remains same)

Acceleration in Curved Motion

If a particle is moving in some curved path then its net acceleration is neither parallel nor perpendicular to the velocity.

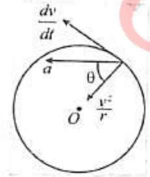
We can resolve the acceleration vector into two components:

(a) **Normal Acceleration**

The component of the acceleration perpendicular to velocity is equal to $\frac{v^2}{r}$

where, r = radius of curvature of the path at that point.

This acceleration (a_r) is responsible for change of direction of velocity. In case of circular path $a_r = \omega^2 r$. This is called as centripetal (or radial) acceleration. r = radius of circle.



$$a_c = \frac{v^2}{r} = \omega^2 r\ [a_c = a_r]$$

(b) **Tangential Acceleration**

The component along the tangent (along the direction of motion) is $a_t = \frac{dv}{dt}$ parallel to the velocity (since it is tangent to the path). Note that v is speed, not velocity.

This component changes magnitude of the velocity and acts just like one-dimensional acceleration.

$$\Rightarrow a_t = \frac{dv}{dt}$$

In case of circular motion, $v = \frac{ds}{dt} = \frac{d}{dt}$(arc length)

Net acceleration vector is obtained by vector addition of these two components.

The magnitude of the net acceleration

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

Relation between Angular Acceleration and Tangential Acceleration in Circular Motion

We know that

$v = r\omega$

Here, v is the linear speed of the particle.

Differentiating again with respect to time, we have

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} \text{ or } a_t = r\alpha$$

Here, $a_t = \frac{dv}{dt}$ is the rate of change of speed (not the rate of change of velocity).

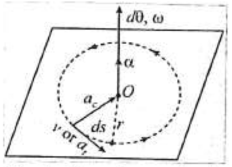
Relations Among Angular Variables for any Path

Using the relations $\vec{\omega} = \frac{d\vec{\theta}}{dt}$ and $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$, following relations may be derived.

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0{}^2 + 2\alpha\theta$



These are valid only if angular acceleration is constant and are analogous to equations of translatory motion, i.e.,

$v = u + at; s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$

Train Your Brain

Example 53: A particle travels in a circle of radius 20 cm at a speed that uniformly increases. If the speed changes from 5 m/s to 6 m/s in 2 s, find the angular acceleration.

Sol. The tangential acceleration is given by

$$a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$\left(\because \text{ Here speed increases uniformly, } a_t = \frac{\Delta v}{\Delta t} = \frac{dv}{dt}\right)$$

$$= \frac{6.0 - 5.0}{2}\ \text{m/s}^2 = 0.5\ \text{m/s}^2$$

The angular acceleration is $\alpha = a_t / r$

$$= \frac{0.5\ \text{m/s}^2}{20\ \text{cm}} = 2.5 \text{rad/s}^2$$

Example 54: A particle moves in a circle of radius 20 cm. Its linear speed at any time is given by $v = 2t$ where v is in m/s and t is in seconds. Find the radial and tangential acceleration at $t = 3$ seconds and hence calculate the total acceleration at this time.

Sol. The linear speed at 3 seconds is

$v = 2 \times 3 = 6$ m/s

The radial acceleration at t = 1 second

$= \frac{v^2}{r} = \frac{6\times 6}{0.2} = 180\ m\ s^{-2}$

The tangential acceleration is given by

$\frac{dv}{dt} = 2$

Tangential acceleration is 2 m s^{-2}

Net acceleration

$= \sqrt{a_r^2 + a_t^2} = \sqrt{(180)^2 + (2)^2} = 180.01\ m/s^2$

Think: Is it possible for a car to move in a circular path in such a way that it has a tangential acceleration but no centripetal acceleration?

Example 55: A particle moves in a circle of radius 2 cm at a speed given by $v = 4t$, where v is in cm/s and t in seconds.

(i) Find the tangential acceleration at $t = 1$ s.

(ii) Find total acceleration at $t = 1$s.

Sol. (i) Tangential acceleration

$$a_t = \frac{dv}{dt} \text{ or } a_t = \frac{d}{dt}(4t) = 4\ cm/s^2$$

(ii) $$a_c = \frac{v^2}{R} = \frac{(4)^2}{2} = 8\ cm\ s^{-2}$$

$$\Rightarrow a = \sqrt{a_t^2 + a_c^2} = \sqrt{(4)^2 + (8)^2} = 4\sqrt{5}\ cm/s^2$$

Example 56: A boy whirls a stone in a horizontal circle of radius 1.5 m and at height 2.0 m above level ground. The string breaks, and the stone files off horizontally and strikes the ground after traveling a horizontal distance of 10 m. What is the magnitude of the centripetal acceleration of the stone while in circular motion?

Sol. $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 2}{9.8}} = 0.64\ s$

$v = \frac{10}{t} = 15.63\ m/s$

$a = \frac{v^2}{R} = 162.86\ m/s^2$

Example 57: Find the magnitude of the acceleration of a particle moving in a circle of radius 10 cm with uniform speed completing the circle in 4 s.

Sol. The distance covered in completing the circle is $2\pi r = 2\pi \times 10$ cm. The linear speed is

$v = \frac{2\pi r}{t} = \frac{2\pi \times 10\ cm}{4\ s} = 5\pi\ cm/s$

The acceleration is $a_r = \frac{v^2}{r} = \frac{(5\pi)^2}{10} = 2.5\pi^2\ cm/s^2$

Example 58: A [illegible] moving in a circular path [illegible] $1.5t^2 + 2t$. If the [illegible] acceleration at t = 2 [illegible] will be

(a) [illegible] (b) 40 rad/sec

(c) [illegible] (d) [illegible] rad/sec

Sol. Given [illegible]

Tangential [illegible]

$\frac{dv}{dt}$ [illegible]

[illegible]

Angular [illegible]

$= 400$ rad/sec^2

Example 59: The [illegible] rest and [illegible] acceleration given by [illegible] after it starts [illegible] 6 sec will be

(a) [illegible] rad/sec (b) [illegible] rad/sec

(c) [illegible] rad/sec (d) [illegible] rad/sec

Sol. Given [illegible]

[illegible]

Angular velocity [illegible]

Since there is no angular [illegible]

The angular [illegible]

Concept Application

63. [illegible]

64. A

65.

66.

67.

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64. A grindstone starts from rest and has a constant angular acceleration of 4.0 rad/sec^2. The angular displacement and angular velocity, after 4 sec. will respectively be

(a) 32 rad, 16 rad/sec (b) 16 rad, 32 rad/s

(c) 64 rad, 32 rad/sec (d) 32 rad, 64 rad/sec

65. A ball is fixed to the end of a string and is rotated in a horizontal circle of radius 5 m with a speed of 10 m/sec. The acceleration of the ball will be

(a) 20 m/s^2 (b) 10 m/s^2

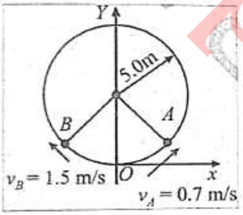
(c) 30 m/s^2 (d) 40 m/s^2

66. A body of mass 2 kg lying on a smooth surface is attached to a string 3 m long and then whirled round in a horizontal circle making 60 revolutions per minute. The centripetal acceleration will be

(a) 118.4 m/s^2 (b) 1.18 m/s^2

(c) 2.368 m/s^2 (d) 23.68 m/s^2

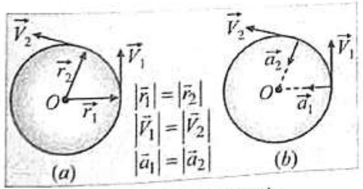
67. Two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7$ m/s and $v_B = 1.5$ m/s, respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens.



UNIFORM CIRCULAR MOTION

Motion of a particle along the circumference of a circle with a constant speed is called uniform circular motion.

Uniform circular motion is an accelerated motion because in uniform circular motion the direction of velocity vector keeps on changing.



Speed of the particle remains constant.

The position vector and velocity vector keep on changing, though their magnitudes remain constant.

The acceleration of the particle is always towards center which is constant in magnitude but variable in direction.

- ❖ In general circular motion, position vector measured from centre of circle, is always perpendicular to the velocity vector, i.e. $\vec{r} \cdot \vec{v} = 0$.
- ❖ In general circular motion, the centripetal acceleration vector is directed opposite to the radius vector.
- ❖ In uniform circular motion, there is no tangential acceleration, so total acceleration is only along radius (centripetal acceleration).
- ❖ In uniform circular motion, velocity vector is always perpendicular to the acceleration vector, i.e., $\vec{v} \cdot \vec{a} = 0$.

Non-uniform circular motion

If the speed of the particle moving in a circle is not constant, the acceleration has both the radial and the tangential components.

Examples of non uniform circular motion

A merry-go-round spinning up from rest to full speed, or a ball whirling around in a vertical circle.

In non-uniform circular motion

Speed $|\vec{v}| \neq$ constant

angular velocity $\omega \neq$ constant

If at any instant

v = magnitude of velocity of particle

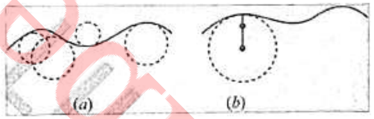
ω = angular velocity of particle,

then, at that instant $v = r\omega$

where r = radius of circular path

RADIUS OF CURVATURE

Any curved path can be assume to be made of infinite circular arcs. Radius of curvature at a point is defined as the radius of that circular arc which fits at that particular point on the curve.


(a) (b)

We know that centripetal acceleration, $a_c = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_c}$. This is the expression for radius of curvature.

For finding the radius of curvature first take the component of net acceleration perpendicular to the velocity, then use the expression

$$R = \frac{v^2}{a_\perp}$$

If a particle moves in a trajectory given by $y = f(x)$ then radius of curvature at any point (x, y) of the trajectory is given by

$$\Rightarrow R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

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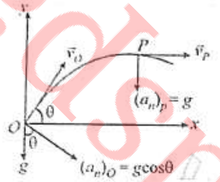
Example 60: Find the ratio of the radius of curvature at the highest point of projectile to that just after its projection if the angle of projection is 30°?

Sol. If $\vec{v}_0$ is the initial velocity, $v_P = V_0\cos\theta$. Normal acceleration at $O = g\cos\theta$. Hence, if r_0 and r_P are radii of curvature at O and P, respectively, then $r_0 = \frac{v_0^2}{g\cos\theta}$ and $r_P = \frac{v_0^2\cos^2\theta}{g}$



Hence, the required ratio $= \frac{r_P}{r_0} = \cos^3\theta = \frac{3\sqrt{3}}{8}$

Example 61: A stone is projected with speed u and angle of projection is θ. Find radius of curvature at $t = 0$.

(*a*) $\frac{u^2\cos^2\theta}{g}$ (*b*) $\frac{u^2}{g\sin\theta}$

(*c*) $\frac{u^2}{g\cos\theta}$ (*d*) $\frac{u^2\sin^2\theta}{g}$

Sol. At $t = 0$

$a_\perp = g\cos\theta$,

$R = \frac{v^2}{a_\perp} = \frac{u^2}{g\cos\theta}$

Concept Application

68. A ball is projected making an angle θ with the vertical. Consider a small part of the trajectory near the highest position and take it approximately to be a circular arc. What is the radius of this circle? This radius is called the radius of curvature of the curve at the point.

Aarambh (Solved Examples)

1. The sum of the magnitudes of two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the, magnitudes of forces

(*a*) 12, 5 (*b*) 14, 4 (*c*) 5, 13 (*d*) 10, 8

Sol. Let P be the smaller force and Q be the greater force then according to problem: $P + Q = 18$ ...(i)

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = 12 \qquad ...(ii)$$

$$\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta} = \tan 90° = \infty$$

$$\therefore P + Q\cos\theta = 0 \qquad ...(iii)$$

By solving (i), (ii) and (iii)

we will get $P = 5$ and $Q = 13$.

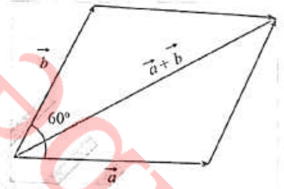
Therefore, option (*c*) is the correct answer.

2. Two vectors of equal magnitude 2 are at an angle of 60° to each other find magnitude of their sum and difference.

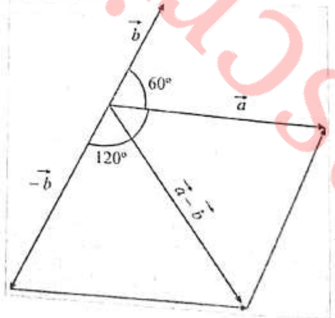
(*a*) 4, 0 (*b*) 2, 2

(*c*) $2\sqrt{3}$, 2 (*d*) 2, $2\sqrt{3}$

Sol. $|\vec{a} + \vec{b}| = \sqrt{2^2 + 2^2 + 2\times2\times2\cos60°} = \sqrt{4+4+4} = 2\sqrt{3}$

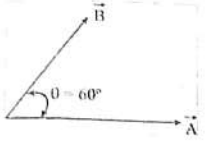


$$|\vec{a} - \vec{b}| = \sqrt{2^2 + 2^2 - 2\times2\times2\cos120°} = \sqrt{4+4-4} = 2$$

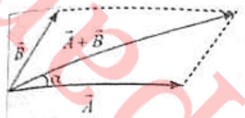


Therefore, option (*c*) is the correct answer.

3. Find $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ in the diagram shown in figure. Given $A = 4$ units and $B = 3$ units.



Sol.



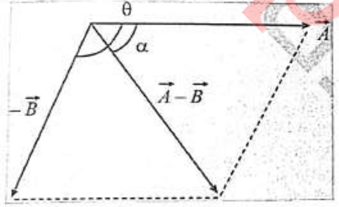
Addition: $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{16+9+2\times4\times3\cos60^\circ} = \sqrt{37}$ units

$\tan\alpha = \dfrac{B\sin\theta}{A+B\cos\theta} = \dfrac{3\sin60^\circ}{4+3\cos60^\circ} = 0.472$

$\therefore \alpha = \tan^{-1}(0.472) = 25.3^\circ$

Thus, resultant of $\vec{A}$ and $\vec{B}$ is $\sqrt{37}$ units at angle 25.3° from $\vec{A}$ in the direction shown in figure.



Subtraction: $S = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$= \sqrt{16+9-2\times4\times3\cos60^\circ} = \sqrt{13}$ *units*

and $\tan\theta = \dfrac{B\sin\theta}{A-B\cos\theta}$

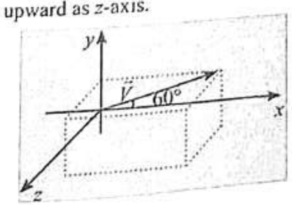
$= \dfrac{3\sin60^\circ}{4-3\cos60^\circ} = 1.04$

$\therefore \alpha = \tan^{-1}(1.04) = 46.1^\circ$

Thus, $\vec{A}-\vec{B}$ is $\sqrt{13}$ units at 46.1° from $\vec{A}$ in the direction shown in figure.

4. A bird moves with velocity 30 m/s in a direction making an angle of 60° with the eastern line and 60° with the vertical upward. Represent the velocity vector in rectangular form.

Sol. Let eastern line be taken as x-axis, northern as y-axis and vertical upward as z-axis.



Let the velocity $\vec{v}$ makes angle a, b and g with x, y and z axis respectively, then $a = 60^\circ$, $g = 60^\circ$ we have

$\cos^2 a + \cos^2 b + \cos^2 g = 1$

$\cos^2 60^\circ + \cos^2 b + \cos^2 60^\circ = 1$

$\cos^2 b = \dfrac{1}{2}; \cos b = \dfrac{1}{\sqrt{2}}$

so $\vec{v} = v\cos a\hat{i} + v\cos b\hat{j} + v\cos g\hat{k}$

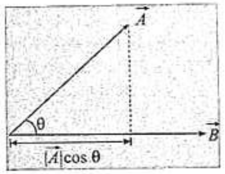
$= 30\left[\dfrac{1}{2}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j} + \dfrac{1}{2}\hat{k}\right] = 15\hat{i} + 15\sqrt{2}\hat{j} + 15\hat{k}$

5. If $\vec{A} = 2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{B} = -\hat{i}+3\hat{j}+4\hat{k}$ then projection of $\vec{A}$ on $\vec{B}$ will be

(*a*) $\dfrac{3}{\sqrt{13}}$ (*b*) $\dfrac{3}{\sqrt{26}}$ (*c*) $\sqrt{\dfrac{3}{26}}$ (*d*) $\sqrt{\dfrac{3}{13}}$

Sol. $|\vec{A}| = \sqrt{2^2+3^2+(-1)^2} = \sqrt{4+9+1} = \sqrt{14}$

$|\vec{B}| = \sqrt{(-1)^2+3^2+4^2} = \sqrt{1+9+16} = \sqrt{26}$



$\vec{A}.\vec{B} = 2(-1)+3\times3+(-1)(4) = 3$

The projection of $\vec{A}$ on $\vec{B}$, $|\vec{A}|\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{B}|} = \dfrac{3}{\sqrt{26}}$

Therefore, option (*b*) is the correct answer.

6. The vectors from origin to the points A and B are $\vec{A} = 3\hat{i}-6\hat{j}+2\hat{k}$ and $\vec{B} = 2\hat{i}+\hat{j}-2\hat{k}$ respectively. The area of the triangle OAB will be

(*a*) $\dfrac{5}{2}\sqrt{17}$ sq.unit (*b*) $\dfrac{2}{5}\sqrt{17}$ sq.unit

(*c*) $\dfrac{3}{5}\sqrt{17}$ sq.unit (*d*) $\dfrac{5}{3}\sqrt{17}$ sq.unit

Sol. Given $\overline{OA} = \vec{a} = 3\hat{i}-6\hat{j}+2\hat{k}$

and $\overline{OB} = \vec{b} = 2\hat{i}+\hat{j}-2\hat{k}$

$$\therefore (\vec{a}\times\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix}$$

$= (12-2)\hat{i} + (4+6)\hat{j} + (3+12)\hat{k}$

$= 10\hat{i} + 10\hat{j} + 15\hat{k}$

$\Rightarrow |\vec{a}\times\vec{b}| = \sqrt{10^2+10^2+15^2}$

$= \sqrt{425} = 5\sqrt{17}$

Area of $\Delta OAB = \dfrac{1}{2}|\vec{a}\times\vec{b}| = \dfrac{5\sqrt{17}}{2}$ sq.unit.

Therefore, option (*a*) is the correct answer.

7. If at an instant the velocity of a projectile be u and its inclination to the horizontal be θ, at what time interval after that instant will the particle be moving at right angles to its former direction

(*a*) $g\,u\sin\theta$ (*b*) $\dfrac{u}{g\sin\theta}$ (*c*) $g\,u\cos\theta$ (*d*) $\dfrac{u}{g\cos\theta}$

Sol. At $t = 0$, $u_x = u\cos\theta$, $u_y = u\sin\theta$

$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Let after time 't' the velocity of projectile be perpendicular to its initial velocity u.

At time t

$v_x = u\cos\theta$, $v_y = u\sin\theta - gt$

$\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$

$\vec{u} \perp \vec{v}$

$\vec{u}\cdot\vec{v} = 0$

$(u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j})\cdot(u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}) = 0$

$u^2\cos^2\theta + (u\sin\theta)^2 - gt\,u\sin\theta = 0$

$u^2(\cos^2\theta + \sin^2\theta) = gt\,u\sin\theta$

$t = \dfrac{u}{g\sin\theta}$

Therefore, option (*b*) is the correct answer.

8. A vertical pole has a black mark at some height. A stone is projected from a fixed point on the ground. When projected at an angle of 45° it hits the pole orthogonally 1 m above the mark. When projected with a different speed at an angle of $\tan^{-1}(3/4)$, it hits the pole orthogonally 1.5 m below the mark. Find the speed and angle of projection so that it hits the mark orthogonally to the pole. [g = 10 m/sec^2]

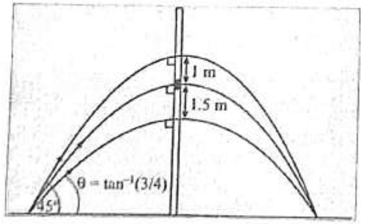
(*a*) $\dfrac{\sqrt{3620}}{3}\text{ ms}^{-1}, \tan^{-1}\left(\dfrac{9}{10}\right)$

(*b*) 160 ms^{-1}, 30°

(*c*) $\dfrac{3620}{9}\text{ ms}^{-1}, \tan^{-1}\left(\dfrac{9}{\sqrt{181}}\right)$

(*d*) None of these

Sol. $H = \dfrac{u^2\sin^2\theta}{2g}$, $R = \dfrac{u^2\sin 2\theta}{g}$



So, $\dfrac{H}{R} = \left(\dfrac{\tan\theta}{4}\right)$ ...(i)

$\dfrac{H+1}{R} = \left(\dfrac{\tan 45°}{4}\right) = \dfrac{1}{4}$ ...(ii)

$\dfrac{H-1.5}{R} = \dfrac{\tan\left[\tan^{-1}(3/4)\right]}{4}$

$\dfrac{H-1.5}{R} = \dfrac{3/4}{4} = \dfrac{3}{16}$ (iii)

$\dfrac{H+1}{H-1.5} = \dfrac{4}{3}$

$3H + 3 = 4H - 6$

$H = 9$ m

Now, $\dfrac{10}{R} = \dfrac{1}{4} \Rightarrow R = 40$ m

So, $\dfrac{9}{40} = \dfrac{\tan\theta}{4}$

$\tan\theta = \dfrac{9}{10} \Rightarrow \theta = \tan^{-1}\left(\dfrac{9}{10}\right)$

$\tan\theta = \dfrac{9}{10} \Rightarrow \sin\theta = \dfrac{9}{\sqrt{181}}$ and $\cos\theta = \dfrac{10}{\sqrt{181}}$

$\Rightarrow \tan\theta = \dfrac{9}{10}$

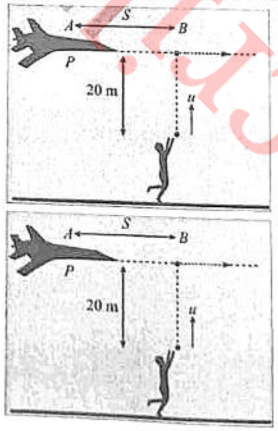
Using $R = \dfrac{u^2\cdot 2\sin\theta\cos\theta}{g}$

$$\frac{u^2 2\left(\frac{9}{\sqrt{181}}\right)\left(\frac{10}{\sqrt{181}}\right)}{10} = 40$$

$u^2 = \dfrac{3620}{9} \Rightarrow u = \dfrac{\sqrt{3620}}{3}$ m/s

Therefore, option (*a*) is the correct answer.

9. A toy plane P starts flying from point A along a straight horizontal line 20 m above ground level starting with zero initial velocity and acceleration 2 m/s^2 as shown. At the same instant, a man P throws a ball vertically upwards with initial velocity 'u'. Ball touches (coming to rest) the base of the plane at point B of plane's journey when it is vertically above the man. 's' is the distance of point B from point A. Just after the contact of ball with the plane, acceleration of plane increases to 4 m/s^2. Answer the following questions:



(*i*) Find
(*a*)
(*c*)
(*ii*) Find
(*a*)
(*c*)
(*iii*) Finc
catc
(*a*)
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Sol. (*i*) Ma
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(*a*)
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Sol. Ho
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iii)

(*i*) Find initial velocity u of the ball.

(*a*) 10 ms^{-1} (*b*) 20 ms^{-1}

(*c*) 50 ms^{-1} (*d*) 30 ms^{-1}

(*ii*) Find distance 's' shown in the figure.

(*a*) 10 m (*b*) 5 m

(*c*) 4 m (*d*) 2 m

(*iii*) Find the distance between man and plane when the man catches the ball back. ($g = 10$ m/s^2)

(*a*) 16 m (*b*) 4 m

(*c*) 20 m (*d*) $4\sqrt{41}$ m

Sol. (*i*) Maximum height reached by ball = 20 m.

So, taking upward direction as positive, $v^2 = u^2 + 2as$

So, $0 = u^2 - 2 \times 10 \times 20$ or $u = 20$ m/sec

Therefore, option (*b*) is the correct answer.

(*ii*) (*c*) Time taken by ball = $t = u/g = 20/10 = 2$ s

(for touching the plane)

Horizontal distance travelled by plane in this time

$t = s = u_x t + 1/2\, a_x t^2$

$\Rightarrow s = 0 \times 2 + 1/2 \times 2 \times 2^2 = 4$ m

Therefore, option (*c*) is the correct answer.

(*iii*) Man catches the ball back 2 seconds after it touches the plane.

Velocity of plane when ball touches it

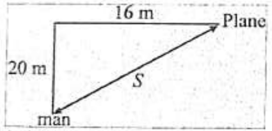
$v_x = u_x + a_x t = 0 + 2 \times 2 = 4$ m/s

Now, acceleration of plane becomes : $a'_x = 4$ m/sec^2

so, s'_x = horizontal distance travelled by plane after touching of the ball

$= u'_x t + 1/2\, a'_x t^2 = 4 \times 2 + 1/2 \times 4 \times 4$

$= 8 + 8 = 16$ m



Final distance between man and plane,

$s = \sqrt{(20)^2 + (16)^2} = \sqrt{656}$ m

Therefore, option (*d*) is the correct answer.

10. A body is so projected in the air that the horizontal range covered by the body is equal to the maximum vertical height attained by the body during the motion. Find the angle of projection?

(*a*) 45° (*b*) $\tan^{-1}\left(\frac{4}{5}\right)$

(*c*) $\tan^{-1}(4)$ (*d*) 37°

Sol. Horizontal Range $R = \frac{u^2 \sin 2\theta}{g}$

Vertical height $H = \frac{u^2 \sin^2 \theta}{2g}$

Given $R = H$

So, $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$

$2 \times 2 \sin\theta \cos\theta = \sin^2\theta$

$\tan\theta = 4$

Therefore, option (*c*) is the correct answer.

11. A projectile can have the same range R for two angles of projection at a given speed. If T_1 and T_2 be the times of flight in two cases, then find out relation between T_1, T_2 and R?

(*a*) $T_1T_2 = \frac{R}{g}$ (*b*) $\frac{T_1}{T_2} = \frac{R}{g}$ (*c*) $\frac{T_1}{T_2} = \frac{2R}{g}$ (*d*) $T_1T_2 = \frac{2R}{g}$

Sol. R is same for θ_1 and θ_2

$\theta_2 = 90 - \theta_1$

$T = \frac{2u\sin\theta}{g}$ $\therefore T_1 = \frac{2u\sin\theta}{g}$; $T_2 = \frac{2u\sin(90-\theta)}{g}$

and, $R = \frac{u^2\sin 2\theta}{g}$ $\therefore T_1T_2 = \frac{2^2 \times u^2 \sin\theta\cos\theta}{g^2}$

$= \frac{2}{g}\left(\frac{u^2 \sin 2\theta}{g}\right)$

$T_1T_2 = \frac{2R}{g}$

Therefore, option (*d*) is the correct answer.

12. A player kicks a football at an angle of 45° with an initial speed of 20 m/s. A second player on the goal line 60 m away in the direction of kick starts running to receive the ball at that instant. Find the constant speed of the second player with which he should run to catch the ball before it hits the ground [$g = 10$ m/s^2].

(*a*) $5\sqrt{2}$ ms^{-1} (*b*) 10 ms^{-1} (*c*) $2\sqrt{2}$ ms^{-1} (*d*) 5 ms^{-1}

Sol. $\theta_1 = 45°$ $u = 20$m/s

$T = \frac{2u_y}{g} = \frac{2 \times 20\frac{1}{\sqrt{2}}}{10} = 2\sqrt{2}s,$ $u_x = 20\frac{1}{\sqrt{2}}$

Now, $R = \left(20\frac{1}{\sqrt{2}}\right) \times 2\sqrt{2} = 40$m

$\Rightarrow$ The second player should run a distance $= 60 - 40$
$= 20$ m

Time is $2\sqrt{2}$ s $\Rightarrow v = \frac{20m}{2\sqrt{2}s} = 5\sqrt{2}$ m/s

Therefore, option (*a*) is the correct answer.

13. A gun kept on a straight horizontal road is used to hit a car, traveling along the same road away from the gun with a uniform speed of $72 \times \sqrt{2}$ km/hour. The car is at a distance of 50 metre from the gun, when the gun is fired at an angle of 45° with the horizontal. Find

(*i*) the distance of the car from the gun when the bullet hits it.

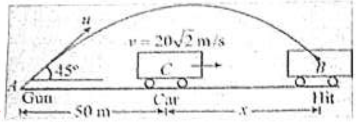
(*a*) 200 m (*b*) 250 m

(*c*) 100 m (*d*) 150 m

(*ii*) the speed of projection of the bullet from the gun. [$g = 10$ m/s^2]

(*a*) 10 m/s (*b*) 20 m/s

(*c*) 40 m/s (*d*) 50 m/s



Sol. range = 50 + x

$$\frac{u^2}{g} = 50 + x \qquad \text{...(i)}$$

Time taken by gun from A to B = time taken by car from C to B

$$\frac{2u}{\sqrt{2}g} = \frac{x}{20\sqrt{2}} \qquad \text{..(ii)}$$

by solving equations (i) and (ii) we have

$u = 50$ m/s

(*i*) Distance of car from gun $= \frac{u^2}{g} = \frac{50^2}{10} = 250$ m

Therefore, option (*b*) is the correct answer.

(*ii*) Speed of projection of shell = 50 m/s

Therefore, option (*d*) is the correct answer.

14. From the top of a tower of height 50 m a ball is projected upwards with a speed of 30 m/s at an angle of 30° to the horizontal. Calculate

(*i*) Maximum height from the ground

(*a*) 61.25 m (*b*) 100 m

(*c*) 20 m (*d*) 21.75 m

(*ii*) Time of flight.

(*a*) 10 s (*b*) 8 s

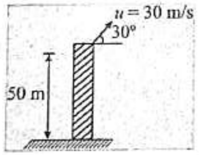
(*c*) 5 s (*d*) 4 s

(*iii*) At what distance from the foot of the tower does the projectile hit the ground?

(*a*) 75 m (*b*) $75\sqrt{3}$ m

(*c*) 50 m (*d*) $20\sqrt{3}$ m

Sol. (*i*)



$$H = \frac{u^2\sin^2\theta}{2g} = \frac{30\times30\times\frac{1}{2}\times\frac{1}{2}}{2\times10} = \frac{90}{8} = 11.25 \text{ m}$$

Maximum height from ground = 50 + 11.25 = 61.25 m.

Therefore, option (*a*) is the correct answer.

(*ii*) $s_y = u_y T + \frac{1}{2}a_y T^2$

where, $s_y = -50$ (vertical displacement)

$u_y = u\sin 30° = 15$ m/s, $a_y = -g = -10$ m/s²

Substituting these values,

$-50 = 15T + \frac{1}{2}(-10)T^2$; or $T^2 - 3T - 10 = 0$

or $T^2 - 5T + 2T - 10 = 0$

or, $T(T - 5) + 2(T - 5) = 0$

or $(T - 5)(T + 2) = 0$

$T = 5$ or $T = -2$

$\Rightarrow T = 5$ sec

Therefore, option (*c*) is the correct answer.

(*iii*) $s_x = u\cos\theta \cdot T = 30 \times \cos 30° \times T = 30 \times \frac{\sqrt{3}}{2} \times 5$

$= 75\sqrt{3}$ m

Therefore, option (*b*) is the correct answer.

15. The equation of a projectile is $y = \sqrt{3}x - \frac{gx^2}{2}$ where at $t = 0$, $x = 0$ and $y = 0$. Also $\frac{d^2x}{dt^2} = 0$, $\frac{d^2y}{dt^2} = -g$. Find the angle (from x-axis) and speed of projection.

(*a*) 45°, 2 ms⁻¹ (*b*) 60°, 4 ms⁻¹

(*c*) 45°, 4 ms⁻¹ (*d*) 60°, 2 ms⁻¹

Sol. $y = \sqrt{3}x - g\frac{x^2}{2}$

Comparing the given (above) equation with the standard equation of trajectory $y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

We get, $\sqrt{3} = \tan\theta$ and $u^2\cos^2\theta = 1$

$\Rightarrow \theta = 60°$

Putting $\theta = 60°$, we get $u^2 = \frac{1}{(1/2)^2}$

$\Rightarrow u = 2$m/s

Alternate Solution

$y = \sqrt{3}x - \frac{gx^2}{2}$

$\Rightarrow \frac{dy}{dt} = \sqrt{3}\frac{dx}{dt} - \frac{1}{2}g\left[2x\left(\frac{dx}{dt}\right)\right]$

$V_y = \sqrt{3}\ V_x - gx\ V_x$

At $t = 0$, $x = 0$, $V_y = u_y$ & $V_x = u_x$; $u_y = \sqrt{3}\ u_x$

$\Rightarrow u_y = \sqrt{3}u_x$

$\Rightarrow \frac{u_y}{u_x} = \sqrt{3} \Rightarrow \tan\theta = \sqrt{3}$

Now,

$\frac{d^2y}{dt^2} = \sqrt{3}\frac{d^2x}{dt^2} - g\left[x\frac{d^2x}{dt^2} + \frac{dx}{dt}\times\frac{dx}{dt}\right]$ here $\frac{d^2x}{dt^2} = 0$

$\Rightarrow \frac{d^2y}{dt^2} = \sqrt{3}\times0 - g[0 + v_x]^2$

But, $\frac{d^2y}{dt^2} = -g$

$\Rightarrow -g = -gv_x^2 \Rightarrow V_x^2 = 1 \Rightarrow V_x = \pm 1$

Since, $V_x = \pm1$ = constant

$\Rightarrow u_x = V_x = \pm1$

Now, $\frac{u_y}{u_x} = \sqrt{3}$

16.

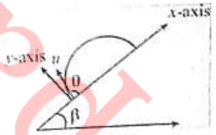
$u_y = \pm\sqrt{3}$

$\therefore$ Speed $= u = \sqrt{u_x^2 + u_y^2} = \sqrt{(\pm 1)^2 + (\pm\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4}$

$u = 2$ m/s

Therefore, option (*d*) is the correct answer.

16. A particle is projected at an angle θ with an inclined plane which is making an angle β with the horizontal as shown in figure. Speed of projection of the particle is *u*. After time *t*, find:



(*i*) *x* component of acceleration.

(*a*) gsin β (*b*) –gsin β

(*c*) gcos β (*d*) –gcos β

(*ii*) *y* component of acceleration.

(*a*) gsin β (*b*) –gsin β

(*c*) gcos β (*d*) –gcos β

(*iii*) *x* component of velocity.

(*a*) $u\cos\theta - g\cos\beta t$ (*b*) $u\sin\theta - g\sin\beta t$

(*c*) $u\cos\theta - g\sin\beta t$ (*d*) $u\sin\theta - g\cos\beta t$

(*iv*) *y* component of velocity.

(*a*) $u\cos\theta - g\cos\beta t$ (*b*) $u\cos\theta - g\sin\beta t$

(*c*) $u\sin\theta - g\cos\beta t$ (*d*) $u\sin\theta - g\sin\beta t$

(*v*) *x* component of displacement.

(*a*) $u\cos\theta t - \frac{1}{2} g\cos\beta t^2$

(*b*) $u\cos\theta t - \frac{1}{2} g\sin\beta t^2$

(*c*) $u\sin\theta t - \frac{1}{2} g\sin\beta t^2$

(*d*) $u\sin\theta t - \frac{1}{2} g\cos\beta t^2$

(*vi*) *y* component of displacement.

(*a*) $u\cos\theta t - \frac{1}{2} g\cos\beta t^2$

(*b*) $u\cos\theta t - \frac{1}{2} g\sin\beta t^2$

(*c*) $u\sin\theta t - \frac{1}{2} g\sin\beta t^2$

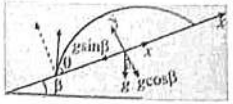
(*d*) $u\sin\theta t - \frac{1}{2} g\cos\beta t^2$

(*vii*) *y* component of velocity when particle is at maximum distance from the inclined plane.

(*a*) Zero (*b*) $u\cos\theta$

(*c*) $u\sin\theta$ (*d*) $u\tan\theta$

Sol.



(*i*) a_x = *x* component of acceleration = $-g\sin\beta$

Therefore, option (*b*) is the correct answer.

(*ii*) a_y = *y* – component of acceleration = $a_y = -g\cos\beta$

Therefore, option (*d*) is the correct answer.

(*iii*) Let *x* – component of velocity = V_x

$V = u + at$

$\Rightarrow V_x = u_x + a_x t = (u\cos\theta) + (-g\sin\beta)\cdot t$

$\Rightarrow V_x = u\cos\theta - g\sin\beta . t$

Therefore, option (*c*) is the correct answer.

(*iv*) Let *y* – component of velocity = V_y

$V_y = u_y + a_y t$

$\Rightarrow V_y = (u\sin\theta) + (-g\cos\beta\, t)$

$\Rightarrow V_y = u\sin\theta - g\cos\beta\cdot t$

Therefore, option (*c*) is the correct answer.

(*v*) Let *x* – component of displacement = s_x

$s = ut + \frac{1}{2}at^2$

$\Rightarrow s_x = u_x t + \frac{1}{2}a_x t^2$

$= (u\cos\theta)\, t + \frac{1}{2}(-g\sin\beta)\, t^2$

$\Rightarrow s_x = u\cos\theta\, t - \frac{1}{2} g\sin\beta t^2$

Therefore, option (*b*) is the correct answer.

(*vi*) Let s_y = *y* – component of displacement

$s_y = u_y t + \frac{1}{2}a_y t^2$

$\Rightarrow s_y = u\sin\theta\, t - \frac{1}{2} g\cos\beta\, t^2$

Therefore, option (*d*) is the correct answer.

(*vii*) $V_y = ?$ when $s_y = (s_y)_{\max}$.

$\frac{d(s_y)}{dt} = V_y$

For maximum s_y, $\frac{d(s_y)}{dt} = 0$ i.e., $V_y = 0$

Therefore, option (*a*) is the correct answer.

17. A particle *A* moves with a velocity $4\hat{i}$ and another particle *B* moves with a velocity $-3\hat{j}$. What is the magnitude of $\vec{V}_{AB}$ and $\vec{V}_{BA}$?

(*a*) $\sqrt{7}\text{ms}^{-1}$, $\sqrt{7}\text{ms}^{-1}$ (*b*) $\sqrt{5}\text{ms}^{-1}$, $\sqrt{5}\text{ms}^{-1}$

(*c*) 7 ms^{-1}, 7 ms^{-1} (*d*) 5 ms^{-1}, 5 ms^{-1}

Sol. $\vec{v}_A = 4\hat{i}$, $\vec{v}_B = -3\hat{j}$

$\vec{v}_{AB} = \vec{v}_A - \vec{V}_B = 4\hat{i} - (-3\hat{j}) = 4\hat{i} + 3\hat{j}$

$\vec{v}_{BA} = \vec{v}_B - \vec{V}_A = -3\hat{j} - 4\hat{i}$

$|\vec{v}_{AB}| = \sqrt{4^2 + 3^2} = 5$ m/s

$|\vec{v}_{BA}| = \sqrt{3^2 + 4^2} = 5$ m/s

Therefore, option (*d*) is the correct answer.

18. A motorboat is observed to travel 10 km h^{-1} relative to the earth in the direction 37° north of east. If the velocity of the boat due to the wind only is 2 km h^{-1} westward and that due to the current only is 4 km h^{-1} southward, what is the magnitude and direction of the velocity of the boat due to its own power?

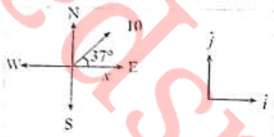
(*a*) 10 kmh^{-1}, 45° north of east

(*b*) 10 kmh^{-1}, 60° west of north

(*c*) $10\sqrt{2}$ kmh^{-1}, 45° north of east

(*d*) $10\sqrt{2}$ kmh^{-1}, 60° west of north

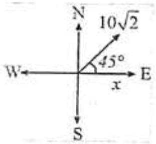
Sol. $\vec{V} = 10\cos 37°\,\hat{i} + 10\sin 37°\,\hat{j}$



$= 10 \times \frac{4}{5}\hat{i} + 10 \times \frac{3}{5}\hat{j} = 8\hat{i} + 6\hat{j}$

$8\hat{i} + 6\hat{j} = \vec{V}_b + \vec{V}_w + \vec{V}_c = \vec{V}_b + (-2\hat{i}) + (-4\hat{j})$

$\vec{V}_b = 10\hat{i} + 10\hat{j}$

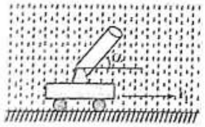


$V_b = 10\sqrt{2}$ km/h

$\tan\theta = \frac{10}{10}$ direction = 45° north of East

Therefore, option (*c*) is the correct answer.

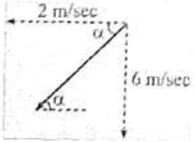
19. A pipe which can rotate in a vertical plane is mounted on a cart. The cart moves uniformly along a horizontal path with a speed $v_1 = 2$ m/s. At what angle α to the horizontal should the pipe be placed so that drops of rain falling vertically with a velocity $v_2 = 6$m/s enter the pipe without touching its walls? Consider the velocity of the drops as constant.



(*a*) $\alpha = \tan^{-1}\left(\frac{1}{3}\right)$ (*b*) $\alpha = \tan^{-1}(3)$

(*c*) $\alpha = \tan^{-1}(\sqrt{3})$ (*d*) $\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Sol.



$\tan\alpha = \frac{6}{2} = 3 \quad \Rightarrow \quad \alpha = \tan^{-1} 3$

Therefore, option (*b*) is the correct answer.

20. An aeroplane has to go from a point *A* to another point *B*, 1000 km away due 30° west of north. A wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. Find the direction in which the pilot should head the plane to reach the point *B* and the time taken by the plane to go from *A* to *B*.

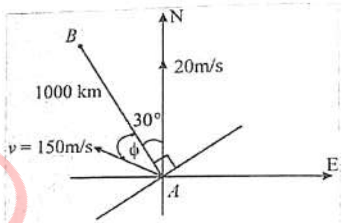
(*a*) $\phi = \sin^{-1}\left(\frac{1}{15}\right)$ east of line *AB*, $t = 100$ min

(*b*) $\phi = \sin^{-1}\left(\frac{1}{15}\right)$ west of line *AB*, $t = 100$ min

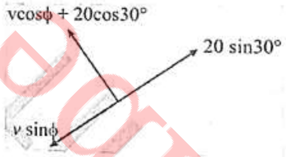
(*c*) $\phi = \sin^{-1}\left(\frac{1}{15}\right)$ east of line *AB*, $t = 100$ sec

(*d*) $\phi = \sin^{-1}\left(\frac{1}{15}\right)$ west of line *AB*, $t = 100$ sec

Sol.



Velocity of plane w.r.t. ground



(*a*) Velocity of plane w.r.t ground is along *AB* so component of velocity perpendicular to line *AB* is zero.

$v\sin\phi = 20\sin 30°$

$\sin\phi = \frac{10}{150} = \frac{1}{15}$

$\phi = \sin^{-1}\left(\frac{1}{15}\right)$ west of line *AB*

(*b*) $t = \frac{1000 \times 10^3}{v\cos\phi + 10\sqrt{3}}\ s = \frac{1000 \times 1000}{150 \times \sqrt{\frac{224}{225}} + 10\sqrt{3}}\ s$

$= \frac{1000 \times 1000}{10 \times 60(\sqrt{3} + \sqrt{224})}$ min

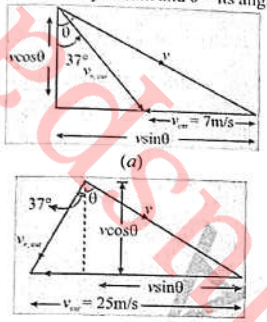
$= \frac{5000}{3(\sqrt{3} + \sqrt{224})}$ min ≈ 100 min

Therefore, option (*b*) is the correct answer.

21. Rain appears to be falling at an angle of 37° with vertical to the driver of a car moving with a velocity of 7 m/sec. When he increases the velocity of the car to 25 m/sec, the rain again appears to fall at an angle 37° with vertical. What is the actual velocity of rain and its direction with vertical?

(*a*) 20 ms^{-1}, 53° (*b*) 20 ms^{-1}, 37°
(*c*) 10 ms^{-1}, 37° (*d*) 10 ms^{-1}, 53°

Sol. Let v = actual velocity of rain and θ = its angle with vertical:



(*a*)

(*b*)

In figure (*a*)

$$v\sin\theta = 7 + v\cos\theta.\tan 37° = 7 + \frac{3}{4}v\cos\theta$$

$$\Rightarrow 4\,v\sin\theta - 3\,v\cos\theta = 28 \qquad \text{...(i)}$$

In figure (*b*)

$$25 = v\sin\theta + v\cos\theta.\tan 37° = v\sin\theta + \frac{3}{4}v\cos\theta$$

$$\Rightarrow 4\,v\sin\theta + 3\,v\cos\theta = 100 \qquad \text{...(ii)}$$

Solving (i) and (ii)

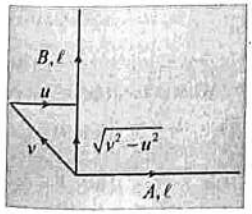
v = 20m/s and θ = 53°

Therefore, option (*a*) is the correct answer.

22. Two boats *A* and *B* move away from a buoy anchored at the middle of a river along the mutually perpendicular straight lines; the boat *A* along the river and the boat *B* across the river. Having moved off an equal distance from the buoy the boats returned. Then the ratio of time of motion of boats τ_A/τ_B if the velocity of each boat with respect to water is η times that of the stream velocity.

(*a*) $\frac{\eta-1}{\eta}$ (*b*) $\frac{\eta^2}{(\eta^2-1)}$ (*c*) $\frac{\sqrt{\eta^2-1}}{\eta}$ (*d*) $\frac{\eta}{\sqrt{\eta^2-1}}$

Sol. $V_{b,r} = v$, $V_r = u$



Time taken by boat *A*

$$\tau_A = \frac{\ell}{v+u} + \frac{\ell}{v-u} = \frac{2v\ell}{v^2-u^2} = \frac{2\eta\ell}{(\eta^2-1)u}$$

Time taken by boat *B*

$$\tau_B = \frac{2\ell}{\sqrt{v^2-u^2}} = \frac{2\ell}{\sqrt{\eta^2-1}\,u}$$

Hence $\frac{\tau_A}{\tau_B} = \frac{\eta}{\sqrt{\eta^2-1}}$

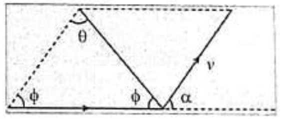
Therefore, option (*d*) is the correct answer.

23. '*n*' number of particles are located at the vertices of a regular polygon of '*n*' sides having the edge length '*a*'. They all start moving simultaneously with equal constant speed '*v*' heading towards the particle next to it all the time. How long will the particles take to collide?

(*a*) $t = \frac{a}{v\left(1-\cos\frac{2\pi}{n}\right)}$ (*b*) $t = \frac{a}{v\sin\left(\frac{2\pi}{n}\right)}$

(*c*) $t = \frac{a}{v\cos\frac{2\pi}{n}}$ (*d*) $t = \frac{a}{\left(1-\sin\frac{2\pi}{n}\right)}$

Sol. For a regular polygon of side n, the angle subtended by a side at the centre:



$$\theta = \frac{2\pi}{n}$$

$$\therefore\ \alpha = \pi - 2\left(\frac{\pi}{2}-\frac{\theta}{2}\right) = \theta \Rightarrow \alpha = \theta = \frac{2\pi}{n}$$

Relative velocity of approach

$= v - v\cos\alpha = v(1-\cos\alpha)$

Time taken for collision

$$t = \frac{a}{v(1-\cos\alpha)} \Rightarrow t = \frac{a}{v\left(1-\cos\frac{2\pi}{n}\right)}$$

Therefore, option (*a*) is the correct answer.

24. A particle moves clockwise in a circle of radius 1 m with centre at $(x, y) = (1\text{m}, 0)$. It starts at rest at the origin at time $t = 0$. Its speed increases at the constant rate of $\left(\frac{\pi}{2}\right)$ m/s^2.

(*a*) How long does it take to travel halfway around the circle?

(*b*) What is the speed at that time?

(*c*) What is the net acceleration at that time?

Sol. R = 1m,

$$a_t = \frac{dv}{dt} = \frac{\pi}{2}\ \text{m/s}^2$$

at $t = 0, u = 0, \omega_0 = 0$

$\alpha = \frac{a_t}{R} = \frac{\pi}{2}$ rad/s^2

(*a*) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow \pi = 0 + \frac{1}{2}\frac{\pi}{2}t^2 \Rightarrow t = 2$ sec

(*b*) $v = u + a_t t = 0 + \frac{\pi}{2} \times 2 = \pi$ m/s

(*c*) $a_t = \frac{\pi}{2}$ m/s^2, $a_c = \frac{v^2}{r} = \pi^2$ m/s^2

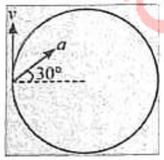
$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{\frac{\pi^2}{4} + \pi^4} = \frac{\pi}{2}\sqrt{1 + 4\pi^2}\ \text{m/s}^2$$

25. Figure shows the direction of total acceleration and velocity of a particle moving clockwise in a circle of radius 2.5 m at a given instant of time. At this instant if magnitude of net acceleration is 25 m/sec^2, find:

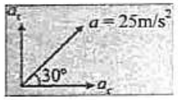
(*i*) The radial acceleration.

(*ii*) The speed of the particle and

(*iii*) Its tangential acceleration



Sol.



(i) $a_c = a\cos 30° = 25\frac{\sqrt{3}}{2}$ m/s^2

(ii) $a_c = \frac{v^2}{R} \Rightarrow v^2 = a_c R = 25\frac{\sqrt{3}}{2} \times 2.5$

$v = \left(125\frac{\sqrt{3}}{4}\right)^{1/2}$ m/s

(iii) $a_t = a\sin 30° = \frac{25}{2}$ m/s^2

26. Two particles *A* and *B* move anticlockwise with the same speed v in a circle of radius *R* and are diametrically opposite to each other. At $t = 0$, A is imparted a tangential acceleration of constant magnitude $a_t = \frac{72v^2}{25\pi R}$. Calculate the time in which *A* collides with *B*, the angle traced by *A* during this time, its angular velocity and radial acceleration of *A* at the time of collision.

Sol. $\omega_{0(\text{rel})} = 0,\ \theta_{\text{rel}} = \pi,\ \alpha_{\text{rel}} = \frac{72v^2}{25\pi R^2}$

$\theta_{\text{rel}} = \omega_{0(\text{rel})}t + \frac{1}{2}\alpha_{\text{rel}}t^2$

$\pi = 0 + \frac{1}{2}\frac{72v^2}{25\pi R^2}t^2 \Rightarrow t = \frac{5\pi R}{6v}$

Angle traced by *A*, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta = \frac{v}{R}\cdot\frac{5\pi R}{6v} + \frac{1}{2}\frac{72v^2}{25\pi R^2}\cdot\left(\frac{5\pi R}{6v}\right)^2$

$= \frac{5\pi}{6} + \pi = \frac{11}{6}\pi$

Angular velocity $\omega = \omega_0 + \alpha t = \frac{v}{R} + \frac{72v^2}{25\pi R^2}\cdot\left(\frac{5\pi R}{6v}\right)$

$= \frac{v}{R} + \frac{12v}{5R} = \frac{17v}{5R}$

$a_c = \omega^2 R = \left(\frac{17v}{5R}\right)^2 R = \frac{289v^2}{25R}$

School Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. Two bullets are fired horizontally with different velocities from the same height. Which will reach the ground first?

(*a*) Slower one

(*b*) Faster one

(*c*) Both will reach simultaneously

(*d*) Cannot be predicted

2. A projectile can have same range *R* for two angles of projection. If t_1 and t_2 are the times of flight in the two cases, then what is the product of two times of flight?

(*a*) $t_1t_2 \propto R^2$ (*b*) $t_1t_2 \propto R$

(*c*) $t_1t_2 \propto \frac{1}{R}$ (*d*) $t_1t_2 \propto \frac{1}{R^2}$

3. The range *R* of projectile is same when its maximum heights are h_1 and h_2. What is the relation between *R*, h_1 and h_2?

(*a*) $R = \sqrt{h_1h_2}$ (*b*) $R = \sqrt{2h_1h_2}$

(*c*) $R = 2\sqrt{h_1h_2}$ (*d*) $R = 4\sqrt{h_1h_2}$

4. Rain is falling vertically downwards with a speed of 4 km h^{-1}. A girl moves on a straight road with a velocity of 3 km h^{-1}. The apparent velocity of rain with respect to the girl is
(*a*) 3 km h^{-1}
(*b*) 4 km h^{-1}
(*c*) 5 km h^{-1}
(*d*) 7 km h^{-1}

5. In going from one city to another, a car travels 75 km north, 60 km north-west and 20 km east. The magnitude of displacement between the two cities is (take $1/\sqrt{2} = 0.7$)
(*a*) 170 km
(*b*) 137 km
(*c*) 119 km
(*d*) 140 km

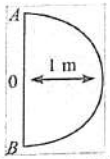
6. Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$. What is the angle between $\vec{a}$ and $\vec{b}$?
(*a*) 0°
(*b*) 90°
(*c*) 60°
(*d*) 180°

7. Given $\vec{A} = 2\hat{i} + p\hat{j} + q\hat{k}$ and $\vec{B} = 5\hat{i} + 7\hat{j} + 3\hat{k}$. If $\vec{A} \parallel \vec{B}$, then the values of p and q are, respectively,
(*a*) $\frac{14}{5}$ and $\frac{6}{5}$
(*b*) $\frac{14}{3}$ and $\frac{6}{5}$
(*c*) $\frac{6}{5}$ and $\frac{1}{3}$
(*d*) $\frac{3}{4}$ and $\frac{1}{4}$

8. A rotating stone has an angular velocity of 11 rad/s. In 0.5 seconds, how much angular displacement is covered?
(*a*) 5.5 rad (*b*) 0.55 rad (*c*) 55 rad (*d*) 0.5 rad

9. What could be the reason a car moving on a horizontal road gets thrown out of the road while taking a turn
(*a*) Due to the reaction of the ground
(*b*) Due to rolling frictional force between tyre and road
(*c*) By the gravitational force
(*d*) Due to lack of sufficient centripetal force

10. In 1 s, a particle goes from point *A* to point *B* moving in a semicircle as shown in figure. The magnitude of the average velocity is



(*a*) Zero (*b*) 1 m/s (*c*) 2 m/s (*d*) 3.14 m/s

Directions: These questions consist of two statements each, printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.
(*a*) Both Assertion and Reason are True and the Reason is a correct explanation of the Assertion.
(*b*) Both Assertion and Reason are True but Reason is not a correct explanation of the Assertion.
(*c*) Assertion is True but the Reason is False.
(*d*) Assertion is False but Reason is True.

11. **Assertion:** The cross product of two vectors is commutative, meaning $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$.
Reason: The cross product is defined as the product of magnitudes of the two vectors, the sine of the angle between them, and the unit vector perpendicular to the plane containing the two vectors.

12. **Assertion:** In the case of ground-to-ground projection of a projectile from ground the angle of projection with horizontal is 0 – 30°. There is no point on its path such that instantaneous velocity is normal to the initial velocity.
Reason: Maximum deviation of the projectile is 2θ = 60°.

VERY SHORT ANSWER TYPE QUESTIONS

13. A man *A* is going due east with a speed of 5 m/s. Another man *B* is going due north with a speed of 7 m/s. Find
(*a*) the relative velocity of *B* w.r.t. *A*.
(*b*) the relative velocity of *A* w.r.t. *B*.

14. A cricket ball is thrown at a speed of 28 ms^{-1} in a direction 30° above the horizontal. Calculate its maximum height.

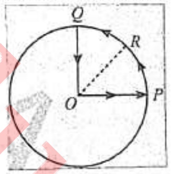
15. State the condition (regarding the value of dot or cross product) for which two vectors are:
(*a*) parallel to each other.
(*b*) perpendicular to each other.

16. If two vectors $\vec{a} = 4$ ms^{-1} and $\vec{b} = 7$ ms^{-1} be inclined at an angle of 60° to each other, then determine the direction and magnitude of their resultant.

17. A player kicks a ball at an angle of 37° to the horizontal with an initial speed of 15 ms^{-1}. Find its time of flight.

18. A circular road of radius 40 m has the angle of banking equal to 45°. At what speed should a vehicle go on this road so that the friction is not used?

19. A cyclist starts from centre O of a circular park of radius 1 km and moves along the path *OPRQO* as shown. If he maintains constant speed of 10ms^{-1}. What is his acceleration at point *R* in magnitude & direction?



SHORT ANSWER TYPE QUESTIONS

20. The equation of projectile is $y = 16x - \frac{5x^2}{4}$. Find the horizontal range.

21. A car with a vertical wind shield moves in a rain storm at the speed of 40 km/h. The rain drops fall vertically with a terminal speed of 20 m/s. Find the angle with the vertical at which the rain drops strike the wind shield.

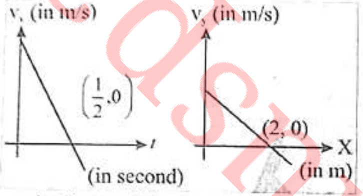
LONG ANSWER TYPE QUESTIONS

22. (*i*) Derive expression for the horizontal range of a projectile.
(*ii*) The maximum range of a projectile is $\frac{2}{\sqrt{3}}$ times the actual range for a given velocity of projection. What is the angle of projection for the actual range?

23. Given two vectors. $\vec{A}=3\hat{i}+\hat{j}+\hat{k}$ and $\vec{B}=\hat{i}-\hat{j}-\hat{k}$. Find the
(*a*) area of the triangle whose two sides are represented by the vectors $\vec{A}$ and $\vec{B}$.
(*b*) area of the parallelogram whose two adjacent sides are represented by the vectors $\vec{A}$ and $\vec{B}$.
(*c*) area of the parallelogram whose diagonals are represented by the vectors $\vec{A}$ and $\vec{B}$.

CASE STUDY BASED QUESTIONS

24. Two graphs of the same projectile motion (in the xy-plane) projected from origin are shown. x-axis is along horizontal direction and y-axis is vertically upwards. Take $g = 10\ ms^{-2}$.



(*i*) The projection speed is:
(*a*) $\sqrt{37}\ ms^{-1}$ (*b*) $\sqrt{41}\ ms^{-1}$
(*c*) $\sqrt{14}\ ms^{-1}$ (*d*) $\sqrt{40}\ ms^{-1}$

(*ii*) Projection angle with the horizontal is:
(*a*) $\tan^{-1}\left(\frac{4}{5}\right)$ (*b*) $\tan^{-1}\left(\frac{2}{3}\right)$
(*c*) $\tan^{-1}\left(\frac{5}{4}\right)$ (*d*) $\tan^{-1}\left(\frac{1}{2}\right)$

(*iii*) Maximum height attained from the point of projection is
(*a*) 1.25 m (*b*) 12.5 m
(*c*) 2.25 m (*d*) None of these

25. A uniform circular motion is the motion of a particle travelling at a constant (uniform) speed along circular path and hence its kinetic energy remains same everywhere. But when a particle moves in a vertical circle completing the loop, then its speed goes on changing at every point & hence, its kinetic energy goes on changing but total mechanical energy remains constant.

(*i*) Uniform circular motion is a example of
(*a*) accelerated motion
(*b*) uniform motion
(*c*) non-accelerated motion
(*d*) None of the above

(*ii*) An insect trapped in a circular groove of radius 14 cm moves along the groove steadily and completes 10 revolutions in 100 s. The linear speed of the insect is
(*a*) 4.3 cm s^{-1} (*b*) 8.8 cm s^{-1}
(*c*) 6.3 cm s^{-1} (*d*) 7.3 cm s^{-1}

(*iii*) What is the magnitude of average velocity after half rotation, if a particle is moving with constant speed v in a circle?
(*a*) $2v$ (*b*) $\frac{2v}{\pi}$ (*c*) $\frac{v}{\pi}$ (*d*) $\frac{v}{2\pi}$

(*iv*) A body moving along a circular path of radius R with velocity v, has centripetal acceleration a. If its velocity is made equal to $2v$. What will be the centripetal acceleration?
(*a*) $4a$ (*b*) $2a$ (*c*) $\frac{a}{4}$ (*d*) $\frac{a}{2}$

(*v*) A point moves along a circle with a velocity $v = at$, where $a = 0.50\ m/s^2$. Find the total acceleration of the point at the moment when it has covered the n^{th} ($n = 0.10$) fraction of the circle after beginning of the motion.
(*a*) $0.8ms^{-2}$ (*b*) $0.6ms^{-2}$
(*c*) $0.7ms^{-2}$ (*d*) $0.9ms^{-2}$

Prarambh (Topicwise)

RESULTANT OF TWO VECTORS

1. Two vectors $\vec{A}$ and $\vec{B}$ lie in a plane. Another vector $\vec{C}$ lies outside this plane. The resultant $\vec{A}+\vec{B}+\vec{C}$ of these three vectors
(*a*) Can be zero
(*b*) Cannot be zero
(*c*) Lies in the plane of $\vec{A}$ and $\vec{B}$
(*d*) Lies in the plane of $\vec{A}$ and $\vec{A}+\vec{B}$

2. The vector sum of the forces of 10 N and 6 N can be
(*a*) 2 N (*b*) 8 N (*c*) 18 N (*d*) 20 N

3. A set of vectors taken in a given order gives a closed polygon. Then the resultant of these vectors is a
(*a*) Scalar quantity (*b*) Pseudo vector
(*c*) Unit vector (*d*) Null vector.

4. A car drives 6.0 km east, then 8 km north, and then 21 km west. The magnitude of the resulting displacement from origin is
(*a*) 35 km (*b*) 23 km
(*c*) 21 km (*d*) 17 km

DIFFERENCE OF TWO VECTORS

5. Two vectors $\vec{a}$ and $\vec{b}$ inclined at an angle θ w.r.t. each other have a resultant $\vec{c}$ which makes an angle β with $\vec{a}$. If the directions of $\vec{a}$ and $\vec{b}$ are interchanged, then the resultant will have the same
(*a*) Magnitude
(*b*) Direction
(*c*) Magnitude as well as direction
(*d*) Neither magnitude nor direction

6. If $\vec{P}+\vec{Q}=\vec{P}-\vec{Q}$, and θ is the angle between $\vec{P}$ and $\vec{Q}$, then
(a) $\theta = 0^\circ$ (b) $\theta = 90^\circ$ (c) $\vec{P}=0$ (d) $\vec{Q}=0$

7. A particle is moving eastward with a speed of 5 m/s. In 10 s the velocity changes to 5 m/s northwards. The average acceleration in this time is
(a) Zero
(b) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-west.
(c) 1/2 m/s^2 towards north-west.
(d) 1/2 m/s^2 towards north.

8. The magnitudes of two forces $\vec{F_1}$ and $\vec{F_2}$ are 10 N and 8 N. The angle between them is 120°. The magnitude of their difference is
(a) $\sqrt{244}$ N (b) $\sqrt{84}$ N
(c) $\sqrt{164}$ N (d) None of these

VECTORS IN UNIT VECTOR FORM

9. The vector joining the points A (1, 1, −1) and B (2, −3, 4) and pointing from A to B is
(a) $-\hat{i}+4\hat{j}-5\hat{k}$ (b) $\hat{i}+4\hat{j}+5\hat{k}$
(c) $\hat{i}-4\hat{j}+5\hat{k}$ (d) $-\hat{i}-4\hat{j}-5\hat{k}$

10. The x and y components of a force are 2N and −3N. The force is
(a) $2\hat{i}-3\hat{j}$ (b) $2\hat{i}+3\hat{j}$ (c) $-2\hat{i}-3\hat{j}$ (d) $3\hat{i}+2\hat{j}$

11. The vector $\hat{i}+x\hat{j}+3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i}+(4x-2)\hat{j}+2\hat{k}$. The values of x are
(a) $-\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1

DOT PRODUCT

12. Three non zero vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}\cdot\vec{B}=0$ and $\vec{A}\cdot\vec{C}=0$. Then $\vec{A}$ can be parallel to:
(a) $\vec{B}$ (b) $\vec{C}$ (c) $\vec{B}\cdot\vec{C}$ (d) $\vec{B}\times\vec{C}$

13. The value of scalar product of two vectors is 8 and that of vector product is $8\sqrt{3}$. The angle between them is:
(a) 30° (b) 60° (c) 120° (d) 150°

14. A particle is moving under the influence of force $\vec{F}=[\hat{i}-2\hat{j}]N$, is now moved from the point (x, y, z) = (2, 1, 3) m to the point (x, y, z) = (3, 2, 4) m. How much work is done by the force $\vec{F}$ during this time period? ($W=\vec{F}.\vec{s}$)
(a) 0 J (b) −1 J
(c) 1 J (d) None of these

15. The angle between $(\hat{i}+\hat{j}+\hat{k})$ and $(2\hat{i}+2\hat{j}-2\hat{k})$ is
(a) $\cos^{-1}\frac{1}{3}$ (b) $\cos^{-1}\frac{1}{\sqrt{3}}$
(c) $\text{Sin}^{-1}\frac{1}{3}$ (d) None of these

16. If $\vec{A}\cdot\vec{B}=\vec{C}\cdot\vec{B}$, which of the following can not be a possible case?
(a) $\vec{A}=\vec{C}$
(b) angle between $\vec{A}$ and $\vec{B}$ = 30° and angle between $\vec{B}$ and $\vec{C}$ = 150° all three vectors are of equal magnitude.
(c) $\vec{B}$ is a null vector
(d) $\vec{A}$, $\vec{B}$ and $\vec{C}$ are along x, y and z-axis respectively

17. The vector $5\hat{i}+2\hat{j}-t\hat{k}$ is perpendicular to the vector $3\hat{i}+\hat{j}+2\hat{k}$ for t =
(a) 1 (b) 4.7 (c) 6.3 (d) 8.5

CROSS PRODUCT

18. What is the torque of the force $\vec{F}=(2\vec{i}-3\vec{j}+4\vec{k})\,N$ acting at the point $\vec{r}=(3\vec{i}+2\vec{j}+3\vec{k})$ m about the origin:
(a) $6\vec{i}-6\vec{j}+12\vec{k}$ (b) $17\vec{i}-6\vec{j}-13\vec{k}$
(c) $-6\vec{i}+6\vec{j}-12\vec{k}$ (d) $-17\vec{i}+6\vec{j}+13\vec{k}$

19. If $\vec{A}\times\vec{B}=\vec{C}$, then which of the following statements is wrong?
(a) $\vec{C}\perp\vec{A}$ (b) $\vec{C}\perp\vec{B}$
(c) $\vec{C}\perp(\vec{A}+\vec{B})$ (d) $\vec{C}\perp(\vec{A}\times\vec{B})$

20. Two vectors $\vec{P}$ and $\vec{Q}$ are inclined at an angle θ to each other. Which of the following is the unit vector perpendicular to $\vec{P}$ and $\vec{Q}$?
(a) $\frac{\vec{P}\times\vec{Q}}{PQ}$ (b) $\frac{\hat{P}\times\hat{Q}}{\sin\theta}$ (c) $\frac{\hat{P}\times\hat{Q}}{PQ\sin\theta}$ (d) $\frac{\vec{P}\times\vec{Q}}{PQ\sin\theta}$

21. If $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{B}=-\hat{i}+\hat{j}+4\hat{k}$, $\vec{C}=3\hat{i}-3\hat{j}-12\hat{k}$, then find the angle between the vectors $(\vec{A}+\vec{B}+\vec{C})$ and $(\vec{A}\times\vec{B})$ in degrees.
(a) θ = 90° (b) θ = 60° (c) θ = 45° (d) θ = 30°

22. Find a unit vector perpendicular to both the vectors $(2\hat{i}+3\hat{j}+\hat{k})$ and $(\hat{i}-\hat{j}+2\hat{k})$.
(a) $\pm\frac{1}{\sqrt{83}}(7\hat{i}-3\hat{j}-5\hat{k})$ (b) $\pm\frac{1}{\sqrt{63}}(7\hat{i}-3\hat{j}-5\hat{k})$
(c) $\pm\frac{1}{\sqrt{83}}(3\hat{i}-7\hat{j}-5\hat{k})$ (d) $\pm\frac{1}{\sqrt{38}}(7\hat{i}-5\hat{j}-3\hat{k})$

23. Find $\vec{A}.\vec{B}$ if $|\vec{A}|=2, |\vec{B}|=5$ and $(\vec{A}\times\vec{B})=8$
(a) $\vec{A}\cdot\vec{B}=\pm 6$ (b) $\vec{A}\cdot\vec{B}=\pm 12$
(c) $\vec{A}\cdot\vec{B}=\pm 8$ (d) $\vec{A}\cdot\vec{B}=\pm 16$

24. If $\sqrt{3}|\vec{A}\times\vec{B}|=\vec{A}.\vec{B}$, then find the angle between $\vec{A}$ and $\vec{B}$.
(a) 75° (b) 30° (c) 65° (d) 45°

25. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along X, Y and Z axis respectively, then tick the wrong statement
(a) $\hat{i}.\hat{i}=1$ (b) $\hat{i}\times\hat{j}=\hat{k}$
(c) $\hat{i}.\hat{j}=0$ (d) $\hat{i}\times\hat{k}=-\hat{i}$

PROJECTILE MOTION: GROUND TO GROUND PROJECTION

26. The point from where a ball is projected is taken as the origin of the coordinate axes. The x and y components of its displacement are given by x = 6t and y = 8t − 5t². What is the velocity of projection?
(a) 6 m s⁻¹ (b) 8 m s⁻¹ (c) 10 m s⁻¹ (d) 14 m s⁻¹

27. At an instant t, the coordinates of a particle are $x = at^2$, $y = bt^2$ and $z = 0$, then its speed at the instant t will be
(a) $t\sqrt{a^2+b^2}$ (b) $2t\sqrt{a^2+b^2}$
(c) $\sqrt{a^2+b^2}$ (d) $2t^2\sqrt{a^2+b^2}$

28. A ball is projected from a certain point on the surface of a planet at a certain angle with the horizontal surface. The horizontal and vertical displacements x and y vary with time t in second as:
$x = 10\sqrt{3}t$ and $y = 10t - t^2$
The maximum height attained by the ball is
(a) 100 m (b) 75 m (c) 50 m (d) 25 m

29. A projectile fired with initial velocity u at some angle θ has a range R. If the initial velocity be doubled at the same angle of projection, then the range will be
(a) 2R (b) R/2 (c) R (d) 4R

30. If the initial velocity of a projectile be doubled, keeping the angle of projection same, the maximum height reached by it will
(a) remain the same (b) be doubled
(c) be quadrupled (d) be halved

31. At the top of the trajectory of a projectile, the directions of its velocity and acceleration are
(a) perpendicular to each other
(b) parallel to each other
(c) inclined to each other at an angle of 45°
(d) antiparallel to each other

32. The range of a particle when launched at an angle of 15° with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle of 45° to the horizontal?
(a) 1.5 km (b) 3.0 km (c) 6.0 km (d) 0.75 km

33. A stone is projected from the ground with velocity 25 m/s. Two seconds later, it just clears a wall 5 m high. The angle of projection of the stone is ($g = 10 \text{m/sec}^2$)
(a) 30° (b) 45° (c) 37° (d) 60°

34. A projectile thrown with a speed v at an angle θ has a range R on the surface of earth. For same v and θ, its range on the surface of moon will be $\left[g_{moon} = \frac{g_{earth}}{6}\right]$
(a) R/6 (b) 6R (c) R/36 (d) 36R

35. A particle reaches its highest point when it has covered exactly one half of its horizontal range. The corresponding point on the displacement (in vertical direction) time graph is characterised by
(a) Negative slope (b) Zero slope
(c) Positive slope (d) either (a) or (c)

36. If the range of a gun which fires a shell with muzzle speed V is R, then the angle of elevation of the gun is
(a) $\cos^{-1}\left(\frac{V^2}{Rg}\right)$ (b) $\cos^{-1}\left(\frac{gR}{V^2}\right)$
(c) $\frac{1}{2}\left(\frac{V^2}{Rg}\right)$ (d) $\frac{1}{2}\sin^{-1}\left(\frac{gR}{V^2}\right)$

37. A ball is thrown upwards. It returns to ground describing a parabolic path. Which of the following remains constant?
(a) Speed of the ball
(b) Kinetic energy of the ball
(c) Vertical component of velocity
(d) Horizontal component of velocity.

38. A bullet is fired horizontally from a rifle at a distant target. Ignoring the effect of air resistance, which of the following is correct?

	Horizontal Acceleration	Vertical Acceleration
(a)	10 ms⁻²	10 ms⁻²
(b)	10 ms⁻²	0 ms⁻²
(c)	0 ms⁻²	10 ms⁻²
(d)	0 ms⁻²	0 ms⁻²

39. A particle is projected with a velocity of 50 m/s at 37° with horizontal. Find the co-ordinates of the particle (w.r.t. the starting point) after 2 s.
Given, g = 10 m/s², sin 37° = 0.6 and cos 37° = 0.8
(a) (40, 80) (b) (80, 40) (c) (60, 80) (d) (80, 60)

40. Find the angle of projection of a projectile for which the horizontal range and maximum height are equal.
(a) 45° (b) tan⁻¹(4)
(c) tan⁻¹(2) (d) None of these

41. A projectile is thrown with velocity v making an angle θ with the horizontal. It just crosses the top of two poles, each of height h, after 1 second and 3 seconds respectively. The time of flight of the projectile is
(a) 1 s (b) 3 s (c) 4 s (d) 7.8 s

42. It was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $\frac{5\pi}{36}$ radians should strike a given target in the same horizontal plane. In actual practice, it was found that a hill just prevented the trajectory. At what angle of elevation (in radians) should the gun be fired to hit the target?
(a) $\frac{5\pi}{36}$ (b) $\frac{11\pi}{36}$ (c) $\frac{7\pi}{36}$ (d) $\frac{13\pi}{36}$

43. A projectile is thrown with a speed v at an angle θ with the vertical. Its average velocity between the instants it crosses half the maximum height is
(a) v sin θ, horizontal and in the plane of projection
(b) v cos θ, horizontal and in the plane of projection
(c) 2v sin θ, horizontal and perpendicular to the plane of projection
(d) 2v cos θ, vertical and in the plane of projection.

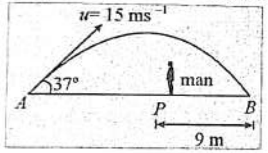
44. During projectile motion, acceleration of a particle at the highest point of its trajectory is
(a) g
(b) zero
(c) less than g
(d) dependent upon projection velocity

45. The maximum range of a projectile is 22 m. When it is thrown at an angle of 15° with the horizontal, its range will be-
(a) 22 m (b) 6 m (c) 15 m (d) 11 m

46. The equation of projectile is $y = 16x - \frac{5x^2}{4}$. The horizontal range is:
(a) 16 m (b) 8 m (c) 3.2 m (d) 12.8 m

47. If four balls A, B, C and D are projected with same speed at the angles of 15°, 30°, 45° and 60° with the horizontal respectively. The two balls which will fall at the same place will be-
(a) A and B (b) A and D
(c) B and D (d) A and C

48. A ball is hit by a batsman at an angle of 37° as shown in figure. The man standing at P should run at what minimum velocity so that he catches the ball before it strikes the ground? Assume that height of man is negligible in comparison to maximum height of projectile.



(a) $3\ \text{ms}^{-1}$ (b) $5\ \text{ms}^{-1}$
(c) $9\ \text{ms}^{-1}$ (d) $12\ \text{ms}^{-1}$

49. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?
(a) The one with the farthest range.
(b) The one which reaches maximum height
(c) The one with the greatest initial velocity
(d) The one leaving the bat at 45° with respect to the ground.

50. A particle is projected from the ground with velocity u at angle θ with horizontal. The horizontal range, maximum height and time of flight are R, H and T respectively. They are given by,

$$R = \frac{u^2 \sin 2\theta}{g},\ H = \frac{u^2 \sin^2\theta}{2g} \text{ and } T = \frac{2u \sin\theta}{g}$$

Now keeping u as fixed, θ is varied from 30° to 60°. Then,
(a) R will first increase then decrease, H will increase and T will decrease
(b) R will first increase then decrease while H and T both will increase
(c) R will decrease while H and T will increase
(d) R, H and T will increase

51. A point mass is projected, making an acute angle with the horizontal. If angle between velocity $\vec{v}$ and acceleration $\vec{g}$ is θ at any time t during the motion, then θ is given by
(a) $0° < \theta < 90°$ (b) $\theta = 90°$
(c) $\theta < 90°$ (d) $0° < \theta < 180°$

52. The velocity at the maximum height of a projectile is half of its initial velocity u. Its range on the horizontal plane is:
(a) $\frac{2u^2}{3g}$ (b) $\frac{\sqrt{3}u^2}{2g}$ (c) $\frac{u^2}{3g}$ (d) $\frac{u^2}{2g}$

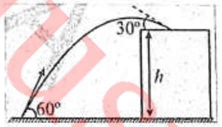
PROJECTILE THROWN FROM SOME HEIGHT ABOVE GROUND

53. A stone is just released from the window of a train moving along a horizontal straight track. The stone will hit the ground following
(a) Straight path (b) Circular path
(c) Parabolic path (d) Hyperbolic path

54. An aeroplane moving horizontally with a speed of 720 km/h drops a food packet, while flying at a height of 396.9 m. The time taken by a food packet to reach the ground and its horizontal range is
(Take $g = 9.8\ \text{m/sec}^2$)
(a) 3 s and 2000 m (b) 5 s and 500 m
(c) 8 s and 1500 m (d) 9 s and 1800 m

55. A plane flying horizontally at a height of 1500 m with a velocity of $200\ \text{ms}^{-1}$ passes directly overhead an antiaircraft gun. The angle with the horizontal at which the gun should be fired so that the shell with a muzzle velocity of $400\ \text{ms}^{-1}$ hits the plane, is:
(a) 90° (b) 60° (c) 30° (d) 45°

56. A stone projected at an angle of 60° from the ground level strikes at an angle of 30° on the roof of a building of height 'h'. Then the speed of projection of the stone is:



(a) $\sqrt{2gh}$ (b) $\sqrt{6gh}$ (c) $\sqrt{3gh}$ (d) $\sqrt{gh}$

57. One stone is projected horizontally from a 20 m high cliff with an initial speed of $10\ \text{ms}^{-1}$. A second stone is simultaneously dropped from that cliff. Which of the following is true?
(a) Both strike the ground with the same speed.
(b) The ball with initial speed $10\ \text{ms}^{-1}$ reaches the ground first.
(c) Both the balls hit the ground at different times.
(d) Both the balls strike the ground with different speeds.

58. A ball is projected from top of a tower with a velocity of 5 m/s at an angle of 53° to horizontal. Its speed when it is at a height of 0.45 m from the point of projection is
(a) 2 m/s (b) 3 m/s
(c) 4 m/s (d) Data insufficient

59. An aeroplane flying at a constant velocity releases a bomb. As the bomb drops down from the aeroplane

(a) it will always be vertically below the aeroplane

(b) it will always be vertically below the aeroplane only if the aeroplane is flying horizontally

(c) it will always be vertically below the aeroplane only if the aeroplane is flying at an angle of 45° to the horizontal.

(d) it will gradually fall behind the aeroplane if the aeroplane is flying horizontally

60. A body is projected horizontally from the top of a tower with initial velocity 18 ms^{-1}. It hits the ground at an angle of 45° with horizontal. What is the vertical component of velocity when the body strikes the ground?

(a) $18\sqrt{2}$ ms^{-1} (b) 18 ms^{-1}

(c) $9\sqrt{2}$ ms^{-1} (d) 9 ms^{-1}

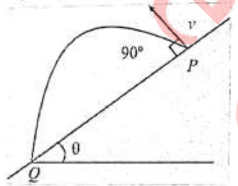
PROJECTILE MOTION ON AN INCLINED PLANE

61. Find time of flight of projectile thrown horizontally with speed 10 ms^{-1} from a long inclined plane which makes an angle of 45° with the horizontal.

(a) $\sqrt{2}$ sec (b) $2\sqrt{2}$ sec

(c) 2 sec (d) None of these

62. If time taken by the projectile to reach Q is T, then PQ =



(a) $Tv\sin\theta$ (b) $Tv\cos\theta$

(c) $Tv\sec\theta$ (d) $Tv\tan\theta$

63. A ball is horizontally projected with a speed v from the top of a plane inclined which is at an angle 45° with the horizontal. How far from the point of projection will the ball strike the plane?

(a) $\frac{v^2}{g}$ (b) $\frac{\sqrt{2}v^2}{g}$ (c) $\frac{2v^2}{g}$ (d) $\left[\frac{2\sqrt{2}v^2}{g}\right]$

64. A particle is projected at 37° with the incline plane in upward direction with speed 10 m/s. The angle of incline plane with the horizontal is 53°. Then the maximum height above the incline plane attained by the particle will be

(a) 3m (b) 4 m (c) 5 m (d) Zero

TWO DIMENSIONAL RELATIVE MOTION

65. A bird is flying towards south with a velocity 40km/hr and a train is moving with a velocity 40 km/hr towards east. What is the velocity of the bird w.r.t. an observer in train

(a) $40\sqrt{2}$ km/hr. North – East

(b) $40\sqrt{2}$ km/hr. South – East

(c) $40\sqrt{2}$ km/hr. South – West

(d) $40\sqrt{2}$ km/hr. North – West

66. A train is moving towards east and a car is along north, both with same speed. The observed direction of car to the passenger in the train is

(a) East-north direction (b) West-north direction

(c) South-east direction (d) None of these

67. A ship is sailing towards north at a speed of $\sqrt{2}$ m/s. The current is taking it towards East at the rate of 1 m/s and a sailor is climbing a vertical pole on the ship at the rate of 1 m/s. Find the velocity of the sailor with respect to ground

(a) $2\hat{i}+\sqrt{2}\hat{j}+\hat{k}$ (b) $\hat{i}+\sqrt{2}\hat{j}+\hat{k}$

(c) $\sqrt{2}\hat{i}+\hat{j}+\hat{k}$ (d) $\hat{i}+\hat{j}+\sqrt{2}\hat{k}$

68. A helicopter is flying south with a speed of 50 kmh^{-1}. A train is moving with the same speed towards east. The relative velocity of the helicopter as seen by the passengers in the train will be towards.

(a) North east (b) South east

(c) North west (d) South west

69. Wind is blowing in the north direction at speed of 2 ms^{-1} which causes the rain to fall at some angle with the vertical. With what velocity should a cyclist drive so that the rain appears vertical to him:

(a) 2 ms^{-1} south (b) 2 ms^{-1} north

(c) 2 ms^{-1} west (d) 4 ms^{-1} south

RAIN MAN PROBLEM

70. A man is walking on a road with a velocity 3 km/hr. Suddenly rain starts falling. The velocity of rain is 10 km/hr in vertically downward direction. the relative velocity of the rain with respect to man is:

(a) $\sqrt{13}$ km/hr (b) $\sqrt{7}$ km/hr

(c) $\sqrt{109}$ km/hr (d) 13 km/hr

71. A boy is running on the plane road with velocity (v) with a long hollow tube in his hand. The water is falling vertically downwards with velocity (u). At what angle to the vertical, he must incline the tube so that the water drops enters in it without touching its side

(a) $\tan^{-1}\left(\frac{v}{u}\right)$ (b) $\sin^{-1}\left(\frac{v}{u}\right)$ (c) $\tan^{-1}\left(\frac{u}{v}\right)$ (d) $\cos^{-1}\left(\frac{v}{u}\right)$

72. A man standing on a road has to hold his umbrella at 37° with the vertical to keep the rain away. He throws the umbrella and starts running at 6 km/hr then he finds that rain drops are hitting his head vertically, then speed of rain drops with respect to moving man:

(a) 20 km/hr. (b) 8 km/hr.

(c) $\frac{10}{\sqrt{3}}$ km/hr. (d) 10 km/hr.

73. A man walks in rain with a velocity of 5 kmh^{-1}. The rain drops strike at him at an angle of 45° with the horizontal. Velocity of rain if it is falling vertically downward -

(a) 5 km h^{-1} (b) 4 km h^{-1}

(c) 3 km h^{-1} (d) 1 km h^{-1}

74. To man running at a speed of 25 m/sec,the rain drops appear to be falling at an angle of 45° from the vertical. If the rain drops are actually falling vertically downwards, then velocity of rain w.r.t. man (in m/sec) is

(a) 25 (b) $25\sqrt{3}$ (c) $25\sqrt{2}$ (d) 4

RIVER MAN PROBLEM

75. A swimmer crosses the river along the line making an angle of 45° with the direction of flow. Velocity of the river water is 5 m/s. Swimmer takes 6 seconds to cross the river of width 60 m. The velocity of the swimmer with respect to water will be:

(a) 10 m/s (b) 12 m/s (c) $5\sqrt{5}$ m/s (d) $10\sqrt{2}$ m/s

76. A boat is rowed across a river (perpendicular to river flow) at the rate of 4.5 km/hr. The river flows at the rate of 6 km/hr. The velocity of boat in m/s is:

(a) 3.1 (b) 2.1 (c) 2.9 (d) 5

77. A boat is sent across a river with a velocity of 8 km/hr. If the resultant velocity of boat is 10 km/hr, then velocity of the river is

(a) 10 km/hr (b) 8 km/hr
(c) 6 km/hr (d) 4 km/hr

78. A boat is moving with velocity of $3\hat{i}+4\hat{j}$ in river and water is moving with a velocity of $-3\hat{i}-4\hat{j}$ with respect to ground. Relative velocity of boat with respect to water is:

(a) $-6\hat{i}-8\hat{j}$ (b) $6\hat{i}+8\hat{j}$
(c) $8\hat{i}$ (d) $6\hat{i}$

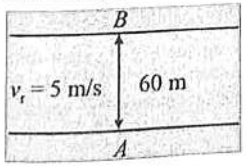
79. A river is flowing from west to east with a speed of 5 m/min. A man can swim in still water with a velocity 10 m/min. In which direction should the man swim so as to take the shortest possible path to go to the south.

(a) 30° with downstream (b) 60° with downstream
(c) 120° with downstream (d) South

80. A river is flowing from west to east at a speed of 5 meters per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction

(a) due north (b) 30° east of north
(c) 30° north of west (d) 60° east of north

81. A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across at a distance of 60 meter in 5 sec. His velocity in still water should be



(a) 12 m/s (b) 13 m/s
(c) 5 m/s (d) 10 m/s

82. To cross the river in shortest distance, a swimmer should swim making angle θ with the upstream. What is the ratio of the time taken to swim across in the shortest time to that in swimming across over shortest distance. [Assume speed of swimmer in still water is greater than the speed of river flow]

(a) cosθ (b) sin θ (c) tan θ (d) cot θ

83. A man can swim with a speed of 4 km h^{-1} in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km h^{-1} and he makes his strokes normal to the river current?

(a) $\frac{1}{3}$hr (b) $\frac{1}{2}$hr
(c) $\frac{1}{4}$hr (d) 1hr

84. A swimmer's speed in the direction of flow of river is 16 km h^{-1}. Swimmer's speed against the direction of flow of river is 8 km h^{-1}. Calculate the swimmer's speed in still water and the velocity of flow of the river (in km).

(a) 2, 8 (b) 12, 4 (c) 4, 8 (d) 8, 2

85. A motorboat going down stream overcome a float at a point M which is fixed with respect to ground. 60 minutes later it turned back and after some time passed meets the float at a distance of 6 km from the point *M*. Find the velocity of the stream assuming a constant velocity for the motorboat in still water.

(a) 4 km/h (b) 7 km/h (c) 3 km/h (d) 6 km/h

86. A ship goes along a river and returns in time t_0 at a speed 2 m/s. On a particular day, a uniform current at speed 1 m/s is present to help the onward journey and oppose the return journey. If the time taken to go along and return on the rough day be *t*, then find the value of t/t_0.

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{5}{4}$ (d) $\frac{1}{2}$

87. Two boats were going down stream with different velocities. When one overtook the other a plastic ball was dropped from one of the boats into water. Some time later both boats turned back simultaneously and went at the same speeds as before (relative to the water) towards the spot where the ball had been dropped. Which boat will reach the ball first?

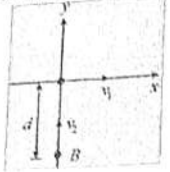
(a) The boat which has greater velocity (relative to water).
(b) The boat which has lesser velocity (relative to water).
(c) Both will reach the ball simultaneously.
(d) Cannot be decided unless we know the actual values of the velocities and the time after which they turned around.

DISTANCE OF NEAREST APPROACH

88. Four particles situated at the corners of a square of side '*a*' move at a constant speed *v*. Each particle maintains a direction towards the next particle in succession. Calculate the time the particles will take to meet each other.

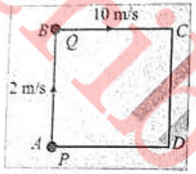
(a) *a*/*v* (b) 2*a*/*v* (c) *a*/2*v* (d) *a*/3*v*

89. Two particles A and B move with velocities v_1 and v_2 respectively along the x and y axis. The initial separation between them is 'd' as shown in the figure. Find the least distance between them during their motion.



(a) $\frac{dv_1^2}{v_1^2+v_2^2}$ (b) $\frac{dv_2^2}{v_1^2+v_2^2}$ (c) $\frac{dv_1}{\sqrt{v_1^2+v_2^2}}$ (d) $\frac{dv_2}{\sqrt{v_1^2+v_2^2}}$

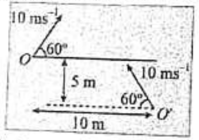
90. Two men P and Q are standing at corners A and B of square $ABCD$ of side 8 m. They start moving along the track with constant speed 2 m/s and 10 m/s respectively. Find the time when they will meet for the first time.



(a) 2 sec (b) 3 sec (c) 1 sec (d) 6 sec

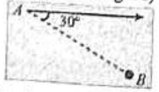
RELATIVE MOTION BETWEEN TWO PROJECTILES

91. Two particles are projected simultaneously from two points O and O' such that 10 m is the horizontal and 5 m is the vertical distance between them as shown in the figure. They are projected at the same inclination 60° to the horizontal with the same velocity 10 ms^{-1}. The time after which their separation becomes minimum is



(a) 2.5 sec (b) 1 sec (c) 5 sec (d) 10 sec

92. A particle A is moving with a constant velocity of 10 m/sec. Another particle B is moving with a constant but unknown velocity. At an instant, the line joining A and B makes an angle of 30° with velocity of A. Find the minimum possible magnitude of velocity of B, if they collide after some time. (see figure)



(a) 5 m/s (b) 4 m/s (c) 10 m/s (d) 6 m/s

93. Two particles are moving with velocities v_1 and v_2. Their relative velocity is the maximum, when the angle between their velocities are:

(a) Zero (b) $\pi/4$ (c) $\pi/2$ (d) π

94. Two billiard balls are rolling on a flat table. One has velocity components $v_x = 1$m/s, $v_y = \sqrt{3}$ m/s and the other has components $v_x = 2$m/s and $v_y = 2$ m/s. If both the balls start moving from the same point, the angle between their path is

(a) 60° (b) 45° (c) 22.5° (d) 15°

KINEMATICS OF CIRCULAR MOTION

95. If a particle moves in a circle describing equal angles in equal times, its velocity vector
(a) Remains constant
(b) Changes in magnitude
(c) Changes in direction
(d) Changes both in magnitude and direction

96. A particle moves with constant angular velocity in a circle. During the motion its
(a) Kinetic energy remains constant
(b) Momentum is constant
(c) Both kinetic energy and momentum both are constant
(d) None of the above is true

97. The length of second's hand in a watch is 1 cm. The change in velocity of its tip in 15 seconds is

(a) Zero (b) $\frac{\pi}{30\sqrt{2}}$cm/sec

(c) $\frac{\pi}{30}$cm/sec (d) $\frac{\pi\sqrt{2}}{30}$cm/sec

98. What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$?

(a) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (b) $-18\hat{i} - 13\hat{j} + 2\hat{k}$

(c) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (d) $6\hat{i} - 2\hat{j} + 8\hat{k}$

99. A wheel is of diameter 1m. If it makes 30 revolutions/sec., then the linear speed of a point on its circumference will be

(a) 30π m/s (b) π m/s (c) 60π m/s (d) $\pi/2$ m/s

100. In uniform circular motion (angular momentum $\vec{L} = m\vec{r} \times \vec{v}$)
(a) Both the angular velocity and the angular momentum vary
(b) The angular velocity varies but the angular momentum remains constant.
(c) Both the angular velocity and the angular momentum stay constant
(d) The angular momentum varies but the angular velocity remains constant.

CENTRIPETAL/TANGENTIAL/NET ACCELERATION

101. A body is moving in a circular path with a constant speed. It has
(a) A constant velocity
(b) A constant acceleration
(c) An acceleration of constant magnitude
(d) An acceleration which varies in magnitude and direction

102. Two bodies of mass 10 kg and 5 kg are moving in concentric orbits of radii R and r such that their periods are the same. Then the ratio between their centripetal acceleration is

(a) R/r (b) r/R (c) R^2/r^2 (d) r^2/R^2

103. A stone is tied to one end of a string 50 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 10 revolutions in 20 s, what is the magnitude of acceleration of the stone?

(a) 493 cm/s^2 (b) 720 cm/s^2

(c) 860 cm/s^2 (d) 990 cm/s^2

104. Two particles P and Q are located at distances r_P and r_Q respectively from the axis of a rotating disc such that $r_P > r_Q$, then

(a) Both P and Q have the same acceleration

(b) Both P and Q do not have any acceleration

(c) P has greater acceleration than Q

(d) Q has greater acceleration than P

105. The formula for centripetal acceleration in a circular motion is

(a) $\vec{\alpha} \times \vec{r}$ (b) $\vec{\omega} \times \vec{v}$ (c) $\vec{\alpha} \times \vec{v}$ (d) $\vec{\omega} \times \vec{r}$

106. A particle is moving in a horizontal circle with constant speed. It has constant

(a) Velocity (b) Acceleration

(c) Kinetic energy (d) Displacement

107. What happens to the centripetal acceleration of a revolving body if you double the orbital speed v and halve the angular velocity ω?

(a) The centripetal acceleration remains unchanged.

(b) The centripetal acceleration is halved.

(c) The centripetal acceleration is doubled.

(d) The centripetal acceleration is quadrupled.

108. A particle is moving along a circular path. The angular velocity, linear velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are $\vec{\omega}, \vec{v}, \vec{\alpha}$ and $\vec{a}_c$. Which of the following relations is not correct?

(a) $\vec{\omega} \perp \vec{v}$ (b) $\vec{\omega} \perp \vec{\alpha}$ (c) $\vec{\omega} \perp \vec{a}_c$ (d) $\vec{v} \perp \vec{a}_c$

109. A particle is moving in circular path with constant tangential acceleration. Time t after the beginning of motion the direction of net acceleration is at 45° to radius vector at the instant. The angular acceleration of the particle at time 't' is proportional to:

(a) $\frac{1}{t}$ (b) $\frac{1}{t^2}$ (c) $\frac{3}{t}$ (d) t^0

110. A car is travelling with linear velocity v on a circular road of radius r. If it is increasing its speed at the rate of 'a' m/s^2, then the resultant acceleration will be:

(a) $\sqrt{\left(\frac{v^2}{r^2} - a^2\right)}$ (b) $\sqrt{\left(\frac{v^4}{r^2} + a^2\right)}$

(c) $\sqrt{\left(\frac{v^4}{r^2} - a^2\right)}$ (d) $\sqrt{\left(\frac{v^2}{r^2} + a^2\right)}$

Prabal (JEE Main Level)

1. If the angle between two forces increases, the magnitude of their resultant

(a) decreases

(b) increases

(c) remains unchanged

(d) first decreases and then increases

2. Which of the following sets of displacements might be capable of bringing a car to its initial point

(a) 5, 10, 30 and 50 unit

(b) 5, 9, 9 and 16 unit

(c) 40, 40, 90 and 200 unit

(d) 10, 20, 40 and 90 unit

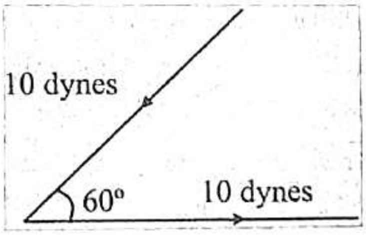
3. If the resultant of two forces of magnitudes P and Q acting at a point at an angle of 60° is $\sqrt{7}Q$, then P/Q is

(a) 1 (b) 3/2 (c) 2 (d) 4

4. The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces. The angle between the two forces is

(a) 150° (b) 90° (c) 60° (d) 120°

5. Two forces each numerically equal to 10 dynes are acting as shown in the following figure, then their resultant is:



(a) 10 dynes (b) 20 dynes

(c) $10\sqrt{3}$ dynes (d) 5 dynes

6. The vector $\vec{P}$ makes 120° with the x-axis and vector $\vec{Q}$ makes 30° with the y-axis. What is their resultant? (taking anticlockwise as positive)

(a) $P + Q$ (b) $P - Q$

(c) $\sqrt{P^2 + Q^2}$ (d) $\sqrt{P^2 - Q^2}$

7. A vector $\vec{A}$ is directed along 30° west of north direction and another vector $\vec{B}$ along 15° south of east. Their resultant cannot be in ________ direction.

(a) North (b) East

(c) North-East (d) South

8. The greatest and the least resultant of two forces acting at a point are 12 N and 8 N respectively. Find out the forces.
(*a*) 12 N and 8 N (*b*) 4 N and 8 N
(*c*) 3 N and 9 N (*d*) 10 N and 2 N

9. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}-\vec{c}=\vec{0}$, then the angle between $\vec{a}$ and $\vec{b}$ is
(*a*) 30° (*b*) 60° (*c*) 90° (*d*) 120°

10. A car is moving on a straight road due north with a uniform speed of 50 km h^{-1}. When it turns left through 90°, if the speed remains unchanged after turning, the change in the velocity of the car in the turning process is
(*a*) Zero
(*b*) $50\sqrt{2}$ km h^{-1} S-W direction
(*c*) $50\sqrt{2}$ km h^{-1} N-W direction
(*d*) 50 km h^{-1} due west.

11. A body is moving in circular path with constant speed v and arc AB, describing angle θ, at the centre. The change of velocity will be
(*a*) $\left(4v\sin\frac{\theta}{4}\right)$ (*b*) $\left(6v\sin\frac{\theta}{2}\right)$
(*c*) $\left(3v\sin\frac{\theta}{2}\right)$ (*d*) $\left(2v\sin\frac{\theta}{2}\right)$

12. If the angle between the unit vectors $\hat{a}$ and $\hat{b}$ is 60°, then $|\hat{a}-\hat{b}|$ is
(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 4

13. The angle that the vector $\vec{A}=2\hat{i}+3\hat{j}$ makes with y-axis is:
(*a*) $\tan^{-1}(3/2)$ (*b*) $\tan^{-1}(2/3)$
(*c*) $\sin^{-1}(2/3)$ (*d*) $\cos^{-1}(3/2)$

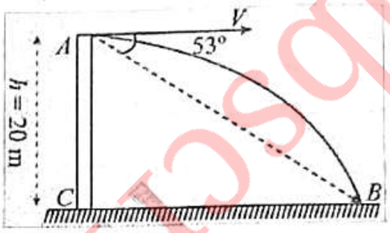
14. If $(\vec{a}+\vec{b})$ is perpendicular to $\vec{b}$ and $\vec{a}+2\vec{b}$ is perpendicular to $\vec{a}$, $|\vec{a}|=a, |\vec{b}|=b$, then
(*a*) $a=b$ (*b*) $a=2b$
(*c*) $b=2a$ (*d*) $a=b\sqrt{2}$

15. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is equal to
(*a*) -1 (*b*) 3 (*c*) 0 (*d*) $-\frac{3}{2}$

16. A vector $\vec{A}$ points vertically downward and $\vec{B}$ points towards east, then the vector product $\vec{A}\times\vec{B}$ is
(*a*) Along west (*b*) Along east
(*c*) Zero (*d*) Along south

17. The sum, difference and cross product of two vectors $\vec{A}$ and $\vec{B}$ are mutually perpendicular only if:
(*a*) $\vec{A}$ and $\vec{B}$ are perpendicular to each other and modulus of $\vec{A}$ is equal to modulus of $\vec{B}$.
(*b*) $\vec{A}$ and $\vec{B}$ are perpendicular to each other.
(*c*) $\vec{A}$ and $\vec{B}$ are perpendicular but magnitudes are arbitrary.
(*d*) modulus of $\vec{A}$ = modulus of $\vec{B}$ and their directions are arbitrary.

18. The line from 'A' to 'B' makes an angle of 53° with the horizontal. The initial velocity of the object is : (take g = 10 m/s^2)



(*a*) 7.5 m/s (*b*) 15 m/s (*c*) 5 m/s (*d*) 20 m/s

19. Two particles are fired from the same point with same speeds 100 m/s and their firing angles with horizontal are 60° and 120° respectively. The time after which their velocity vectors become perpendicular to each other, is
(*a*) 5 s (*b*) $5(\sqrt{3}-1)$ s (*c*) $5\sqrt{3}$ s (*d*) $5\sqrt{3}/2$ s

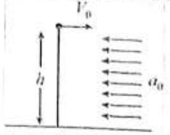
20. The equation of the path of the projectile is $y=0.5x-0.04x^2$. The initial speed of the projectile is
(*a*) 10 m/s (*b*) 15 m/s
(*c*) 12.5 m/s (*d*) None of these

21. A particle starts from the origin at $t=0$ and moves in the xy plane with constant acceleration a in the y-direction. Its equation of path is $y=bx^2$. The x-component of its velocity is
(*a*) variable (*b*) $\sqrt{\left(\frac{2a}{b}\right)}$ (*c*) $\frac{a}{2b}$ (*d*) $\sqrt{\left(\frac{a}{2b}\right)}$

22. The horizontal range of a projectile is R and the maximum height attained by it is H. A strong wind now begins to blow in the direction of motion of the projectile, giving it a constant horizontal acceleration = $g/2$. Under the same conditions of projection, the horizontal range of the projectile will now be.
(*a*) $R+\frac{H}{2}$ (*b*) $R+H$ (*c*) $R+\frac{3H}{2}$ (*d*) $R+2H$

23. A projectile is fired horizontally from an inclined plane (of inclination 45° with horizontal) with a speed of 50 m/s. If g = 10 m/s^2, the range measured along the incline is
(*a*) 500 m (*b*) $500\sqrt{2}$ m
(*c*) $200\sqrt{2}$ m (*d*) None of these

24. A particle is projected horizontally from a tower with velocity V_0. Wind is blowing in opposite direction and is providing a constant horizontal acceleration a_0. If particle strikes the ground vertically then height of building is



(*a*) $\frac{V_0^2 g}{a_0^2}$ (*b*) $\frac{V_0^2 g}{2a_0^2}$ (*c*) $\frac{2V_0^2 g}{a_0^2}$ (*d*) $\frac{V_0^2}{g}$

25. A particle is projected at an angle of 45° from a point lying 2 m from the foot of a wall. It just touches the top of the wall and falls on the ground 4 m from it on the other side of the wall. The height of the wall is

(*a*) 3/4 m (*b*) 2/3 m (*c*) 4/3 m (*d*) 1/3 m

26. At $t = 0$ a particle leaves the origin with a velocity of 6 m/s in the positive y direction. Its acceleration is given by $\vec{a} = 2\hat{i} - 3\hat{j}$ m/s^2. The x and y coordinates of the particle at the instant the particle reaches maximum y coordinate are

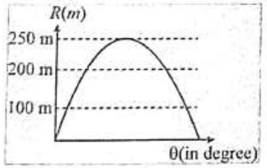
(*a*) 2m, 3m (*b*) 4m, 6m (*c*) 1m, 3m (*d*) 2m, 6m

27. Average velocity of a particle in projectile motion between its starting point and the highest point of its trajectory is: (projection speed = u, angle of projection from horizontal = θ)

(*a*) $u \cos\theta$ (*b*) $\frac{u}{2}\sqrt{1+3\cos^2\theta}$

(*c*) $\frac{u}{2}\sqrt{2+\cos^2\theta}$ (*d*) $\frac{u}{2}\sqrt{1+\cos^2\theta}$

28. From the ground level a ball is to be shot with a certain speed. Graph shows the range R it will have versus the launch angle θ. The least speed the ball will have during its flight if θ is chosen such that the flight time is half of its maximum possible value, is equal to (Take $g = 10$ m/s^2)



(*a*) 250 m/s (*b*) $50\sqrt{3}$ m/s

(*c*) 50 m/s (*d*) $25\sqrt{3}$ m/s

29. A plane surface is inclined making an angle θ with the horizontal. From the bottom of this inclined plane, a bullet is fired with velocity v. The maximum possible range of the bullet on the inclined plane is

(*a*) $\frac{v^2}{g}$ (*b*) $\frac{v^2}{g(1+\sin\theta)}$

(*c*) $\frac{v^2}{g(1-\sin\theta)}$ (*d*) $\frac{v^2}{g(1+\cos\theta)}$

30. Trajectory of particle in a projectile motion is given as $y = x - x^2/80$. Here, x and y are in metres and considered along horizontal and vertical direction respectively ($g = 10$ m/s^2). For this projectile motion.

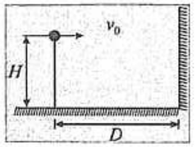
(*a*) angle of projection is 45°

(*b*) angle of velocity with horizontal after 4s is $\tan^{-1}(1/2)$

(*c*) maximum height is 80 m

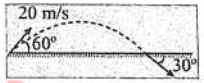
(*d*) horizontal range is 20 m

31. A ball is projected horizontally from a table such that it collides with wall then with ground. If after collision component of velocity perpendicular to the surface is reversed in direction without change in magnitude and component of velocity parallel to the surface remains unchanged. At what distance from the wall does the ball collide with the ground. (H = 500 m, v_0 = 20 m/s, D = 100 m)



(*a*) 100 m (*b*) 200 m (*c*) 50 m (*d*) 150 m

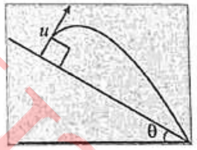
32. A particle is projected from a horizontal surface with a velocity of 20 m/s at an angle of 60° with the horizontal. When it hits the horizontal surface, its velocity makes an angle of 30° with the horizontal. Apart from gravity, a horizontal force acts on the particle during the motion. The speed of the particle when it hits the ground is



(*a*) 20 m/s (*b*) $20\sqrt{3}$ m/s

(*c*) 10 m/s (*d*) $10\sqrt{3}$ m/s

33. A particle is projected perpendicularly to an inclined plane as shown in the figure. If the initial velocity of the particle is u, calculate how far from the point of projection does it hit the plane again if the distance is measured along the plane?



(*a*) $\frac{2u^2}{g}$ (*b*) zero

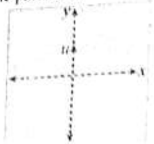
(*c*) $\frac{2u^2}{g}\sin\theta$ (*d*) $\frac{2u^2}{g}\tan\theta\sec\theta$

34. A particle is released from a certain height H = 400 m. Due to the wind the particle gathers the horizontal velocity component $v_x = ay$ where $a = \sqrt{5}\,\text{s}^{-1}$ and y is the vertical displacement of the particle from point of release, then the horizontal drift of the particle when it strikes the ground is

(*a*) 2.67 km (*b*) 8.67 km

(*c*) 1.67 km (*d*) 5.1 km

35. A particle is projected with speed u from O along y-axis. If acceleration of the particle is $(a\hat{i} + a\hat{j})$, then equation of trajectory of the particle is:



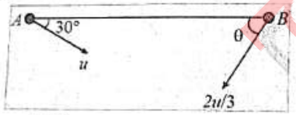
(a) $y = \frac{1}{2}\frac{ax^2}{u^2}$ (b) $(y-x)^2 = \frac{2xu^2}{a}$

(c) $(x-y)^2 = \frac{2yu^2}{a}$ (d) $x^2 + y^2 = \frac{2xu^2}{a}$

36. A boat, which has a speed of 5 km/h in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/h is

(a) 1 (b) 3 (c) 4 (d) $\sqrt{41}$

37. Anoop (A) hits a ball along the ground with a speed u in a direction which makes an angle 30° with the line joining him and the fielder Babul (B). Babul runs to intercept the ball with a speed $\frac{2u}{3}$. At what angle θ should he run to intercept the ball?



(a) $\sin^{-1}\left[\frac{\sqrt{3}}{2}\right]$ (b) $\sin^{-1}\left[\frac{2}{3}\right]$

(c) $\sin^{-1}\left[\frac{3}{4}\right]$ (d) $\sin^{-1}\left[\frac{4}{5}\right]$

38. Two particles are moving along two long straight lines, in the same plane with same speed equal to 20 cm/s. The angle between the two lines is 60° and their intersection point is O. At a certain moment, the two particles are located at distances 3m and 4m from O and are moving towards O. Subsequently, the shortest distance between them will be

(a) 50 cm (b) $40\sqrt{2}$ cm

(c) $50\sqrt{2}$ cm (d) $50\sqrt{3}$ cm

39. A train is moving on a track at 30ms^{-1}. A ball is thrown from it perpendicular to the direction of motion with 30ms^{-1} at 45° from horizontal. Find the distance of ball from the point of projection on train to the point where it strikes the ground.

(a) 90 m (b) $90\sqrt{3}$ m

(c) 60 m (d) $60\sqrt{3}$ m

40. Two bodies were thrown simultaneously from the same point, one straight up, and the other, at an angle of θ = 30° to the horizontal. The initial velocity of each body is 20 ms^{-1}. Neglecting air resistance, the distance between the bodies at $t = 1.2$ later is

(a) 20 m (b) 30 m (c) 24 m (d) 50 m

41. Ship A is travelling with a velocity of 5 kmh^{-1} due east. A second ship is heading 30° east of north. What should be the speed of second ship if it is to remain always due north with respect to the first ship?

(a) 10 kmh^{-1} (b) 9 kmh^{-1} (c) 8 kmh^{-1} (d) 7 kmh^{-1}

42. A river is flowing from west to east at a speed of 5 m/ min. A man on the south bank of the river, capable of swimming at 10 m/min in still water, wants to swim across the river in the shortest time. Finally he will move in a direction

(a) $\tan^{-1}(2)$ E of N (b) $\tan^{-1}(2)$ N of E

(c) 30° E of N (d) 60° E of N

43. A car is moving towards east with a speed of 25 kmh^{-1}. To the driver of the car, a bus appears to move towards north with a speed of $25\sqrt{3}\text{ kmh}^{-1}$. What is the actual velocity of the bus?

(a) 50 kmh^{-1}, 30° E of N (b) 50 kmh^{-1} 30° N of E

(c) 25 kmh^{-1} 30° E of N (d) 25 kmh^{-1} 30° N of E

44. A swimmer wishes to cross a 500 m river flowing at 5 kmh^{-1}. His speed with respect to water is 3 kmh^{-1}. The shortest possible time to cross the river is

(a) 10 min (b) 20 min (c) 6 min (d) 7.5 min

45. A man can swim in still water with a speed of 2 ms^{-1}. If he wants to cross a river of water current speed $\sqrt{3}\text{ ms}^{-1}$ along shortest possible path, then in which direction should he swim?

(a) At an angle 120° to the water current

(b) At an angle 150° to the water current

(c) At an angle 90° to the water current

(d) None of these

46. Rain, driven by the wind, falls on a railway compartment with a velocity of 20 ms^{-1}, at an angle of 30° to the vertical. The train moves, along the direction of wind flow, at a speed of 108 kmh^{-1}. Determine the apparent velocity of rain for a person sitting in the train.

(a) $20\sqrt{7}\text{ ms}^{-1}$ (b) $10\sqrt{7}\text{ ms}^{-1}$

(c) $15\sqrt{7}\text{ ms}^{-1}$ (d) $10\sqrt{7}\text{ kmh}^{-1}$

47. Twelve persons are initially at the twelve corners of a regular polygon of twelve sides of side a. Each person now moves with a uniform speed v in such a manner that 1 is always directed towards 2, 2 towards 3, 3 towards 4, and so on. The time after which they meet is

(a) $\frac{v}{a}$ (b) $\frac{2a}{v}$

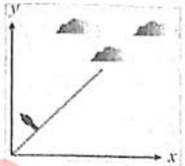
(c) $\frac{2a}{v(2+\sqrt{3})}$ (d) $\frac{2a}{v(2-\sqrt{3})}$

48. A boat is moving towards east with velocity 4m/s with respect to still water and river is flowing towards north with velocity 2 m/s and the wind is blowing towards north with velocity 6 m/s. The direction of the flag blown over by the wind hoisted on the boat is:

(a) North-west (b) South-east

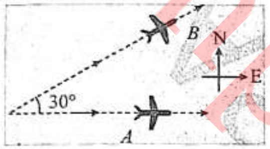
(c) $\tan^{-1}(1/2)$ with east (d) North

49. To a man running upwards on the hill, the rain appears to fall, vertically downwards with 4 m/s. The velocity vector of the man w.r.t. earth is $(2\hat{i} + 3\hat{j})$ m/s. If the man starts running down the hill with the same speed, then determine the relative velocity (in m/s) of the rain w.r.t. man.



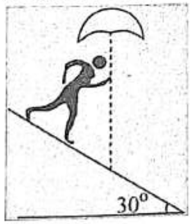
(a) $2\hat{i} + 4\hat{j}$ (b) $4\hat{i} - 2\hat{j}$
(c) $2\hat{i} - 4\hat{j}$ (d) $4\hat{i} + 2\hat{j}$

50. An aeroplane A is flying horizontally due east at a speed of 400 km/hr. Passengers in A, observe another aeroplane B moving perpendicular to direction of motion at A. Aeroplane B is actually moving in a direction 30° north of east in the same horizontal plane as shown in the Figure. Determine the velocity (in m/s) of B.



(a) $400\hat{i} + \frac{400}{\sqrt{3}}\hat{j}$ (b) $400\hat{i} + 400\hat{j}$
(c) $400\sqrt{3}\hat{i} + 400\hat{j}$ (d) $200\sqrt{3}\hat{i} + 200\hat{j}$

51. A man is coming down an incline of-angle 30°. When he walks with speed $2\sqrt{3}$ m/s he has to keep his umbrella vertical to protect himself from rain. The actual speed of rain is 5 m/s, At what angle with vertical should he keep his umbrella where he is at rest so that, he does not get drenched?



(a) 30° (b) 37° (c) 53° (d) 60°

52. The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be:

(a) 2π mm/s and 0 mm/s
(b) $2\sqrt{2}\pi$ mm/s and 4.44 mm/s
(c) $2\sqrt{2}\pi$ mm/s and 2π mm/s
(d) 2π mm/s and $2\sqrt{2}\pi$ mm/s

53. A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 sec, it rotates through an angle θ_1 : in the next 2 sec, it rotates through an additional angle θ_2. The ratio of θ_2 / θ_1 is
(a) 1 (b) 2 (c) 3 (d) 5

54. The linear and angular acceleration of a particle are 10 m/sec² and 5 rad/sec² respectively. It is at a distance from the axis of rotation
(a) 50 m (b) 1/2 m (c) 1 m (d) 2 m

55. The earth (radius 6400 km), makes one revolution about its own axis in 24 hours. The centripetal acceleration of a point on its equator is nearly:

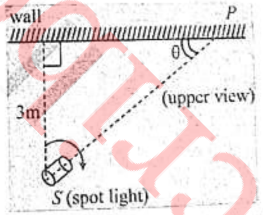
(a) $340\frac{cm}{sec^2}$ (b) $3.4\frac{cm}{sec^2}$
(c) $34\frac{cm}{sec^2}$ (d) $0.34\frac{cm}{sec^2}$

56. If angular velocity of a disc depends on angle rotated θ as $\omega = \theta^2 + 2\theta$, then its angular acceleration α at θ = 1 rad is
(a) 8 rad/sec² (b) 10 rad/sec²
(c) 12 rad/sec² (d) None

57. The velocity and acceleration vectors of a particle undergoing circular motion are $\vec{v} = 2\hat{i}$ m/s and $\vec{a} = 2\hat{i} + 4\hat{j}$ m/s² respectively at an instant of time. The radius of the circle is
(a) 1m (b) 2m (c) 3m (d) 4m

58. A particle is projected horizontally from the top of a tower with a velocity v_0. If v be its velocity at any instant, then the radius of curvature of the path of the particle at that instant is directly proportional to:
(a) v^3 (b) v^2 (c) v (d) $1/v$

59. A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance 3 m. What is the velocity of the spot P when θ = 45°?



(a) 0.6 m/s (b) 0.5 m/s (c) 0.4 m/s (d) 0.3 m/s

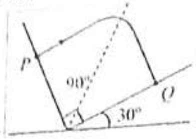
INTEGER TYPE QUESTIONS

60. A ball is projected from a certain point on the surface of a planet at a certain angle with the horizontal surface. The horizontal and vertical displacement x and y varies with time t in second as:

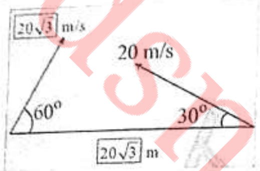
$x = 10\sqrt{3}t$ and $y = 10t - t^2$

The maximum height attained by the ball is_____ m.

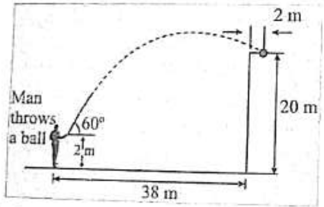
61. A particle is projected from point P with velocity $5\sqrt{3}$ ms^{-1} perpendicular to the surface of a hollow right angled cone as shown in figure. It collides at Q normally. What will be the velocity with which it collides at Q?



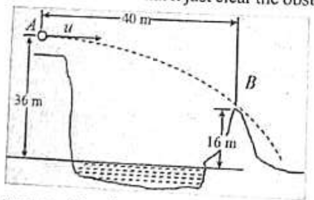
62. In the figure shown, the two projectiles are fired simultaneously. The minimum distance between them during their flight is $a\sqrt{b}$ m. Find $a + b$.



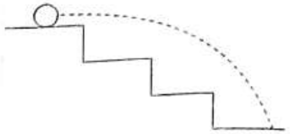
63. A soft ball is thrown at an angle of $\alpha = 60°$ above the horizontal. It lands a distance $d = 2$m from the edge of a flat roof, whose height is $h = 20$ m ; the edge of the roof is $l = 38$ m from the thrower (see fig.). At what speed (in m/s) was the softball thrown? $\left(\text{use } \sqrt{3} = 1.73\right)$



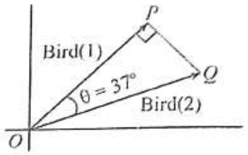
64. With what minimum horizontal velocity 'u', (in m/s) can a boy throw a rock at A so that it just clear the obstruction at B?



65. A staircase contains three steps each 10 cm high and 20 cm wide. What should be the minimum horizontal velocity of the ball rolling off the uppermost plane so as to hit directly the lowest plane (in ms^{-1})?

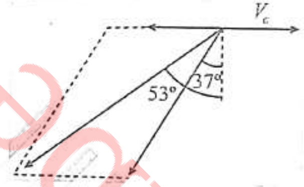


66. Two birds are at origin. They both start to fly in straight line in different directions making an angle $\theta = 37°$ with each other at the same time as shown.



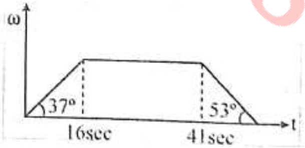
Bird(1) on reaching a point P takes a right angle turn to reach point Q simultaneously with Bird (2). If the bird (1) was flying with a constant speed of 7 km/hr then what must be the constant speed of bird (2) (in km/hr)?

67. A car is travelling in steady rain with constant acceleration in a straight line. When it begins to move the driver sees that the raindrops make track at an angle of 37° with the vertical on the side window. After 20 sec., the raindrops make track at an angle of 53° with vertical in same direction. Find the acceleration of the car in cm/s^2. Rain is falling at 3 m/s.



68. Find the radius (in cm) of a rotating disc if the velocity of a point on rim is 5 times the velocity of a point located 5 m closer to the centre.

69. The angular velocity of a body moving in a circular path is shown in graph below. What is the average angular velocity (in rad/s) for the entire motion? Approximate the answer to nearest integer.

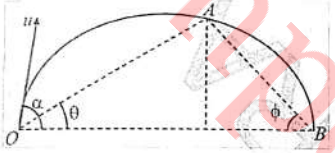


Parikshit (JEE Advanced Level)

MULTIPLE CORRECT TYPE QUESTIONS

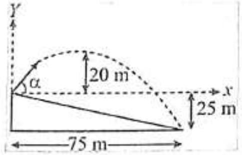
1. Which of the following is a true statement?
   (*a*) A vector cannot be divided by another vector
   (*b*) Angular displacement can either be a scalar or a vector
   (*c*) Since addition of vectors is commutative therefore vector subtraction is also commutative
   (*d*) The resultant of two equal forces of magnitude F acting at a point is F if the angle between the two forces is 120°

2. A football is kicked as shown in figure. When it is at point A, OA and OB make angles θ and ϕ with the horizontal. Find the relation between α, θ and ϕ.



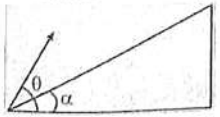
   (*a*) $\cot\alpha = \cot\theta + \cot\phi$
   (*b*) $\tan\alpha = \tan\theta - \tan\phi$
   (*c*) $\cot\alpha = \cot\theta - \cot\phi$
   (*d*) $\tan\alpha = \tan\theta + \tan\phi$

3. A ball thrown down the incline strikes at a point on the incline 25 m below the horizontal as shown in the figure. If the ball rises to a maximum height of 20 m above the point of projection, the angle of projection α (with horizontal x-axis) is



   (*a*) $\tan^{-1}\frac{4}{3}$
   (*b*) $\tan^{-1}\frac{3}{4}$
   (*c*) $\tan^{-1}\frac{3}{2}$
   (*d*) $\tan^{-1}\frac{2}{3}$

4. A projectile is fired at an angle θ with the horizontal. Find the condition under which it lands perpendicular on an inclined plane of inclination α as shown in figure.



   (*a*) $\sin\alpha = \cos(\theta - \alpha)$
   (*b*) $\cos\alpha = \sin(\theta - \alpha)$
   (*c*) $\tan\theta = \cot(\theta - \alpha)$
   (*d*) $\cot(\theta - \alpha) = 2\tan\alpha$

5. A ball is rolled off along the edge of a horizontal table with velocity 4 m/s. It hits the ground after time 0.4s. Which of the following are correct?
   (*a*) The height of the table is 0.8 m
   (*b*) It hits the ground at an angle of 60° with the vertical
   (*c*) It covers a horizontal distance 1.6 m from the table
   (*d*) It hits the ground with vertical velocity 4 m/s

6. A man on a rectilinearly moving cart, facing the direction of motion, throws a ball straight up with respect to himself
   (*a*) The ball will always return to him.
   (*b*) The ball will never return to him.
   (*c*) The ball will return to him if the cart moves with constant velocity.
   (*d*) The ball will fall behind him if the cart moves with some positive acceleration.

7. State which of the following statements is/are false.
   (*a*) If two particles are neither approaching nor separating from each other, then their relative velocity is zero.
   (*b*) If relative velocity of particle B with respect to A is $\vec{v}_1$, relative velocity of particle C with respect to B is $\vec{v}_2$ and particle A moves with velocity $\vec{v}_0$ with respect to ground, then the velocity of C with respect to ground cannot be zero. (assuming $\vec{v}_1, \vec{v}_2$ & $\vec{v}_0$ to be non zero)
   (*c*) Four dogs are running along a line in the same direction, such that each is running relative to the dog in front of him with equal speed. Then the rate of separation between the third and the first dog is same as that of the fourth and the second dog. (where first, second, third and fourth are taken in order)
   (*d*) At some instant of time at a place, two particles are observed and it is found that their relative velocity is zero. Then they will remain stationary with respect to each other.

8. A river is flowing with a velocity of 2 m/s. A boat is moving downstream along the river. Velocity of the boat in still water is 3 m/s. A person standing on the boat throws a ball (w.r.t. himself) in a plane perpendicular to the direction of motion of the boat with 10 m/s at 60° with the horizontal. When the ball reaches highest point of its path.
   (*a*) The speed of ball w.r.t. man standing on boat is 5 m/s
   (*b*) The speed of ball w.r.t. river is 3 m/s
   (*c*) The speed of ball w.r.t. river is 0 m/s
   (*d*) The speed of ball w.r.t. ground is $5\sqrt{2}$ m/s

9. Consider a shell that has a muzzle velocity of 45 ms^{-1} fired from the tail gun of an airplane moving horizontally with a velocity of 215 ms^{-1}. The tail gun can be directed at any angle with the vertical in the plane of motion of the airplane. The shell is fired when the plane is above point A on ground and the plane is above point B on ground when the shell hits the ground. (Assume for simplicity that the Earth is flat)

(*a*) Shell may hit the ground at point A.

(*b*) Shell may hit the ground at point B.

(*c*) Shell may hit a point on earth which is behind point A.

(*d*) Shell may hit a point on earth which is ahead of point B.

10. Choose the correct alternative (s)

(*a*) If the greatest height to which a man can throw a stone is h, then the greatest horizontal distance upto which he can throw the stone is $2h$.

(*b*) The angle of projection for a projectile motion whose range R is n times the maximum height is $\tan^{-1}(4/n)$

(*c*) The time of flight T and the horizontal range R of a projectile are connected by the equation $gT^2 = 2R\tan\theta$ where θ is the angle of projection.

(*d*) A ball is thrown vertically up. Another ball is thrown at an angle θ with the vertical. Both of them remain in air for the same period of time. Then the ratio of heights attained by the two ball 1 : 1.

11. Ship A is located 4 km north and 3 km east of ship B. Ship A has a velocity of 20 kmh^{-1} towards the south and ship B is moving at 40 kmh^{-1} in a direction 37° north of east. X and Y-axes are along east and north directions, respectively

(*a*) Velocity of A relative to B is $(-32\,\hat{i} - 44\,\hat{j})$ km/h

(*b*) Position of A relative to B as a function of time is given by $\vec{r}_{AB} = [(3-32t)\hat{i} + (4-44t)\hat{j}]$ km

(*c*) Velocity of A relative to B is $(32\,\hat{i} - 44\,\hat{j})$ km/h

(*d*) Position of A relative to B as a function of time is given by $(32\,t\,\hat{i} - 44\,t\,\hat{j})$ km

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 12 to 14): Two forces $\vec{F}_1 = 2\hat{i} + 2\hat{j}$ N and $\vec{F}_2 = 3\hat{j} - 4\hat{k}$ N are acting on a particle.

12. The resultant force acting on particle is:

(*a*) $2\hat{i}+5\hat{j}+4\hat{k}$ (*b*) $2\hat{i}-5\hat{j}-4\hat{k}$

(*c*) $\hat{i}-3\hat{j}-2\hat{k}$ (*d*) $\hat{i}-\hat{j}-\hat{k}$

13. The angle between $\vec{F}_1$ and $\vec{F}_2$ is

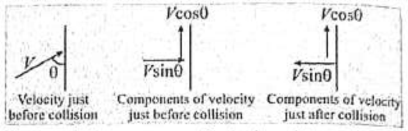
(*a*) $\theta = \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$ (*b*) $\theta = \cos^{-1}\left(\frac{3}{5\sqrt{2}}\right)$

(*c*) $\theta = \cos^{-1}\left(\frac{2}{3\sqrt{5}}\right)$ (*d*) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{5}\right)$

14. The component of force $\vec{F}_1$ along force $\vec{F}_2$ is

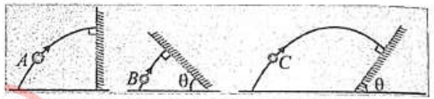
(*a*) $\frac{5}{6}$ (*b*) $\frac{5}{3}$ (*c*) $\frac{6}{5}$ (*d*) $\frac{5}{2}$

Comprehension (Q. 15 to 17): We know how by neglecting the air resistance, the problems of projectile motion can be easily solved and analysed. Now we consider the case of the collision of a ball with a wall. In this case the problem of collision can be simplified by considering the case of elastic collision only. When a ball collides with a wall we can divide its velocity into two components, one perpendicular to the wall and other parallel to the wall. If the collision is elastic then the perpendicular component of velocity of the ball gets reversed with the same magnitude.


V
θ
Velocity just before collision
Vcosθ
Vsinθ
Components of velocity just before collision
Vcosθ
Vsinθ
Components of velocity just after collision

The other parallel component of velocity will remain constant if given wall is smooth.

Now let us take a problem. Three balls 'A' and 'B' & 'C' are projected from ground with same speed at same angle with the horizontal. The balls A, B and C collide with the wall during their flight in air and all three collide perpendicularly with the wall as shown in figure.



15. Which of the following relation about the maximum height H of the three balls from the ground during their motion in air is correct ?

(*a*) $H_A = H_C > H_B$ (*b*) $H_A > H_B = H_C$

(*c*) $H_A > H_C > H_B$ (*d*) $H_A = H_B = H_C$

16. If the time taken by the ball A to fall back on ground from start is 4 seconds and that by ball B is 2 seconds. Then the time taken by the ball C to reach the inclined plane after projection will be :

(*a*) 6 s (*b*) 4 s (*c*) 3 s (*d*) 5 s

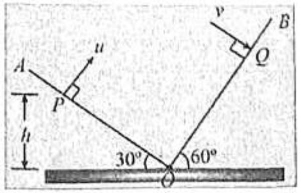
17. The maximum height attained by ball 'A' from the ground is:

(*a*) 10 m (*b*) 15 m

(*c*) 20 m (*d*) Insufficient information

Comprehension (Q. 18 to 21): Two inclined planes OA and OB having inclinations 30° and 60° with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3}$ ms^{-1} along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicular at Q, then

(Take $g = 10$ m/s^2).



18. The time of flight from P to Q is:
(a) 5 s (b) 2 s
(c) 1 s (d) None of these

19. The speed with which the particle strikes the plane OB is:
(a) 10 m/s
(b) 20 m/s
(c) 30 m/s
(d) 40 m/s

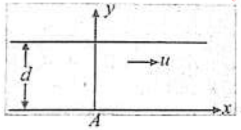
20. The height h of point P from the ground is:
(a) $10\sqrt{3}$ m (b) 10 m
(c) 5 m (d) 20 m

21. The distance PQ is:
(a) 20 m (b) $10\sqrt{3}$ m
(c) 10 m (d) 5 m

Comprehension (Q. 22 to 24): We know that when a boat travels in water, its net velocity w.r.t. ground is the vector sum of two velocities. First is the velocity of boat itself in river and other is the velocity of water w.r.t. ground. Mathematically:

$$\vec{v}_{\text{boat}} = \vec{v}_{\text{boat, water}} + \vec{v}_{\text{water}}$$

Now given that velocity of water w.r.t. ground in a river is u. Width of the river is d.



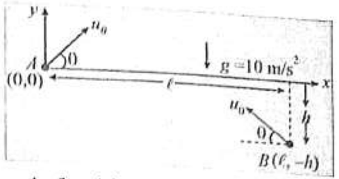
A boat starting from A aims perpendicular to the river with an acceleration of $a = 5t$, where t is time. The boat starts from point (1, 0) of the coordinate system (assume SI units).

22. Obtain the total time taken to cross the river.
(a) $(3d/5)^{1/3}$ (b) $(6d/5)^{1/3}$
(c) $(6d/5)^{1/2}$ (d) $(2d/3)^{1/3}$

23. Find the equation of trajectory of the boat.
(a) $x-1=\left(\frac{3y}{5}\right)^{1/3}$ (b) $x=u\left(\frac{6y}{5}\right)^{1/3}$
(c) $x-1=u\left(\frac{6y}{5}\right)^{1/3}$ (d) None of these

24. Find the drift of the boat when it is in the middle of the river.
(a) $u\left(\frac{3d}{5}\right)^{1/3}$ (b) $u\left(\frac{3d}{5}\right)^{1/3}+1$
(c) $u\left(\frac{6d}{5}\right)^{1/3}$ (d) None of these

Comprehension (Q. 25 to 28): Two particles 'A' and 'B' are projected in the vertical plane with same initial speed u_0 from point (0, 0) and (ℓ, $-h$) towards each other as shown in figure at $t = 0$.

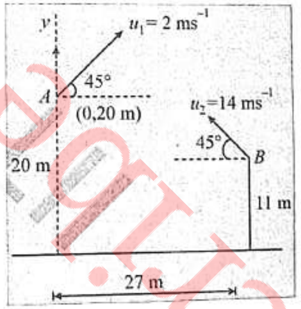


25. The path of particle 'A' with respect to particle 'B' will be
(a) Parabola
(b) Straight line parallel to x-axis
(c) Straight line parallel to y-axis
(d) None of these

26. Minimum distance between particle A and B during motion will be:
(a) ℓ (b) h
(c) $\sqrt{\ell^2+h^2}$ (d) $\ell + h$

27. The time when separation between A and B is minimum is:
(a) $\frac{\ell}{u_0\cos\theta}$ (b) $\sqrt{\frac{2h}{g}}$
(c) $\frac{\ell}{2u_0\cos\theta}$ (d) $\frac{2\ell}{u_0\cos\theta}$

Comprehension (Q. 28 to 30): Two particles are thrown simultaneously from points A and B with velocities $u_1 = 2$ ms^{-1} and $u_2 = 14$ ms^{-1}, respectively, as shown in figure.



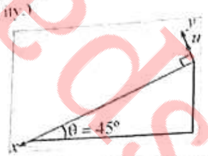
28. The relative velocity of B as seen from A in
(a) $-8\sqrt{2}\hat{i}+6\sqrt{2}\hat{j}$ (b) $4\sqrt{2}\hat{i}+3\sqrt{3}\hat{j}$
(c) $3\sqrt{5}\hat{i}+2\sqrt{3}\hat{j}$ (d) $3\sqrt{2}\hat{i}+4\sqrt{3}\hat{j}$

29. The direction (angle) with horizontal at which B will appear to move as seen from A is
(a) 37° (b) 53°
(c) 15° (d) 90°

30. Minimum separation between A and B is
(a) 3 m (b) 6 m
(c) 12 m (d) 9 m

MATCH THE COLUMN TYPE QUESTIONS

31. An inclined plane makes an angle θ = 45° with horizontal. A stone is projected normally from the inclined plane, with speed u m/s at $t = 0$ sec. x and y axes are drawn from point of projection along and normal to inclined plane as shown. The length of incline is sufficient for stone to land on it and neglect air friction. Match the statements given in Column-I with the results in Column-II. (g in Column-II is acceleration due to gravity.)



	Column-I		Column-II
A.	The instant of time at which velocity of stone is parallel to x-axis	p.	$\frac{2\sqrt{2}u}{g}$
B.	The instant of time at which velocity of stone makes an angle θ = 45° with positive x-axis in clockwise direction.	q.	$\frac{2u}{g}$
C.	The instant of time at which (starting from $t = 0$) component of displacement along x-axis become half the range on inclined plane is	r.	$\frac{\sqrt{2}u}{g}$
D.	Time of flight on inclined plane is	s.	$\frac{u}{\sqrt{2}g}$

(*a*) A-(r); B-(q); C-(s); D-(p)

(*b*) A-(r); B-(s); C-(q); D-(p)

(*c*) A-(r); B-(s); C-(p); D-(q)

(*d*) A-(p); B-(q); C-(r); D-(s)

32. A particle is projected from level ground. Assuming projection point as origin, x-axis along horizontal and y-axis along vertically upwards. If particle moves in x-y plane and its path is given by $y = ax - bx^2$ where a, b are positive constants. Then match the physical quantities given in Column-I with the values given in Column-II. (g in Column II is acceleration due to gravity.)

	Column-I		Column-II
A.	Horizontal component of velocity	p.	$\frac{a}{b}$
B.	Time of flight	q.	$\frac{a^2}{4b}$
C.	Maximum height	r.	$\sqrt{\frac{g}{2b}}$
D.	Horizontal range	s.	$\sqrt{\frac{2a^2}{bg}}$

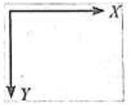
(*a*) A-(q); B-(s); C-(r); D-(p)

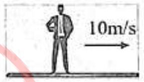
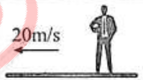
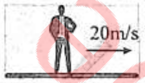
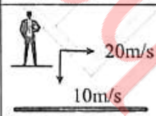
(*b*) A-(s); B-(r); C-(p); D-(q)

(*c*) A-(r); B-(s); C-(q); D-(p)

(*d*) A-(q); B-(s); C-(r); D-(p)

33. Rain is falling at velocity $10\hat{i} + 10\hat{j}$ m/s. Then match the direction of velocity of rain w.r.t. man in Column-II from Column-I.



	Column-I		Column-II
A.		p.	→
B.		q.	↓
C.		r.	↘
D.		s.	↙
		t.	←

(*a*) A-(q); B-(r); C-(s); D-(t)

(*b*) A-(p); B-(s); C-(t); D-(r)

(*c*) A-(s); B-(t); C-(s); D-(r)

(*d*) A-(q); B-(s); C-(r); D-(t)

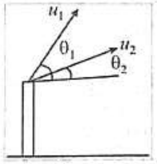
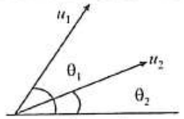
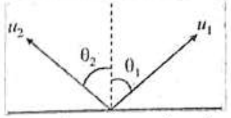
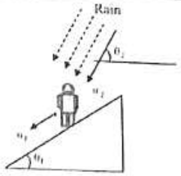
34. Match Column-I with Column-II and select the correct answer using the codes given below the lists.

	Column-I		Column-II
A.	If swimmer can swim at 5m/sec in still water and if velocity of water flow is 4m/sec then angle between direction of swimming and direction of river flow to minimize drift.	p.	53°

B.	If swimmer can swim at 5 m/sec in still water and velocity of flow is 3 m/sec then angle between direction of velocity of swimmer with respect to river and the direction of river flow if swimmer crosses the river in minimum time.	q.	127°
C.	If swimmer can swim at 4 m/sec and velocity of flow is 3m/sec then angle of resultant velocity (w.r.t. ground) with the direction of river flow if swimmer swims perpendicular to flow of river.	r.	143°
D.	Angle between direction of fluttering of flag and north if wind blows towards south west direction with a velocity $3\sqrt{2}$ m/sec. Man moves with a velocity 7 m/sec along west, holding flag in his hand.	s.	90°

(*a*) A-(q); B-(r); C-(s); D-(p)
(*b*) A-(r); B-(s); C-(p); D-(q)
(*c*) A-(s); B-(q); C-(s); D-(r)
(*d*) A-(q); B-(s); C-(r); D-(q)

35. Column-I shows certain situations with certain conditions and Column-II shows the parameters in which situations of Column-I match. Which can be possible combination.

	Column-I		Column-II
A.	$u_1 = u_2; \theta_1 = \theta_2$	p.	 Two projectiles are projected from a height such that they strike ground at the same time.
B.	$u_1 > u_2; \theta_1 > \theta_2$	q.	 Two projectiles under standard ground to ground projection such that horizontal range is same.
C.	$u_1 < u_2; \theta_2 > \theta_1$	r.	 Two swimmer starting from same point on a river bank such that time of crossing is same. u_1 and u_2 are velocities relative to river.
		s.	 Person moving downward along slope in rain such that he observes rain vertically

(*a*) A-(p,s,r); B-(q,s,p); C-(r,s)
(*b*) A-(p,r); B-(q,p,s); C-(q,r)
(*c*) A-(p,s,r); B-(q,r,p); C-(q,p,s)
(*d*) A-(p,q,r); B-(q,r); C-(q,r)

36. A particle is moving with speed $v = 2t^2$ on the circumference of circle of radius R. Match the quantities given in column-I with corresponding results in column-II

	Column-I		Column-II
A.	Magnitude of tangential acceleration of particle	p.	Decreases with time.
B.	Magnitude of centripetal acceleration of particle	q.	Increases with time
C.	Magnitude of angular speed of particle with respect to centre of circle	r.	Remains constant
D.	Angle between the total acceleration vector and centripetal acceleration vector of particle	s.	Depends on the value of radius R

(*a*) A-(q,r); B-(q,s); C-(q,s); D-(p)
(*b*) A-(q,r); B-(q,p); C-(q); D-(p,s)
(*c*) A-(q); B-(q,s); C-(q,s); D-(p,s)
(*d*) A-(s); B-(q,s); C-(q,p); D-(p,r)

37. Match the statements given in Column-I with statements given in Column-II

	Column-I		Column-II
A.	If $\lvert \vec{A} + \vec{B} \rvert = \lvert \vec{A} \rvert = \lvert \vec{B} \rvert$ then angle between $\vec{A}$ and $\vec{B}$ is	p.	90°
B.	Magnitude of resultant of two forces $\lvert \vec{F_1} \rvert = 8\text{N}$ and $\lvert \vec{F_2} \rvert = 4\ N$ may be	q.	120°
C.	Angle between $\vec{A} = 2\hat{i} + 2\hat{j}$ & $\vec{B} = 3\hat{k}$ is	r.	$12\ N$
D.	Magnitude of resultant of vectors $\vec{A} = 2\hat{i} + \hat{j}$ and $\vec{B} = 3\hat{k}$ is	s.	$\sqrt{14}$

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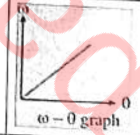
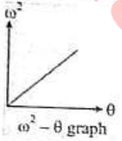
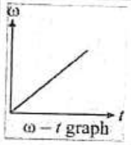
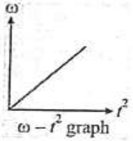
(*a*) A-(s); B-(q); C-(p); D-(r)
(*b*) A-(p); B-(r); C-(s); D-(q)
(*c*) A-(r); B-(s); C-(q); D-(p)
(*d*) A-(q); B-(r); C-(p); D-(s)

38. Each situation in column-I gives graph of a particle moving in circular path. The variables ω, θ and t represent angular speed (at any time t), angular displacement (in time t) and time respectively. Column-II gives certain resulting interpretation. Match the graphs in column-I with statements in column-II.

	Column-I		Column-II
A.		p.	Angular acceleration of particle is uniform
B.		q.	Angular acceleration of particle is non-uniform
C.		r.	Angular acceleration of particle is directly proportional to t.
D.		s.	Angular acceleration of particle is directly proportional to θ.

(*a*) A-(q,r); B-(q); C-(p); D-(r)
(*b*) A-(q,s); B-(p); C-(p); D-(q,r)
(*c*) A-(q,s); B-(r); C-(p,r); D-(q,r)
(*d*) A-(q,s); B-(p); C-(p); D-(q,s)

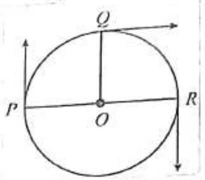
NUMERICAL TYPE QUESTIONS

Answer should be rounded off upto two decimal places

39. The ratio of the distance carried away by the water current, downstream, in crossing a river, by a person, making same angle with downstream and upstream is 2: 1. The ratio of the speed of person to the water current cannot be less than______.

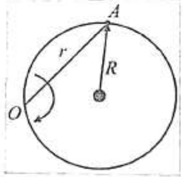
40. When a man moves down the inclined plane with a constant speed 5 ms^{-1} which makes an angle of 37° with the horizontal, he finds that the rain is falling vertically downward. When he moves up the same inclined plane with the same speed, he finds that the rain makes an angle $\theta = \tan^{-1}\left(\frac{7}{8}\right)$ with the horizontal. Find the speed of the rain (in ms^{-1}) $\left(\sqrt{2} = 1.414\right)$.

41. Three point particles P, Q, R move in a circle of radius 'r' with different but constant speeds. They start moving at $t = 0$ from their initial positions as shown in the figure. The angular velocities (in rad/sec) of P, Q and R are 5π, 2π and 3π respectively, in the same sense. The time at which they all meet is ______ s



42. A projectile is projected at an angle of 60° with some initial velocity. Find the ratio of the times at which the component of horizontal velocity of the projectile will be equal to its component of vertical velocity in magnitude.

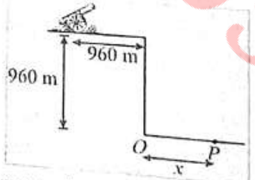
43. A particle A moves along a circle of radius R = 50 cm so that its radius vector r relative to the fixed point O (Fig.) rotates with the constant angular velocity = 0.40 rad/s. Then find the ratio of modulus v of the velocity of the particle and the modulus a of its total acceleration (in s^{-1}).



INTEGER TYPE QUESTIONS

44. Two particles P and Q move with constant velocities $v_1 = 2$ ms^{-1} and $v_2 = 4$ ms^{-1} along two mutually perpendicular straight lines towards the intersection point O. At moment $t = 0$, the particles were located at distances $l_1 = 12$ m and $l_2 = 19$ m from O, respectively. Find the time when they are nearest (nearest integer) (in s).

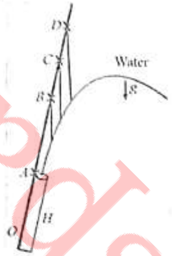
45. A gun is mounted on a plateau 960 m away from its edge as shown. Height of plateau is 960 m. The gun can fire shells with a velocity of 100 m/s at any angle of choice. What is the minimum distance x (OP) from the edge of plateau where the shell of gun cannot reach (in m)?



46. A particle moves along an arc of a circle of radius R = 5m. Its velocity depends on the distance covered s as $v = a\sqrt{s}$, where a is a constant. Then find the angle α between the vector of the total acceleration and the vector of velocity at s = 2 m (in degrees).

SINGLE CORRECT TYPE QUESTIONS

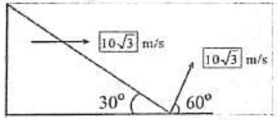
47. A straight rod $OABCD$ is strapped on to the end of a straight hose pipe H with 'A' at the nozzle of the pipe (figure). Strings are tied to the rod at B, C and D and are allowed to drop down vertically. The lengths of the strings at B, C and D to just touch the stream of water, streaming out of the hose pipe from its nozzle, are in the ratio



(a) $\frac{1}{AB}:\frac{1}{BC}:\frac{1}{CD}$ (b) $AB^2 : AC^2 : AD^2$

(c) $\frac{1}{AB^2}:\frac{1}{AC^2}:\frac{1}{AD^2}$ (d) $\sqrt{AB}:\sqrt{AC}:\sqrt{AD}$

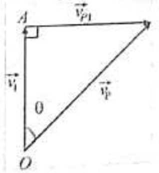
48. A particle is projected at an angle 60° with speed $10\sqrt{3}$ m/s, from the point A, as shown in the figure. At the same time the wedge is made to move with speed $10\sqrt{3}$ m/s towards right as shown in the figure. Then the time after which particle will strike with wedge is



(a) 2s (b) $2\sqrt{3}$ s

(c) $\frac{4}{\sqrt{3}}$ s (d) None of these

49. An airplane is observed by two persons travelling at 60 km/hour in two vehicles moving in opposite directions on a straight road. To an observer in one vehicle the plane appears' to cross the road track at right angles while to the observer in the other vehicle the angle appears to be 45°. At what angle does the plane actually cross the road track?



(a) 45° (b) $\theta = \tan^{-1}(2)$

(c) $\theta = \tan^{-1}(4)$ (d) $\theta = \tan^{-1}(8)$

50. A particle is moving in xy-plane. At certain instant, the components of its velocity and acceleration are as follows $v_x = 3$ m/s, $v_y = 4$m/s, $a_x = 2$m/s^2 and $a_y = 1$m/s^2. The rate of change of speed at this moment is

(a) 4 m/s^2 (b) 2 m/s^2

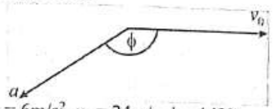
(c) $\sqrt{3}$ m/s^2 (d) $\sqrt{5}$ m/s^2

51. A particle moves with deceleration along the circle of radius R so that at any moment of time its tangential and normal accelerations are equal in moduli. At the initial moment $t = 0$ the speed of the particle equals v_0. The speed of the particle as a function of the distance covered s will be

(a) $v = v_0 e^{-s/R}$ (b) $v = v_0 e^{s/R}$

(c) $v = v_0 e^{-R/s}$ (d) $v = v_0 e^{R/s}$

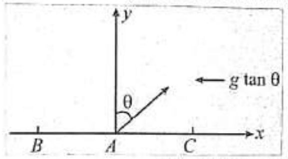
52. The figure shows the velocity and the acceleration of a point-like body at the initial moment of its motion. The direction and the absolute value of the acceleration remain constant. Find the time in seconds when the velocity reach its minimum value?



(Data: $a = 6$m/s^2, $v_0 = 24$m/s, $\phi = 143°$)

(a) 1.6 sec (b) 3.2 sec (c) 4.8 ses (d) 6.4 sec

53. A ball is projected towards right from point A at an angle θ with vertical. The wind blows towards left along negative x-axis with acceleration $g \tan\theta$. The ball will return to point:



(a) A (b) B left to point A

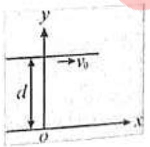
(c) C right to point A (d) None of the above

54. A stone is projected from level ground at $t = 0$ sec such that its horizontal and vertical components of initial velocity are 10 m/s and 20 m/s respectively. Then the instant of time at which magnitude of tangential and magnitude of normal components of acceleration of stone are same is (neglect air resistance) $g = 10$ m/s^2:

(a) $\frac{1}{2}$ sec (b) 1 sec

(c) 3 sec (d) both (b) and (c)

55. Velocity of the river with respect to ground is given by v_0. Width of the river is d. A swimmer swims (with respect to water) perpendicular to the current with acceleration $a = 2t$ (where t is time) starting from rest from the origin O at $t = 0$. The equation of trajectory of the path followed by the swimmer is



(a) $y = \frac{x^3}{3v_0^3}$ (b) $y = \frac{x^2}{2v_0^2}$

(c) $y = \frac{x}{v_0}$ (d) $y = \sqrt{\frac{x}{v_0}}$

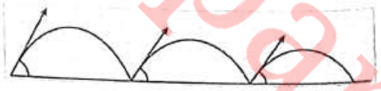
PYQ's (Past Year Questions)

MOTION IN A PLANE WITH CONSTANT ACCELERATION

1. Position of an ant (S in metres) moving in Y-Z plane is given by $\vec{S} = 2t^2\hat{j} + 5\hat{k}$ (where t is in second). The magnitude and direction of velocity of the ant at $t = 1$ s will be: **[27 Jan, 2024 (Shift-I)]**
   (*a*) 16 m/s in y-direction (*b*) 4 m/s in x-direction
   (*c*) 9 m/s in z-direction (*d*) 4 m/s in y-direction
2. A particle starts from origin at $t = 0$ with a velocity $5\hat{i}$ m/s and moves in x-y plane under action of a force which produces a constant acceleration of $(3\hat{i} + 2\hat{j})$m/s^2. If the x-coordinate of the particle at that instant is 84 m, then the speed of the particle at this time is $\sqrt{\alpha}$ m / s. The value of α is _____. **[27 Jan, 2024 (Shift-I)]**
3. The co-ordinates of a particle moving in x-y plane are given by:
   $x = 2 + 4t$, $y = 3t + 8t^2$.
   The motion of the particle is: **[04 April, 2024 (Shift-I)]**
   (*a*) non-uniformly accelerated.
   (*b*) uniformly accelerated having motion along a straight line.
   (*c*) uniform motion along a straight line.
   (*d*) uniformly accelerated having motion along a parabolic path.

"PROJECTILE MOTION: GROUND TO GROUND PROJECTION"

4. Projectiles A and B are thrown at angles of 45° and 60° with vertical respectively from top of a 400 m high tower. If their ranges and times of flight are same, the ratio of their speeds of projection $v_A : v_B$ is: **[30 Jan, 2024 (Shift-II)]**
   (*a*) $1 : \sqrt{3}$ (*b*) $\sqrt{2} : 1$ (*c*) $1 : 2$ (*d*) $1 : \sqrt{2}$
5. A particle moving in a circle of radius R with uniform speed takes time T to complete one revolution. If this particle is projected with the same speed at an angle θ to the horizontal, the maximum height attained by it is equal to $4R$. The angle of projection θ is then given by: **[1 Feb, 2024 (Shift-I)]**
   (*a*) $\sin^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (*b*) $\sin^{-1}\left[\frac{\pi^2 R}{2gT^2}\right]^{\frac{1}{2}}$
   (*c*) $\cos^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (*d*) $\cos^{-1}\left[\frac{\pi R}{2gT^2}\right]^{\frac{1}{2}}$
6. The maximum height reached by a projectile is 64 m. If the initial velocity is halved, the new maximum height of the projectile is _____ m. **[05 April, 2024 (Shift-II)]**
7. The angle of projection for a projectile to have same horizontal range and maximum height is: **[08 April, 2024 (Shift-II)]**
   (*a*) $\tan^{-1}(2)$ (*b*) $\tan^{-1}(4)$ (*c*) $\tan^{-1}\left(\frac{1}{4}\right)$ (*d*) $\tan^{-1}\left(\frac{1}{2}\right)$
8. The range of the projectile projected at an angle of 15° with horizontal is 50 m. If the projectile is projected with same velocity at an angle of 45° with horizontal, then its range will be: **[10 April, 2023 (Shift-I)]**
   (*a*) 50 m (*b*) $50\sqrt{2}\, m$ (*c*) 100 m (*d*) $100\sqrt{2}\, m$
9. Given below are two statements. One is labelled as Assertion A and the other is labelled as **Reason R**.
   Assertion A: Two identical balls A and B thrown with same velocity 'u' at two different angles with horizontal attained the same range R. If A and B reached the maximum height h_1 and h_2 respectively, then $R = 4\sqrt{h_1 h_2}$
   Reason R: Product of said heights.
   $$h_1 h_2 = \left(\frac{u^2 \sin^2\theta}{2g}\right) \cdot \left(\frac{u^2 \cos^2\theta}{2g}\right)$$
   Choose the correct answer: **[25 June, 2022 (Shift-II)]**
   (*a*) Both A and R are true and R is the correct explanation of A.
   (*b*) Both A and R are true but R is NOT the correct explanation of A.
   (*c*) A is true but R is false.
   (*d*) A is false but R is true.
10. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1 - x)$. The y component vector of the initial velocity is _____ $\hat{j}$
    (Take $g = 10$ m/s^2) **[JEE Adv, 2022]**
11. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is $0.8\,V_1$, the value of α is _____ **[JEE Adv, 2019]**



PROJECTILE MOTION ON AN INCLINED PLANE, KINETIC ENERGY OF A PROJECTILE

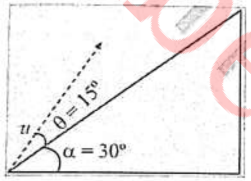
12. A ball rolls off the top of a stairway with horizontal velocity u. The steps are 0.1 m high and 0.1 m wide. The minimum velocity u with which that ball just hits the step 5 of the stairway will be $\sqrt{x}$ ms^{-1} where x = _____ [use $g = 10$ m/s^2]. **[29 Jan, 2024 (Shift-I)]**

13. A body of mass M thrown horizontally with velocity v from the top of the tower of height H touches the ground at a distance of 100m from the foot of the tower. A body of mass $2\,M$ thrown at a velocity $v/2$ from the top of the tower of height $4H$ will touch the ground at a distance of .... m. **[08 April, 2024 (Shift-II)]**

14. A stone is projected at angle 30° to the horizontal. The ratio of kinetic energy of the stone at point of projection to its kinetic energy at the highest point of flight will be **[29 Jan, 2023 (Shift-I)]**

(*a*) 1 : 2 (*b*) 1 : 4
(*c*) 4 : 1 (*d*) 4 : 3

15. A plane is inclined at an angle $\alpha = 30°$ with a respect to the horizontal. A particle is projected with a speed $u = 2\ \text{ms}^{-1}$ from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to: **[10 April, 2019 (Shift-II)]**
(Take $g = 10\ \text{ms}^{-2}$)



(*a*) 14 cm (*b*) 20 cm
(*c*) 18 cm (*d*) 26 cm

TWO DIMENSIONAL RELATIVE MOTION

16. The speed of a swimmer is 4 km h^{-1} in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km, he reaches a point 750 m down the stream on the opposite bank.

The speed of the river water is ______ km h^{-1}. **[31 Jan, 2023 (Shift-I)]**

17. A fighter jet is flying horizontally at a certain with a speed of 200 ms^{-1}. When it passes directly overhead ad anti-aircraft gun, a bullet is fired from the gun, at an angle θ with the horizontal, to hit the jet. If the bullet speed is 400 m/s, the value of θ will be ________. **[26 June, 2022 (Shift-I)]**

18. A particle is moving along the x-axis with its coordinate with time 't' given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1s$, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v(in m/s) is **[8 Jan, 2020 (Shift-I)]**

19. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is: **[9 Jan, 2019 (Shift-I)]**

(*a*) $y = x^2$ + constant (*b*) $y^2 = x$ + constant
(*c*) $y^2 = x^2$ + constant (*d*) xy = constant

RAIN MAN PROBLEM

20. A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2}\ \text{kmh}^{-2}$, the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is: **[27 June, 2022 (Shift-I)]**

(*a*) 30 kmh^{-1} (*b*) $\frac{25}{\sqrt{2}}$ kmh^{-1}
(*c*) $\frac{30}{\sqrt{2}}$ kmh^{-1} (*d*) 25 kmh^{-1}

RELATIVE MOTION BETWEEN TWO PROJECTILE

21. Two projectiles A and B are thrown with initial velocities of 40 m/s and 60 m/s at angles 30° and 60° with the horizontal respectively. The ratio of their ranges respectively is (g = 10 m/s^2) **[8 April, 2023 (Shift-I)]**

(*a*) $\sqrt{3} : 2$ (*b*) $2 : \sqrt{3}$
(*c*) 1 : 1 (*d*) 4 : 9

22. A ball of mass m is thrown vertically upward. Another ball of mass 2 m is thrown an angle θ with the vertical. Both the balls stay in air for the same period of time. The ratio of the heights attained by the two balls respectively is $\frac{1}{x}$. The value of x is ________. **[JEE Adv, 2022]**

23. An object is projected in the air with initial velocity u at an angle θ. The projectile motion is such that the horizontal range R, is maximum. Another object is projected in the air with a horizontal range half of the range of first object. The initial velocity remains same in both the case. The value of the angle of projection, at which the second object is projected, will be ________ degree. (Mark the smallest angle possible) **[JEE Adv, 2022]**

KINEMATICS OF CIRCULAR MOTION

24. A clock has 75 cm, 60 cm long second hand and minute hand respectively. In 30 minutes duration the tip of second hand will travel x distance more than the tip of minute hand. The value of x in meter is nearly (Take $\pi = 3.14$): **[08 April, 2024 (Shift-I)]**

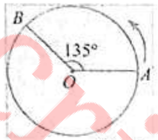
(*a*) 139.4 (*b*) 140.5
(*c*) 220.0 (*d*) 118.9

25. A body is moving with constant speed, in a circle of radius 10m. The body completes one revolution in 4s. At the end of 3rd second, the displacement of body (in m) from its starting point is: **[31 Jan, 2023 (Shift-II)]**

(*a*) 30 (*b*) 15π

(*c*) 5π (*d*) $10\sqrt{2}$

26. A person moved from A to B on a circular path as shown in figure. If the distance travelled by him is 60 m, then the magnitude of displacement would be:
(Given cos135° = – 0.7) **[25 July, 2022 (Shift-I)]**



(*a*) 42 m (*b*) 47 m

(*c*) 19 m (*d*) 40 m

27. A huge circular arc of length 4.4 ly subtends an angle '4s' at the centre of the circle. How long it would take for a body to complete 4 revolution if its speed is 8 AU per second?
Given : 1 ly = 9.46×10^{15} m **[27 Aug, 2021 (Shift-I)]**
1 AU = 1.5×10^{11} m

(*a*) 4.5×10^{10} s (*b*) 4.1×10^{8} s

(*c*) 3.5×10^{6} s (*d*) 7.2×10^{8} s

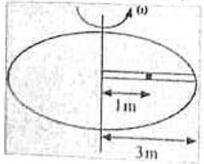
"CENTRIPETAL/TANGENTIAL/NET ACCELERATION"

28. If the radius of curvature of the path of two particles of same mass are in the ratio 3:4, then in order to have constant centripetal force, their velocities will be in the ratio of: **[29 Jan, 2024 (Shift-I)]**

(*a*) $\sqrt{3}:2$ (*b*) $1:\sqrt{3}$ (*c*) $\sqrt{3}:1$ (*d*) $2:\sqrt{3}$

29. A particle is moving in a circle of radius 50 cm in such a way that at any instant the normal and tangential components of its acceleration are equal. If its speed at $t = 0$ is 4 m/s, the time taken to complete the first revolution will be $\frac{1}{\alpha}[1-e^{-2\pi}]s$, where α = ______ . is **[29 Jan, 2024 (Shift-II)]**

30. A circular table is rotating with an angular velocity of ω rad/s about its axis (see figure). There is a smooth groove along a radial direction on the table. A steel ball is gently placed at a distance of 1 m on the groove. All the surface are smooth. If the radius of the table is 3 m, the radial velocity of the ball w.r.t. the table at the time ball leaves the table is $x\sqrt{2}\omega$ m/s, where the value of x is ______ **[08 April, 2024 (Shift-II)]**



31. A child of mass 5kg is going round a merry-go-round that makes 1 rotation in 3.14 s. The radius of the merry-go-round is 2 m. The centrifugal force on the child will be **[6 April, 2023 (Shift-II)]**

(*a*) 80 N (*b*) 50 N (*c*) 100 N (*d*) 40 N

32. A stone tied to 180 cm long string at its end is making 28 revolutions in horizontal circle in every minute. The magnitude of acceleration of stone is $\frac{1936}{x}\text{ms}^{-2}$. The value of x ______.

$\left(\text{Take } \pi = \frac{22}{7}\right)$ **[30 Jan, 2023 (Shift-II)]**

ADDITION AND SUBTRACTION OF TWO VECTORS

33. If two vectors $\vec{A}$ and $\vec{B}$ having equal magnitude R are inclined at an angle θ, then **[31 Jan, 2024 (Shift-II)]**

(*a*) $|\vec{A}-\vec{B}| = \sqrt{2}R\sin\frac{\theta}{2}$ (*b*) $|\vec{A}+\vec{B}| = 2R\sin\frac{\theta}{2}$

(*c*) $|\vec{A}+\vec{B}| = 2R\cos\frac{\theta}{2}$ (*d*) $|\vec{A}-\vec{B}| = 2R\cos\frac{\theta}{2}$

34. A vector has magnitude same as that of $\vec{A} = 3\hat{i}+4\hat{j}$ and is parallel to $\vec{B} = 4\hat{i}+3\hat{j}$. The x and y components of this vector in first quadrant are x and 3 respectively where x = ______.

[30 Jan, 2024 (Shift-II)]

35. Two forces $\vec{F}_1$ and $\vec{F}_2$ are acting on a body. One force has magnitude thrice that of the other force and the resultant of the two forces is equal to the force of larger magnitude. The angle between $\vec{F}_1$ and $\vec{F}_2$ is $\cos^{-1}\left(\frac{1}{n}\right)$. The value of $|n|$ is ______.

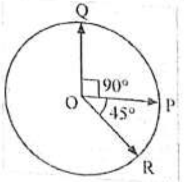
[04 April, 2024 (Shift-I)]

36. The angle between vector $\vec{Q}$ and the resultant of $(2\vec{Q}+2\vec{P})$ and $(2\vec{Q}-2\vec{P})$ is: **[05 April, 2024 (Shift-I)]**

(*a*) 0° (*b*) $\tan^{-1}\frac{(2\vec{Q}-2\vec{P})}{2\vec{Q}+2\vec{P}}$

(*c*) $\tan^{-1}\left(\frac{P}{Q}\right)$ (*d*) $\tan^{-1}\left(\frac{2Q}{P}\right)$

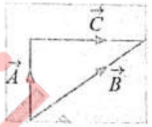
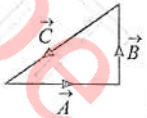
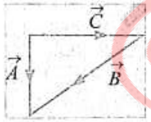
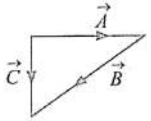
37. Three vectors $\overrightarrow{OP}, \overrightarrow{OQ}$ and $\overrightarrow{OR}$ each of magnitude A are acting as shown in figure. The resultant of the three vectors is $A\sqrt{x}$. The value of x is ______. **[08 April, 2024 (Shift-I)]**



38. If $\vec{a}$ and $\vec{b}$ makes an angle $\cos^{-1}\left(\frac{5}{9}\right)$ with each other, then $|\vec{a}+\vec{b}|=\sqrt{2}\,|\vec{a}-\vec{b}|$ for $|\vec{a}|=n|\vec{b}|$. The integer value of n is ______. **[09 April, 2024 (Shift-I)]**

39. The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$ and its magnitude is half that of $\vec{B}$. The angle between vectors $\vec{A}$ and $\vec{B}$ is $x°$. Find x. **[09 April, 2024 (Shift-II)]**

40. Match List-I with List-II.

List-I		List-II	
A.	$\vec{C}-\vec{A}-\vec{B}=0$	I.	
B.	$\vec{A}-\vec{C}-\vec{B}=0$	II.	
C.	$\vec{B}-\vec{A}-\vec{C}=0$	III.	
D.	$\vec{A}+\vec{B}=-\vec{C}$	IV.	

Choose the correct answer from the options given below: **[25 July, 2021 (Shift-I)]**

(a) A-IV, B-I, C-III, D-II

(b) A-IV, B-III, C-I, D-II

(c) A-I, B-IV, C-II, D-III

(d) A-III, B-II, C-IV, D-I

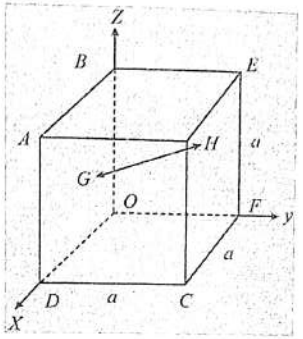
UNIT VECTOR, MAGNITUDE OF VECTOR, ANGLE MADE BY VECTORS

41. Two forces having magnitude A and $\frac{A}{2}$ are perpendicular to each other. The magnitude of their resultant is **[8 April, 2023 (Shift-I)]**

(a) $\frac{\sqrt{5}A}{4}$ (b) $\frac{5A}{2}$

(c) $\frac{\sqrt{5}A^2}{2}$ (d) $\frac{\sqrt{5}A}{2}$

42. In the cube of side 'a' shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be **[10 Jan, 2019 (Shift-I)]**



(a) $\frac{1}{2}a(\hat{k}-\hat{i})$ (b) $\frac{1}{2}a(\hat{i}-\hat{k})$

(c) $\frac{1}{2}a(\hat{j}-\hat{i})$ (d) $\frac{1}{2}a(\hat{j}-\hat{k})$

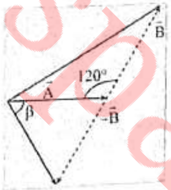
DOT PRODUCT

43. If two vectors $\vec{P}=\hat{i}+2m\hat{j}+m\hat{k}$ and $\vec{Q}=4\hat{i}-2\hat{j}+m\hat{k}$ are perpendicular to each other. Then, the value of m will be: **[24 Jan, 2023 (Shift-II)]**

(a) 1 (b) −1

(c) −3 (d) 2

44. The angle between vector $(\vec{A})$ and $(\vec{A}-\vec{B})$ is: **[26 Aug, 2021 (Shift-II)]**



(a) $\tan^{-1}\left(\frac{\sqrt{3}B}{2A-B}\right)$ (b) $\tan^{-1}\left(\frac{B\cos\theta}{A-B\sin\theta}\right)$

(c) $\tan^{-1}\left(\frac{A}{0.7B}\right)$ (d) $\tan^{-1}\left(\frac{-\frac{B}{2}}{A-B\frac{\sqrt{3}}{2}}\right)$

45. Let $\left|\vec{A}_1\right| = 3, \left|\vec{A}_2\right| = 5$ and $\left|\vec{A}_1 + \vec{A}_2\right| = 5$. The value of $(2\vec{A}_1 + 3\vec{A}_2).(3\vec{A}_1 - 2\vec{A}_2)$ is: **[8 April, 2019 (Shift-II)]**

(*a*) -112.5 (*b*) -106.5

(*c*) -118.5 (*d*) -99.5

CROSS PRODUCT

46. For three vectors $\vec{A} = (-x\hat{i} - 6\hat{j} - 2\hat{k})$, $\vec{B} = (-\hat{i} + 4\hat{j} + 3\hat{k})$ and $\vec{C} = (-8\hat{i} - \hat{j} + 3\hat{k})$, if $\vec{A}\cdot(\vec{B}\times\vec{C}) = 0$, then value of x is ________. **[06 April, 2024 (Shift-I)]**

47. If $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$ then, the uni vector in the direction of $\vec{P}\times\vec{Q}$ is $\frac{1}{x}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right)$. Th value of x is **[25 Jan, 2023 (Shift-I)]**

48. $\vec{A}$ is a vector quantity such that $|\vec{A}|$ = non-zero constan Which of the following expression is true for $\vec{A}$? **[25 June, 2022 (Shift-I)]**

(*a*) $\vec{A}.\vec{A} = 0$ (*b*) $\vec{A}\times\vec{A} < 0$ (*c*) $\vec{A}\times\vec{A} = 0$ (*d*) $\vec{A}\times\vec{A} > 0$

49. If $\vec{P}\times\vec{Q} = \vec{Q}\times\vec{P}$ the angle between $\vec{P}$ and $\vec{Q}$ is $\theta\,(0° < \theta < 360°)$. The value of '$\theta$' will be ________ **[25 Feb, 2021 (Shift-II)]**

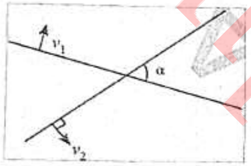
> *"Gratitude is the fairest blossom which springs from the soul."*
>
> **– Henry Ward Beecher**

SINGLE CORRECT TYPE QUESTIONS

1. Water flows out in all directions with the same speed from a sprinkler consisting of a perforated spherical shell fixed at the end of a hose. When the sprinkler is fixed at the ground, maximum height attained by a water stream is h. If the sprinkler is shifted to height h above the ground, by what factor will the watered area on the ground change? Neglect diameter of the spherical shell as compared to the height h.

(*a*) 8 (*b*) 6 (*c*) 4 (*d*) 2

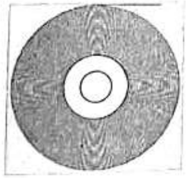
2. A plane contains two straight lines intersecting each other at an angle α. The lines start moving perpendicular to themselves in the same plane as shown in the figure with velocities v_1 and v_2 respectively. Find the velocity of the point of intersection of the two lines.



(*a*) $\sqrt{v_1^2+v_2^2+2v_1v_2\cos\alpha}$ (*b*) $\dfrac{\sqrt{v_1^2+v_2^2+2v_1v_2\sin\alpha}}{\sin\alpha}$

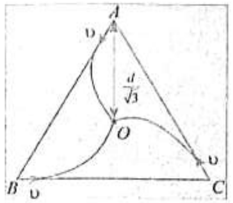
(*c*) $\dfrac{\sqrt{v_1^2+v_2^2+2v_1v_2\sin\alpha}}{\cos\alpha}$ (*d*) $\dfrac{\sqrt{v_1^2+v_2^2+2v_1v_2\cos\alpha}}{\sin\alpha}$

3. An $L = 70$ m long thin tape wound on a spool of radius $r_0 = 10$ mm makes a tape roll of outer radius $R = 25$ mm. A motor used to wind the tape rotates the spool at a constant angular velocity and takes $T = 165$ s to complete the winding. Calculate length of the tape, which has been wound in $t = 110$ s from the beginning of the winding.



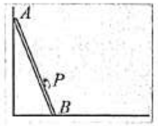
(*a*) 35 m (*b*) 40 m (*c*) 45 m (*d*) 50 m

4. Three particles A, B and C are situated at the vertices of an equilateral triangle ABC of side d at $t = 0$. Each of the particles is moving with constant speed v. A always has its velocity along AB, B along BC and C along CA. Find the initial radius of curvature of the path followed by any of the particle.



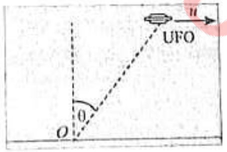
(*a*) $2d$ (*b*) $\dfrac{d}{\sqrt{3}}$ (*c*) $\dfrac{\sqrt{3}d}{2}$ (*d*) $\dfrac{2d}{\sqrt{3}}$

5. A stick AB of length $l = 100$ cm stands vertically on a horizontal floor leaning on a wall. A beetle P starts climbing the stick from the floor. When the beetle starts climbing, the lower end B of the stick is made to move away from the wall with a constant velocity $\upsilon = 4$ m/s. The beetle climbs the stick with a constant speed $u = 3$ m/s relative to the stick. If the upper end A does not leave the wall, what maximum height can the beetle rise?



(*a*) 25 cm (*b*) 37.5 cm (*c*) 50 cm (*d*) 62.5 cm

6. A UFO (un-identified flying object) is believed to be a space ship used by aliens. In a particular scene of a science fiction movie, a UFO is flying horizontally at a very high altitude with a speed u that is η (< 1) times of speed c of light. The UFO is emitting sharp light pulses at regular and very small intervals. Find the speed of the UFO recorded by an observer on the ground at point O when the UFO appears at an angle θ with the vertical. Ignore relativistic corrections.



(*a*) $u(1+\eta\sin\theta)$ (*b*) $\dfrac{u}{1+\eta\cos\theta}$

(*c*) $u(1+\eta\cos\theta)$ (*d*) $\dfrac{u}{1+\eta\sin\theta}$

7. A particle projected from the ground passes two points, which are at heights $h_1 = 12$ m and $h_2 = 18$ m above the ground and a distance $d = 10$ m apart. What could be the minimum speed of projection? Acceleration due to gravity is $g = 10$ m/s^2.

(*a*) 10 m/s (*b*) 12 m/s (*c*) 15 m/s (*d*) 20 m/s

8. A balloon moves up vertically from the ground at $t = 0$ such that if a stone is thrown from it with a horizontal velocity v_0 relative to it, the stone always hits the ground at a fixed point horizontally away $\frac{2v_0^2}{g}$ from it. Find the height of the balloon as a function of time.

(*a*) $\frac{2v_0^2}{g}\left[1-e^{-\frac{gt}{2v_0}}\right]$ (*b*) $\frac{v_0^2}{2g}\left[1-e^{-\frac{gt}{2v_0}}\right]$

(*c*) $\frac{v_0^2}{2g}\left[1-e^{-\frac{gt}{v_0}}\right]$ (*d*) $\frac{2v_0^2}{g}\left[1-e^{-\frac{gt}{v_0}}\right]$

9. A particle A moves in a circular path of radius R with uniform speed v. Another particle B which is initially at the center of the circle chases particle A with uniform speed v such that it always remains collinear with the center of the circle and location of the particle A. The time t after which particle B collides with A is:

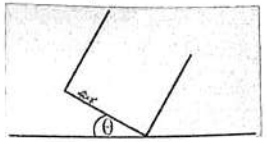
(*a*) $\frac{2\pi R}{v}$ (*b*) $\frac{\pi R}{2v}$

(*c*) $\frac{\pi R}{v}$ (*d*) $\frac{3\pi R}{2v}$

INTEGER TYPE QUESTIONS

10. To cross a river of width $b = 1.8$ km, a boatman steers his boat always aiming toward a point that is directly opposite to the starting point. Velocity of the boat relative to the river current is $\upsilon_{b/R} = 5$ km/hr and river current velocity is $\upsilon_R = 4$ km/hr everywhere. Determine time (in minutes), which the boat will take to cross the river.

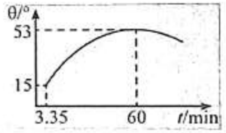
11. A grasshopper on the bottom of a cubical box has to jump out of the box. If each side of the box is $h = 100$ cm and the grasshopper can jump with a maximum initial velocity $u = 4$ m/s. What should the minimum tilt angle θ (in degrees) of the box so that the grasshopper can jump out of the box. Acceleration due to gravity is $g = 10$ m/s^2.



12. A honeybee is flying parallel to a tabletop at a height $h = 3.0$ m with a constant velocity $\upsilon = 4$ m/s. With its wings, it can achieve a maximum acceleration $a = 10$ m/s^2. At an instant when the honeybee is vertically above a honey drop on the tabletop, it decides to reach the honey drop. Neglect the reaction time of the honeybee and find the minimum time (in seconds) in which the honeybee can reach the honey drop.

Direction 13 to 15 Questions

A hot air balloon of constant ascent velocity can be used to investigate wind velocities at various altitudes. Employing this idea, one day a hot air balloon was released at a distance $l = 50$ m from a point, where a telescope was installed to track the balloon. Since wind velocity was almost zero up to a height of approximately 15 m that day, the balloon first rose upwards and then due to horizontal drift caused by the wind it followed a plane curvilinear path. During continuous tracking of the balloon, the telescope has been rotated in a vertical plane. A graph depicting how the angle of elevation varies with time thus obtained is shown in the figure.



13. Ascent velocity of the balloon is closest to

(*a*) 3 m/min (*b*) 4 m/min

(*c*) 5 m/min (*d*) 6 m/min

14. Wind velocity at the altitude of the balloon, where angle of elevation acquires its maximum value is closest to

(*a*) 3 m/min (*b*) 4 m/min

(*c*) 5 m/min (*d*) 6 m/min

15. Horizontal drift of the balloon, when angle of elevation acquires its maximum value is closest to

(*a*) 180 m (*b*) 150 m

(*c*) 120 m (*d*) 110 m

Answer Key

CONCEPT APPLICATION

1. [150°] 2. (*a*) 60°, (*b*) 15°, (*c*) 75° 3. (*a*) 120°, (*b*) 120°, (*c*) 120° 4. $F_1 = 6$ N, $F_2 = 10$ N, 12. (*c*)
5. $R = 7\sqrt{2}$ *cm* at 45° from *x*-axis 6. (*b*) 7. (*d*) 8. $(\pm\sqrt{12})$ 9. (*c*) 10. (*d*) 11. (*b*)
13. (*a*) 14. (*a*) 15. (*d*) 16. $R = \sqrt{2}\,r + r$ along *OB* 17. 2 cm along *x*-axis 18. $(15i + 20j)$
19. [150°] 20. (*b*) 21. (*b*) 22. (*a*) 23. (*c*) 24. (*c*) 25. (*d*) 26. (*b*) 27. (*c*)
28. (*b*) 29. (*a*) 30. (*a*) 31. $\pm\dfrac{\hat{i} - 4\hat{j} - 5\hat{k}}{\sqrt{42}}$ 32. $\hat{i} + 7\hat{j} + 5\hat{k}$ 33. (*c*) 34. (*a*)
35. (*c*) 36. (*d*) 37. (*d*) 38. (*d*) 39. (*c*) 40. (*b*) 41. (*d*) 42. (*c*) 43. (*d*) 44. (*a*)
45. (*c*) 46. (*a*) 47. (*a*) 48. (*a*) 49. (*b*) 50. (i) 10 sec (ii) 980 m (iii) $98\sqrt{2}$ m/s 51. (*c*) 52. (*a*)
53. (*c*) 54. [3 m] 55. (*b*) 56. (*b*) 57. (*d*) 58. (*a*) 59. (*b*) 60. (*b*) 61. [40 m] 62. (*c*)
63. (*c*) 64. (*a*) 65. (*a*) 66. (*a*) 67. 14.29s, 0.45 m/s^2 68. $\dfrac{u^2 \sin^2\theta}{g}$

SCHOOL LEVEL PROBLEMS

1. (*c*) 2. (*b*) 3. (*d*) 4. (*c*) 5. (*c*) 6. (*b*) 7. (*a*) 8. (*a*) 9. (*d*) 10. (*c*)
11. (*d*) 12. (*a*) 24. (i)-(*b*), (ii)-(*a*), (iii)-(*a*), 25.(i)-(*a*), (ii)-(*c*), (iii)-(*a*), (iv)-(*d*), (v)-(*c*)

PRARAMBH (TOPICWISE)

1. (*b*) 2. (*b*) 3. (*d*) 4. (*d*) 5. (*a*) 6. (*d*) 7. (*b*) 8. (*a*) 9. (*c*) 10. (*a*)
11. (*a*) 12. (*d*) 13. (*b*) 14. (*b*) 15. (*a*) 16. (*b*) 17. (*d*) 18. (*b*) 19. (*d*) 20. (*b*)
21. (*a*) 22. (*a*) 23. (*a*) 24. (*b*) 25. (*d*) 26. (*c*) 27. (*b*) 28. (*d*) 29. (*d*) 30. (*c*)
31. (*a*) 32. (*b*) 33. (*a*) 34. (*b*) 35. (*b*) 36. (*d*) 37. (*d*) 38. (*c*) 39. (*b*) 40. (*b*)
41. (*c*) 42. (*d*) 43. (*a*) 44. (*a*) 45. (*d*) 46. (*d*) 47. (*c*) 48. (*b*) 49. (*b*) 50. (*b*)
51. (*d*) 52. (*b*) 53. (*c*) 54. (*d*) 55. (*b*) 56. (*c*) 57. (*d*) 58. (*c*) 59. (*a*) 60. (*b*)
61. (*c*) 62. (*d*) 63. (*d*) 64. (*a*) 65. (*c*) 66. (*b*) 67. (*b*) 68. (*d*) 69. (*b*) 70. (*c*)
71. (*a*) 72. (*b*) 73. (*a*) 74. (*c*) 75. (*c*) 76. (*b*) 77. (*c*) 78. (*b*) 79. (*c*) 80. (*a*)
81. (*b*) 82. (*b*) 83. (*c*) 84. (*b*) 85. (*c*) 86. (*b*) 87. (*c*) 88. (*a*) 89. (*c*) 90. (*b*)
91. (*b*) 92. (*a*) 93. (*d*) 94. (*d*) 95. (*c*) 96. (*a*) 97. (*d*) 98. (*b*) 99. (*a*) 100. (*c*)
101. (*c*) 102. (*a*) 103. (*a*) 104. (*c*) 105. (*b*) 106. (*c*) 107. (*a*) 108. (*b*) 109. (*b*) 110. (*b*)

PRABAL (JEE MAIN LEVEL)

1. (*a*) 2. (*b*) 3. (*c*) 4. (*d*) 5. (*a*) 6. (*a*) 7. (*d*) 8. (*d*) 9. (*d*) 10. (*b*)
11. (*d*) 12. (*b*) 13. (*b*) 14. (*d*) 15. (*d*) 16. (*d*) 17. (*d*) 18. (*a*) 19. (*b*) 20. (*c*)
21. (*d*) 22. (*d*) 23. (*b*) 24. (*b*) 25. (*c*) 26. (*b*) 27. (*b*) 28. (*d*) 29. (*b*) 30. (*a*)
31. (*a*) 32. (*b*) 33. (*d*) 34. (*a*) 35. (*b*) 36. (*b*) 37. (*c*) 38. (*d*) 39. (*a*) 40. (*c*)
41. (*a*) 42. (*b*) 43. (*a*) 44. (*a*) 45. (*b*) 46. (*b*) 47. (*d*) 48. (*a*) 49. (*d*) 50. (*a*)
51. (*b*) 52. (*d*) 53. (*c*) 54. (*d*) 55. (*b*) 56. (*c*) 57. (*a*) 58. (*a*) 59. (*a*) 60. [25]
61. [15] 62. [13] 63. [25] 64. [20] 65. [2] 66. [5] 67. [7] 68. [625] 69. [9]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*a,b,d*)	2. (*d*)	3. (*a*)	4. (*d*)	5. (*a,c,d*)	6. (*c,d*)	.7. (*a,b,d*)	8. (*a, d*)	9. (*b,d*)	10. (*a,b,c,d*)
11. (*a,b*)	12. (*a*)	13. (*b*)	14. (*c*)	15. (*a*)	16. (*c*)	17. (*c*)	18. (*b*)	19. (*a*)	20. (*c*)
21. (*a*)	22. (*b*)	23. (*c*)	24. (*a*)	25. (*b*)	26. (*b*)	27. (*c*)	28. (*a*)	29. (*b*)	30. (*d*)
31. (*b*)	32. (*c*)	33. (*a*)	34. (*b*)	35. (*d*)	36. (*c*)	37. (*d*)	38. (*b*)	39. [0.33]	40. [5.66]
41. [1.50]	42. [3.73]	43. [0.80]	44. [5]	45. [480]	46. [53]	47. (*b*)	48. (*a*)	49. (*b*)	50. (*b*)
51. (*a*)	52. (*b*)	53. (*a*)	54. (*d*)	55. (*a*)					

PYQ's (PAST YEAR QUESTIONS)

1. (*d*)	2. [673]	3. (*d*)	4. (Bonus)	5. (*a*)	6. [16]	7. (*b*)	8. (*c*)	9. (*a*)	10. [5]
11. [4.00]	12. (*d*)	13. [2]	14. [100]	15. (*b*)	16. [3]	17. [60]	18. [580]	19. (*c*)	20. (*c*)
21. (*d*)	22. [1]	23. [15]	24. (*a*)	25. (*d*)	26. (*b*)	27. (*a*)	28. (*a*)	29. [8]	30. [2]
31. (*d*)	32. [125]	33. (c)	34. [4]	35. [6]	36. (*a*)	37. [3]	38. [3]	39. [150]	40. (*b*)
41. (*d*)	42. (*c*)	43. (*d*)	44. (*a*)	45. (*c*)	46. [4]	47. [4]	48. (*c*)	49. [180]	

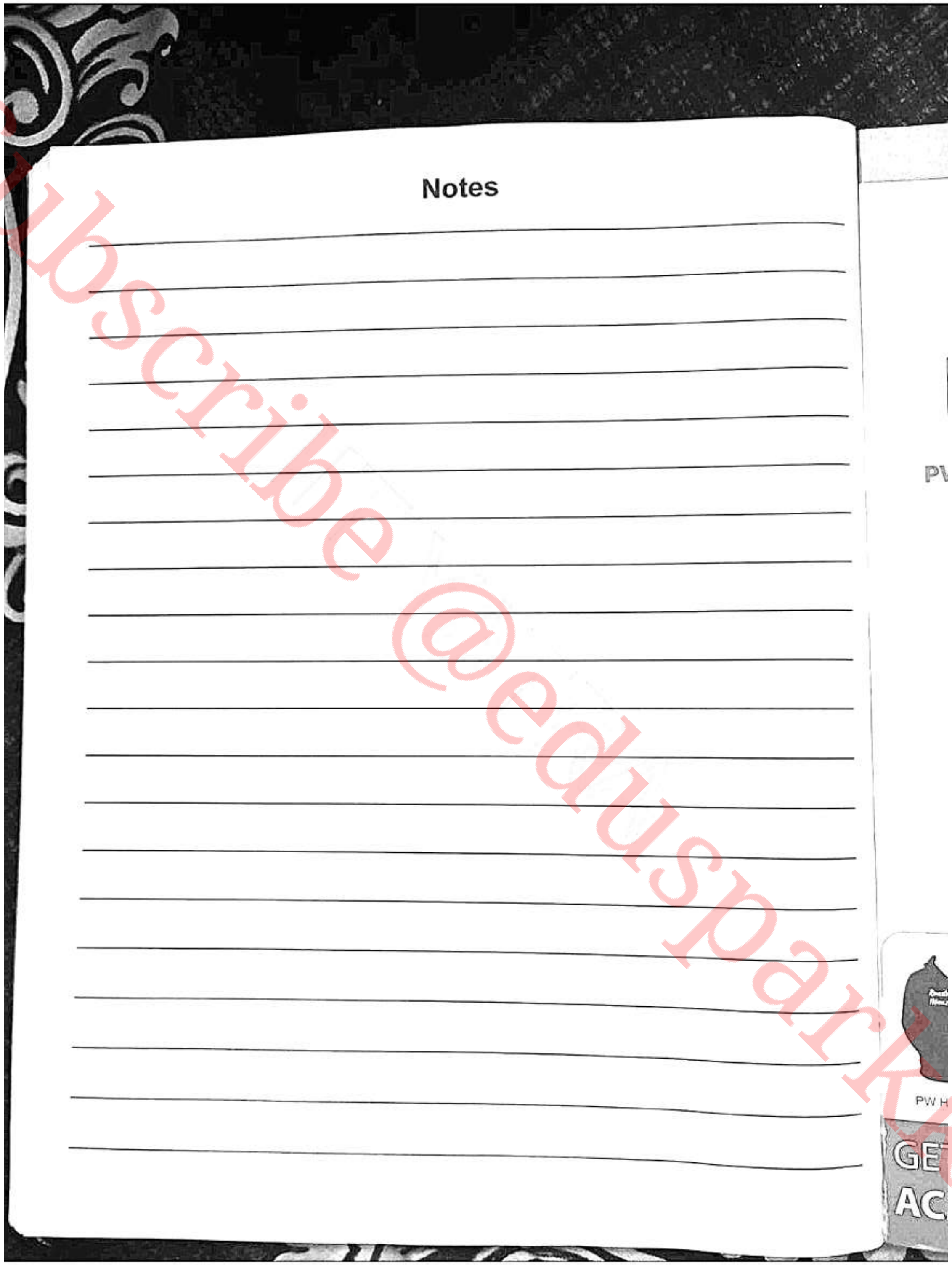
PW CHALLENGERS

1. (*d*)	2. (*d*)	3. (*b*)	4. (*d*)	5. (*b*)	6. (*d*)	7. (*d*)	8. (*a*)	9. (*b*)	10. [60]
11. [37]	12. [1]	13. (*b*)	14. (*a*)	15. (*d*)					

VOLUNTEER OR GIVE BACK

- Helping others can increase your appreciation for what you have. By giving your time, skills, or resources to those in need, you're reminded of your own blessings.
- Volunteer at a local charity or simply help a friend or neighbor. The act of giving can create a deep sense of gratitude for your own abilities and circumstances.

Notes



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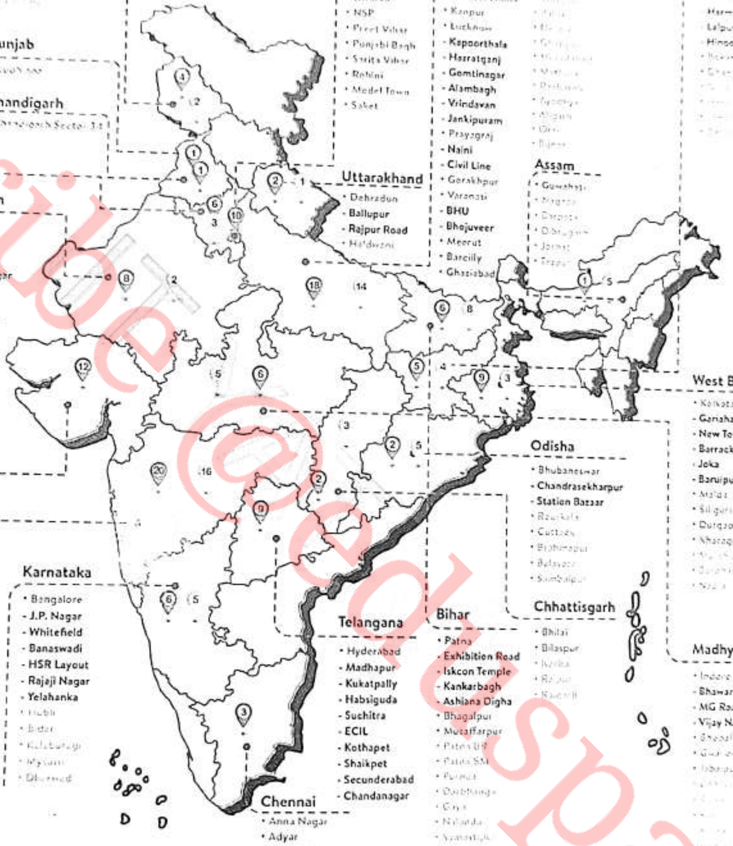

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- Jammu
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  - Channi Rama
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Haryana
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  - South City
  - Sector 14
- [illegible]
- [illegible]
- [illegible]

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